Lecture #5
Signal operations

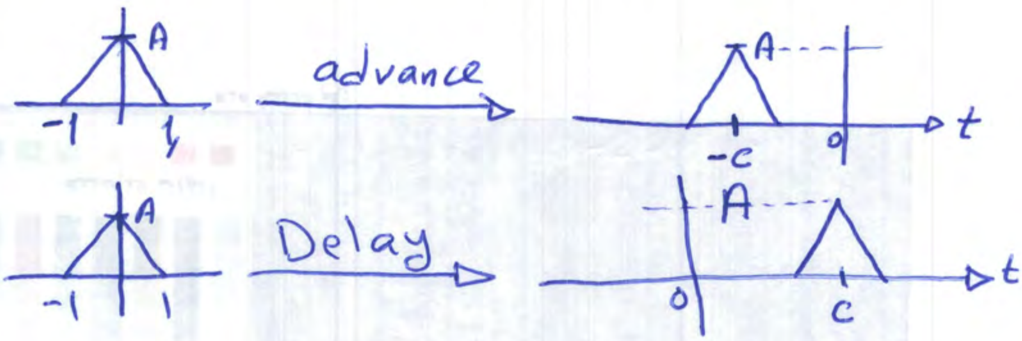
1) Time Shifting :

$x(t) = g(t \mp c)$ where c is constant

$x[n] = g[n \mp c]$

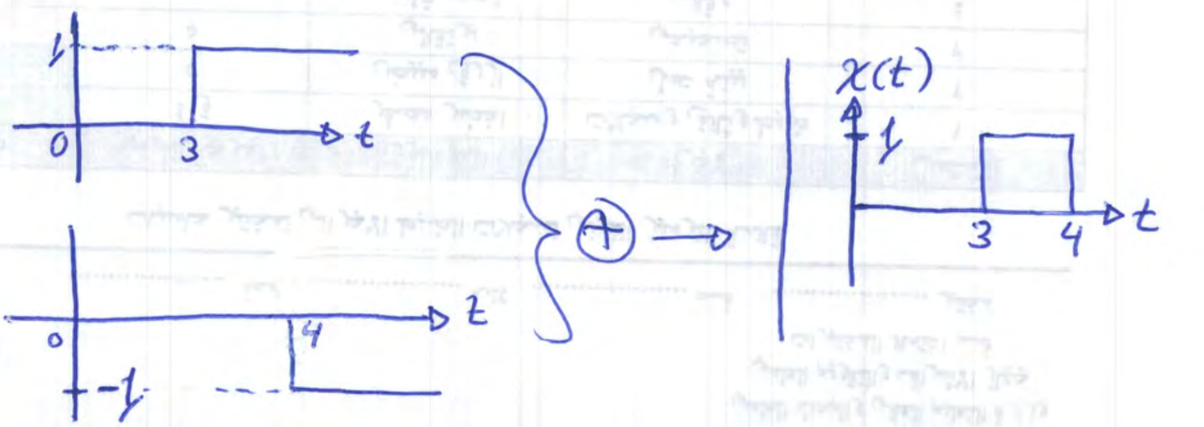
where +ve constant → Advance in time.
-ve constant → Delay in time.

For instance



EX. 27 Sketch $x(t) = u(t-3) - u(t-4)$.

solution

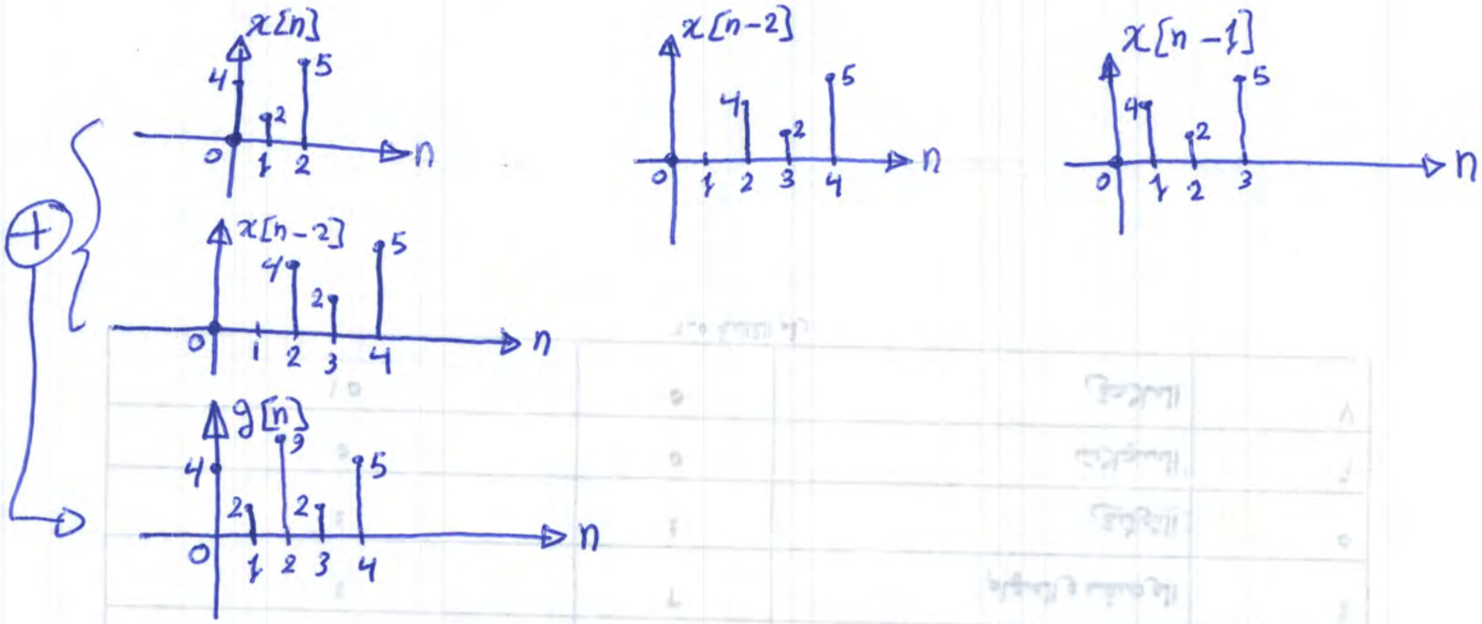


EX. 28 $x[n] = [\underline{4}, 2, 5]$. sketch $x[n-2]$ and $x[n-1]$ then Find $g[n] = x[n] + x[n-2]$ graphically.

solution

$x[n] = [4, 2, 5] = 4\delta[n] + 2\delta[n-2] + 5\delta[n-3]$

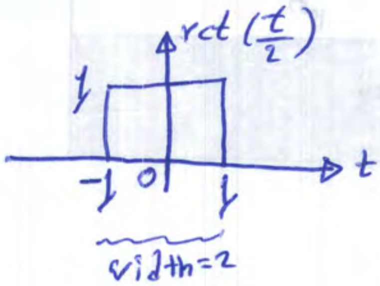
or just plot it



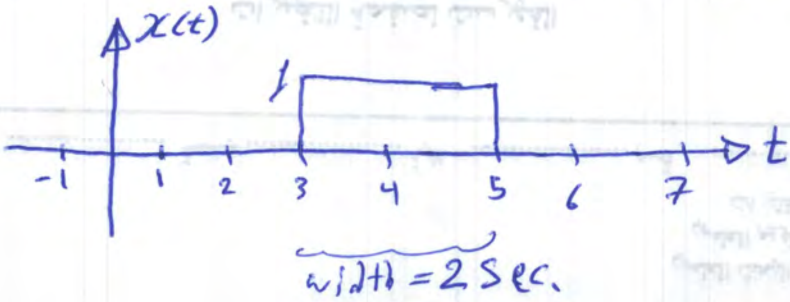
EX. 29 sketch $x(t) = rct\left(\frac{t-4}{2}\right)$.

solution

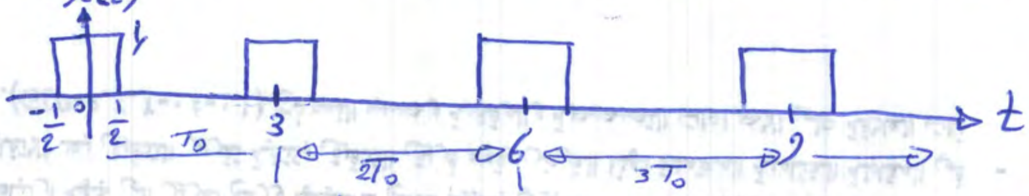
first whe plot $rct\left(\frac{t}{2}\right)$ which means rct of width = 2



next we shift $rct\left(\frac{t}{2}\right)$ by 4 to the right



EX.30 Describe the periodic signal shown below mathematically and determine its fundamental period.



solution $x(t)$ is periodic when it repeats itself every $T_o = 3$ seconds.

The rectangle pulse has width = 1 second, the rest of the period is zero.

$rct(t)$ repeats each T_o

$$rct(t \mp 3) = rct(t)$$

$$\therefore x(t) = rct(t) + rct(t \mp 3) + rct(t \mp 6) + rct(t \mp 9) + rct(t \mp 3K)$$ where K is integer.

$$\therefore T_o = 3 \Rightarrow x(t) = rct(t \mp 0T_o) + rct(t \mp T_o) + rct(t \mp 2T_o) + rct(t \mp 3T_o) + \cdots + rct(t \mp KT_o)$$

$$\therefore x(t) = \sum_{K=-\infty}^{K=\infty} rct(t + KT_o)$$

OR

$$x(t) = \sum_{K=-\infty}^{K=\infty} rct(t + 3K)$$

2) Time scale :

* Time scale is the operation to compress or expand the signal in time.

For instance the signal $x(t)$ scaled in time by the factor γ, then it becomes $x(\gamma t)$.

$\Rightarrow$ if $\gamma > 1 \longrightarrow$ compression.

if $0 < \gamma < 1 \longrightarrow$ Expanssion.

On the other hand, DT signals obay the same rule

$x[\sigma n]$ where σ is integer number.

$\Rightarrow$ if $\sigma > 1 \longrightarrow$ compression. [intermediate samples are lost and the sequence is shorter] (Down sampling - Decimation)

if $x[\frac{n}{\sigma}] \longrightarrow$ Expanssion

the sequence is longer

Up sampling - interpolation.

EX.31 Consider a CT signal $x(t)$ defined as follows:

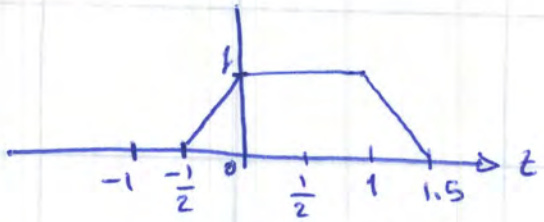
$$x(t) = \begin{cases} t+1 & -1 \leq t \leq 0 \\ 1 & 0 \leq t \leq 2 \\ -t+3 & 2 \leq t \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the expression for time scaled signal $x(2t)$ and $x(t/2)$. Sketch the two signals.

Solution we put $2t$ instead of each t

(1)
$$x(t) = \begin{cases} 2t+1 & -0.5 \leq t \leq 0 \\ 1 & 0 \leq t \leq 1 \\ -2t+3 & 1 \leq t \leq 1.5 \\ 0 & \text{elsewhere} \end{cases}$$ Compression by 2

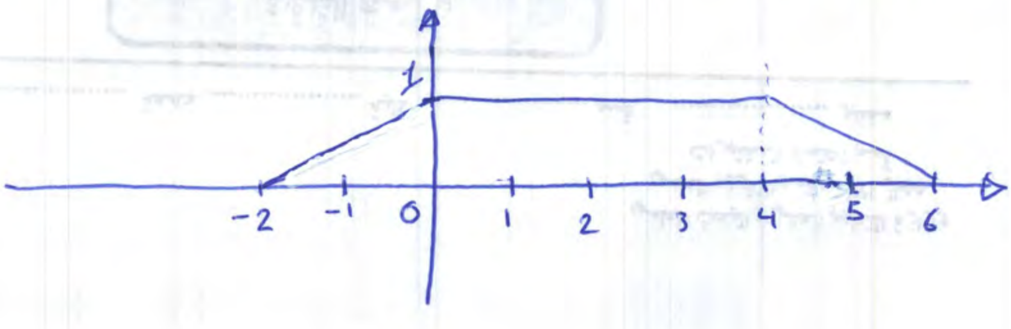
original



compressed by 2

(2) we substitute each t by $\frac{t}{2}$

$$x\left(\frac{t}{2}\right) = \begin{cases} \left(\frac{t}{2}\right)+1 & -1 \leq \frac{t}{2} \leq 0 \\ 1 & 0 \leq \frac{t}{2} \leq 2 \\ -\left(\frac{t}{2}\right)+3 & 2 \leq \frac{t}{2} \leq 3 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} \frac{t}{2}+1 & -2 \leq t \leq 0 \\ 1 & 0 \leq t \leq 4 \\ -\frac{t}{2}+3 & 4 \leq t \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$



* Discrete Time Scaling [Expanssion - Interpolation] :-

⇒ The interpolated (expanded) DT signal inserts $(m-1)$ zeros in between adjacent samples of the original DT sequence $x[n]$

⇒ Interpolation of DT $x[n]$ signal is reversable process.

* For expanssion

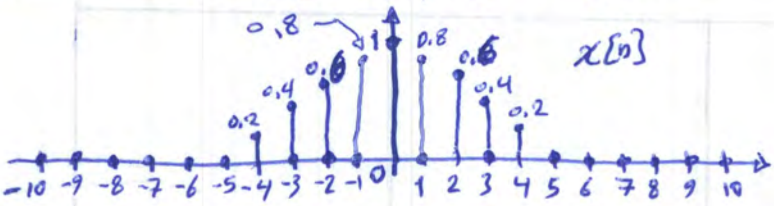
$$x[n] = \begin{cases} x[\frac{n}{m}] & \text{if } n \text{ is a multiple of integer } m \\ 0 & \text{otherwise} \end{cases}$$

⇒ Another interpolation methods will be studied in advanced course.

Ex. 32 Calculate and sketch $p[n] = x[2n]$ and $q[n] = x[\frac{n}{2}]$

where $x[n] = [0, 0, 0, 0, 0, 0, 0.2, 0.4, 0.6, 0.8, \underline{1}, 0.8, 0.6, 0.4, 0.2, 0, 0, 0, 0, 0, 0]$.

Solution $x[n]$ is plotted below

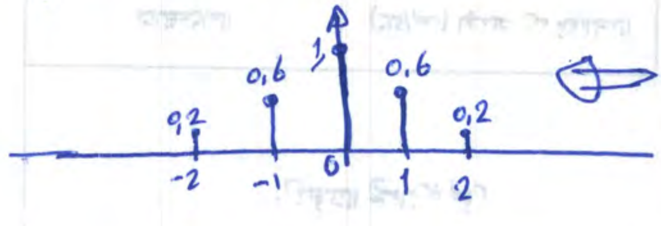


n	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$x[n]$	0	0	0	0	0	0	0.2	0.4	0.6	0.8	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0

Decimation (compression) $p[n] = x[2n]$, $x[n]$ nonzero in $-5 \leq n \leq 5$

$\therefore$ $p[n] = x[2n]$ nonzero in $-3 \leq n \leq 3$

n	-3	-2	-1	0	1	2	3
$p[n]$	$x[-6] = 0$	$x[-4] = 0.2$	$x[-2] = 0.6$	$x[0] = 1$	$x[2] = 0.6$	$x[4] = 0.2$	$x[6] = 0$

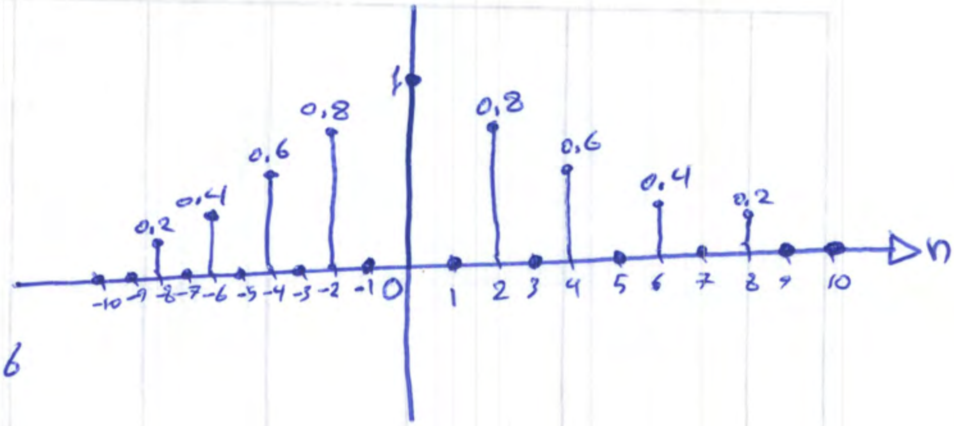


⟸ compressed

* For expansion $q[n] = x\left[\frac{n}{2}\right]$ nonzero in $-10 \leq n \leq 10$

See next page

n	q[n]
-10	$\rightarrow x[-5] = 0$
-9	$\rightarrow 0$
-8	$\rightarrow x[-4] = 0.2$
-7	$\rightarrow 0$
-6	$\rightarrow x[-3] = 0.4$
-5	$\rightarrow 0$
-4	$\rightarrow x[-2] = 0.6$
-3	$\rightarrow 0$
-2	$\rightarrow x[-1] = 0.8$
-1	$\rightarrow 0$
0	$\rightarrow x[0] = 1$
1	$\rightarrow 0$
2	$\rightarrow x[1] = 0.8$
3	$\rightarrow 0$
4	$\rightarrow x[2] = 0.6$
5	$\rightarrow 0$
6	$\rightarrow x[3] = 0.4$
7	$\rightarrow 0$
8	$\rightarrow x[4] = 0.2$
9	$\rightarrow 0$
10	$\rightarrow x[5] = 0$



3) Time inversion (Time Reversal):

Time inversion ≡ Time Reversal ≡ Time Reflection

CT or DT signals $x(t)$ or $x[n]$

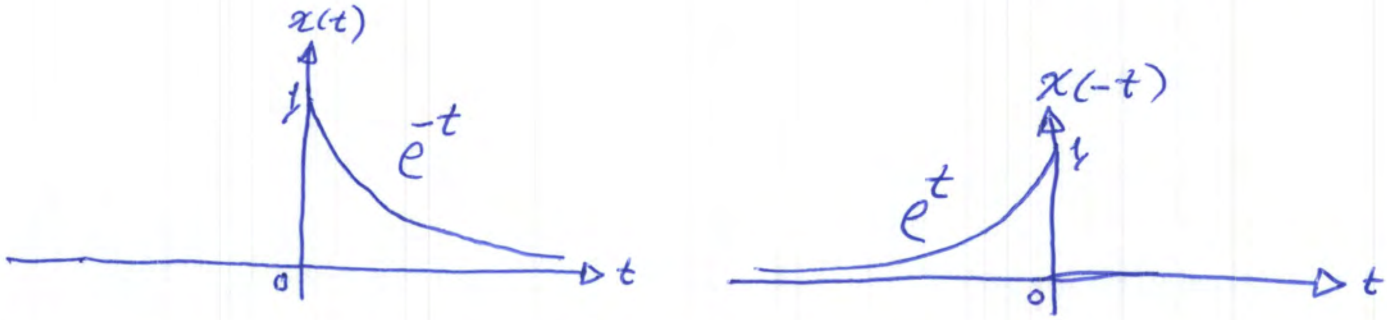
the time reversal is $x(-t)$ or $x[-n]$

Ex. 33 sketch the CT signal $x(t)$ and its time inversion where

$$x(t) = e^{-t} u(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

solution

$$x(-t) = \begin{cases} e^{-(-t)} & -t \geq 0 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} e^{+t} & t \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$

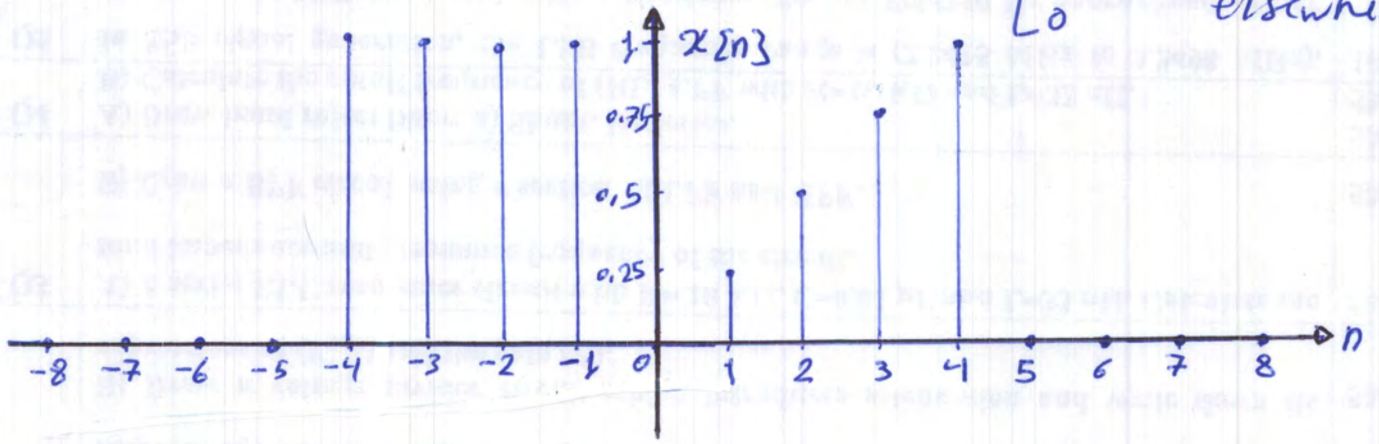


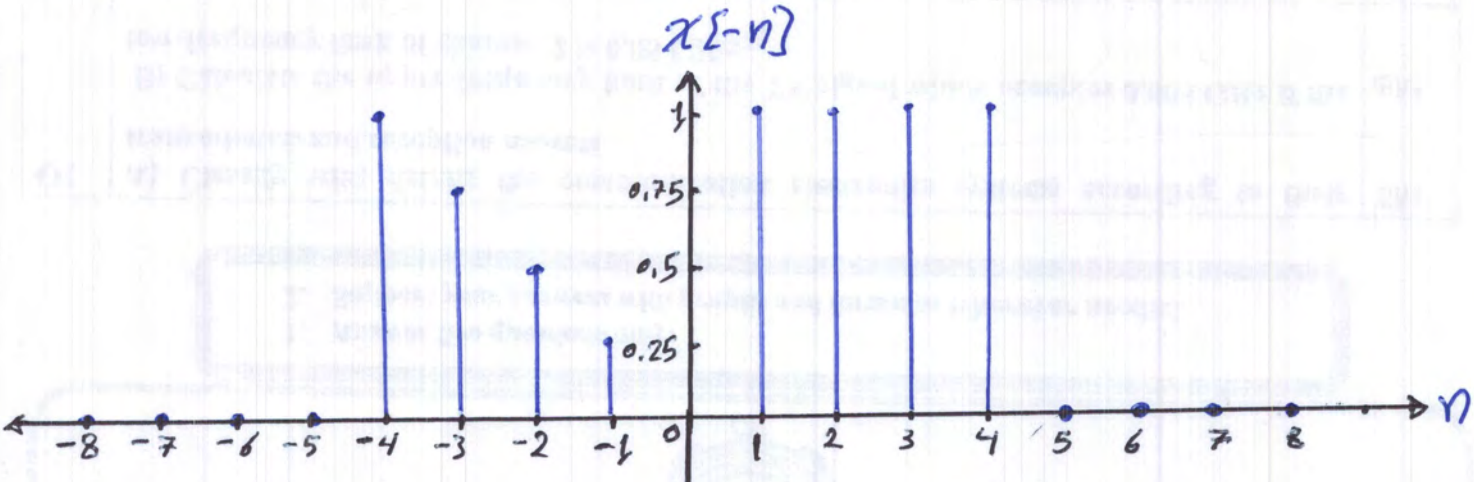
EX.34 Determine $x[-n]$ and sketch $x[n]$ and $x[-n]$

where $x[n] = \begin{cases} 1 & -4 \le n \le -1 \\ 0.25n & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$

solution $x[-n]$ can be obtained by replacing each n by $-n$:-

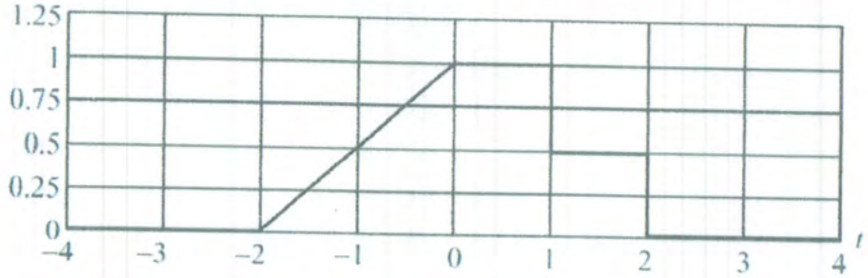
$$x[-n] = \begin{cases} 1 & -4 \le -n \le -1 \\ 0.25(-n) & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} 1 & 1 \le n \le 4 \\ -0.25n & -4 \le n \le 0 \\ 0 & \text{elsewhere} \end{cases}$$





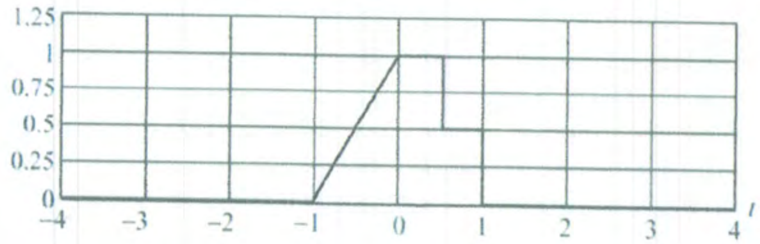
Example 2.8:

Determine $x(4-2t)$, where the waveform for the CT signal $x(t)$ is plotted as follows:

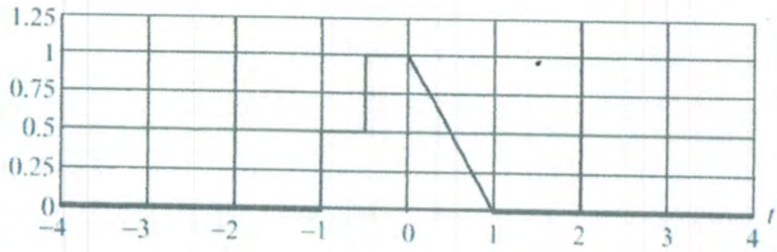


Solution: Express $x(4-2t\ ) = x(-2[t-2])$ and follow steps (i)–(iii) as outlined below.

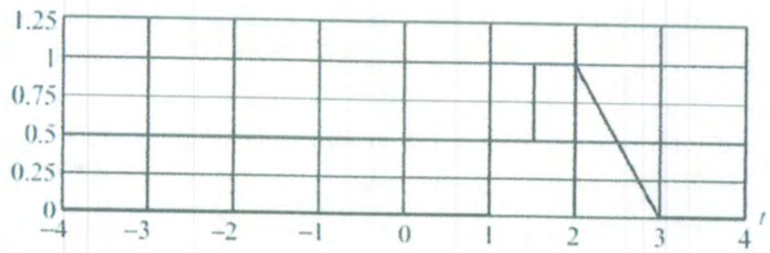
(i) Compress $x(t)$ by a factor of 2 to obtain $x(2t)$. The resulting waveform is shown here:



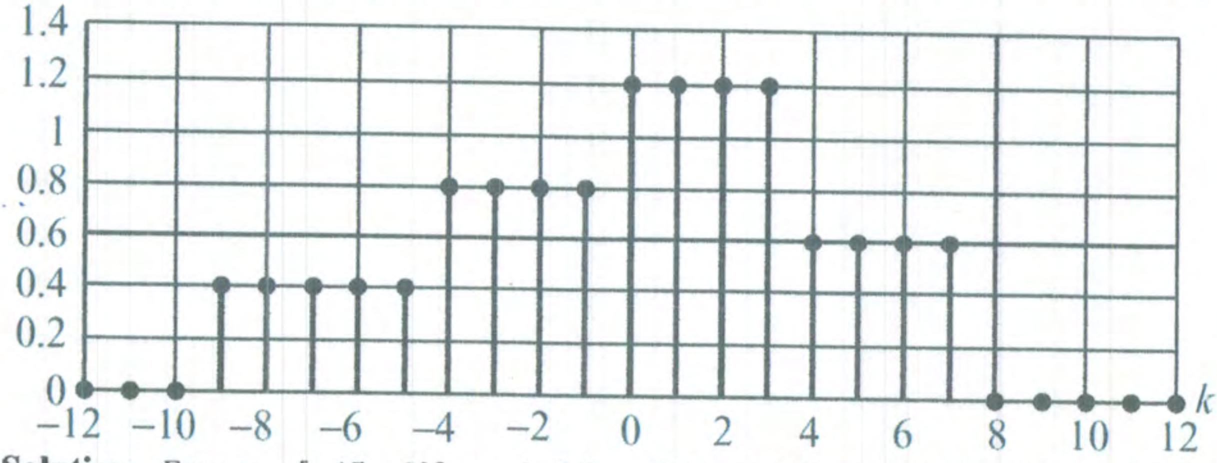
(ii) Time-reverse x(2t) to obtain x(-2t). The waveform for x(-2t)



(iii) Shift $x(-2t)$ towards the right-hand side by ***two time units*** to obtain $x(-2[t-2]) = x(4-2t)$.

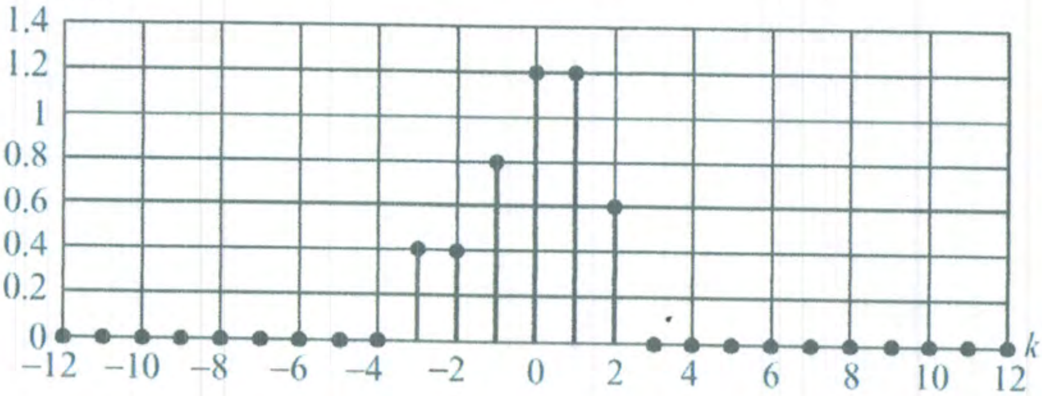


Example 2.9: Sketch the waveform for $x[-15-3k]$ for the DT sequence $x[k]$ plotted below:

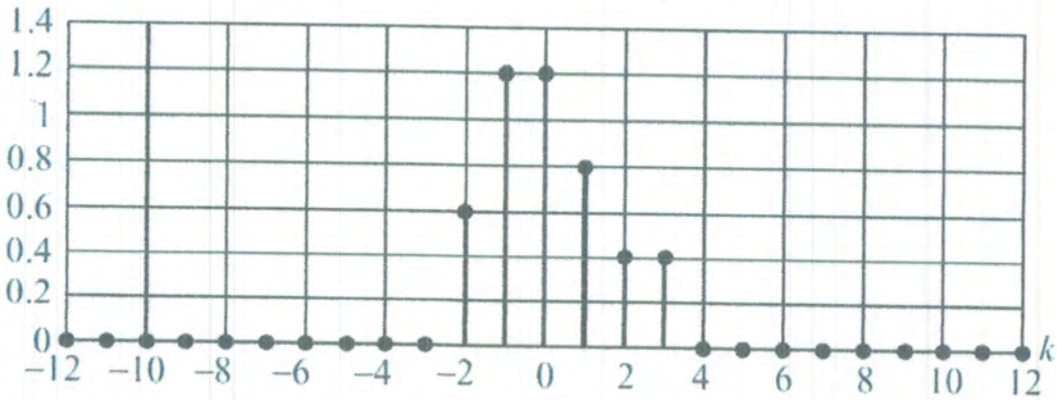


Solution: Express $x[-15-3k] = x[-3(k+5)]$ and follow steps (i)–(iii) as outlined below.

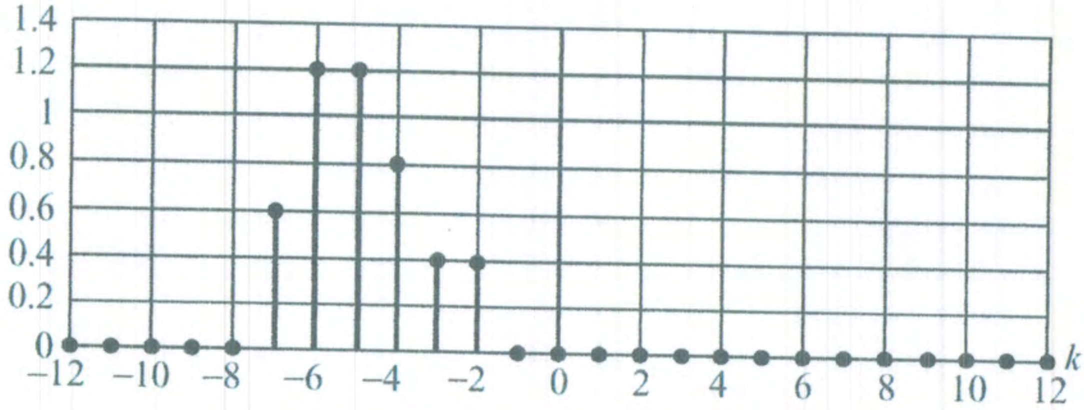
(i) Compress $x[k]$ by a factor of 3 to obtain $x[3k]$. The resulting waveform is shown here



(ii) Time-reverse $x[3k]$ to obtain $x[-3k]$. The waveform for $x[-3k]$ is shown below:



(iii) Shift $x[-3k]$ towards the left-hand side by ***five time units*** to obtain $x[-3(k+5)] = x[-15-3k]$.



69

Continuous and Discrete Time Signals and Systems

Homework

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CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521854559

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN-13 978-0-521-85455-9 hardback

Problems: See Page 53 of the Textbook shown above.

1.2 Sketch each of the following CT signals as a function of the independent variable t over the specified range:

(i) $x_1(t) = \cos(3\pi t/4 + \pi/8)\ for\ -1 \le t \le 2;$
(ii) $x_2(t) = \sin(-3\pi t/8 + \pi/2)\ for\ -1 \le t \le 2;$
(iii) $x_3(t) = 5t + 3\,exp(-t)\ for\ -2 \le t \le 2;$
(iv) $x_4(t) = (\sin(3\pi t/4 + \pi/8))^2\ for\ -1 \le t \le 2;$
(v) $x_5(t) = \cos(3\pi t/4) + \sin(\pi t/2)\ for\ -2 \le t \le 3;$
(vi) $x_6(t) = t\,exp(-2t)\ for\ -2 \le t \le 3.$

1.3 Sketch the following DT signals as a function of the independent variable k over the specified range:

(i) $x_1[k] = \cos(3\pi k/4 + \pi/8)\ for\ -5 \le k \le 5;$
(ii) $x_2[k] = \sin(-3\pi k/8 + \pi/2)\ for\ -10 \le k \le 10;$
(iii) $x_3[k] = 5k + 3-k\ \ for\ -5 \le k \le 5;$
(iv) $x_4[k] = |\sin(3\pi k/4 + \pi/8)|\ for\ -6 \le k \le 10;$
(v) $x_5[k] = \cos(3\pi k/4) + \sin(\pi k/2)\ for\ -10 \le k \le 10;$
(vi) $x_6[k] = k4^{-|k|}\ for\ -10 \le k \le 10.$

1.5 Determine if the following CT signals are periodic. If yes, calculate the fundamental period T_0 for the CT signals:

(i) $x_1(t) = \sin(-5\pi t/8 + \pi/2);$
(ii) $x_2(t) = |\sin(-5\pi t/8 + \pi/2)|;$
(iii) $x_3(t) = \sin(6\pi t/7) + 2\cos(3t/5);$
(iv) $x_4(t) = exp(j(5t + \pi/4));$
(v) $x_5(t) = exp(j3\pi t/8) + exp(\pi t/86);$
(vi) $x_6(t) = 2\cos(4\pi t/5) \times \sin^2(16t/3);$
(vii) $x_7(t) = 1 + \sin(20t) + \cos(30t + \pi/3).$

1.6 Determine if the following DT signals are periodic. If yes, calculate the fundamental period N_0 for the DT signals:

(i) $x_1[k] = 5 \times (-1)^k\ ;$
(ii) $x_2[k] = exp(j(7\pi k/4)) + exp(j(3k/4));$
(iii) $x_3[k] = exp(j(7\pi k/4)) + exp(j(3\pi k/4));$
(iv) $x_4[k] = \sin(3\pi k/8) + \cos(63\pi k/64);$
(v) $x_5[k] = exp\left(j\left(\frac{7\pi k}{4}\right)\right) + \cos(4\pi k/7 + \pi);$
(vi) $x_6[k] = \sin(3\pi k/8)\cos(63\pi k/64).$

1.7 Determine if the following CT signals are energy or power signals or neither. Calculate the energy and power of the signals in each case:

(i) $x_1(t) = \cos(\pi t)\sin(3\pi t);$
(ii) $x_2(t) = exp(-2t);$
(iii) $x_3(t) = exp(-j2t);$

(iv) $x_4(t) = exp(-2t)u(t);$

(v) $x_5(t) = \begin{cases} \cos(3\pi t) & -3 \le t \le 3 \\ 0 & elsewhere \end{cases}$

(vi) $x_6(t) = \begin{cases} t & 0 \le t \le 2 \\ 4-t & 2 \le t \le 4 \\ 0 & elsewhere \end{cases}$

1.8 Repeat Problem 1.7 for the following DT sequences:

(i) $x1[k] = \cos\left(\frac{\pi k}{4}\right)\sin\left(\frac{3\pi k}{8}\right)$

(ii) $x2[k] = \begin{cases} \cos\left(\frac{3\pi k}{16}\right) & -10 \le k \le 0 \\ 0 & elsewhere \end{cases}$

(iii) $x3[k] = (-1)^k$;

(iv) $x4[k] = exp(j(\pi k/2 + \pi/8));$

(v) $x5[k] = \begin{cases} 2^k & 0 \le k \le 10 \\ 1 & 11 \le k \le 15 \\ 0 & elsewhere \end{cases}$

1.9 Show that the average power of the CT periodic signal $x(t) = A\sin(\omega_0 t + \theta)$, with real-valued coefficient A, is given by $A^2/2$.

1.10 Show that the average power of the CT signal $y(t) = A_1\sin(\omega_1 t + \varphi_1) + A_2\sin(\omega_2 t + \varphi_2)$, with real-valued coefficients A_1 and A_2, is given by

$$P_y = \begin{cases} \frac{A_1^2}{2} + \frac{A_2^2}{2} & \omega_1 \ne \omega_2 \\ \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1A_2\cos(\emptyset_1 - \emptyset_2) & \omega_1 = \omega_2 \end{cases}$$

1.11 Show that the average power of the CT periodic signal $x(t) = D\,exp[j(\omega_0 t + \theta)]$ is given by $|D|^2$.

1.12 Show that the average power of the following CT signal:

$$x(t) = \sum_{n=1}^{N} D_n e^{j\omega_n t}, \qquad \omega_p \ne \omega_r \;\; if\; p \ne r$$

for $1 \le p, r \le N$, is given by

$$P_x = \sum_{n=1}^{N} |D_n|^2$$

1.14 Determine if the following CT signals are even, odd, or neither even nor odd. In the latter case, evaluate and sketch the even and odd components of the CT signals:

(i) $x1(t) = 2\sin(2\pi t)[2 + \cos(4\pi t)]$;

(ii) $x2(t) = t^2 + \cos(3t)$;

(iii) $x3(t) = \exp(-3t)\sin(3\pi t)$;

(iv) $x4(t) = t\sin(5t)$;

(v) $x5(t) = tu(t)$;

(vi) $x6(t) = \begin{cases} 3t & 0 \le t < 2 \\ 6 & 2 \le t < 4 \\ 3(-t+6) & 4 \le t \le 6 \\ 0 & \text{elsewhere.} \end{cases}$

1.15 Determine if the following DT signals are even, odd, or neither even nor odd. In the latter case, evaluate and sketch the even and odd components of the DT signals:

(i) $x1[k] = \sin(4k) + \cos(2\pi/k3)$;

(ii) $x2[k] = \sin(\pi k/3000) + \cos(2\pi k/3)$;

(iii) $x3[k] = \exp(j(7\pi k/4)) + \cos(4\pi k/7 + \pi)$;

(iv) $x4[k] = \sin(3\pi k/8)\cos(63\pi k/64)$;

(v) $x5[k] = \begin{cases} (-1)^k & k \ge 0 \\ 0 & k < 0. \end{cases}$

1.16 Consider the following signal:

$$x(t) = 3\sin\left(\frac{2\pi(t-T)}{5}\right).$$

Determine the values of T for which the resulting signal is (a) an even function, and (b) an odd function of the independent variable t.

1.17 By inspecting plots (a), (b), (c), and (d) in Fig. P1.17, classify the CT waveforms as even versus odd, periodic versus aperiodic, and energy versus power signals. If the waveform is neither even nor odd, then determine the even and odd components of the signal. For periodic signals, determine the fundamental period. Also, compute the energy and power present in each case.

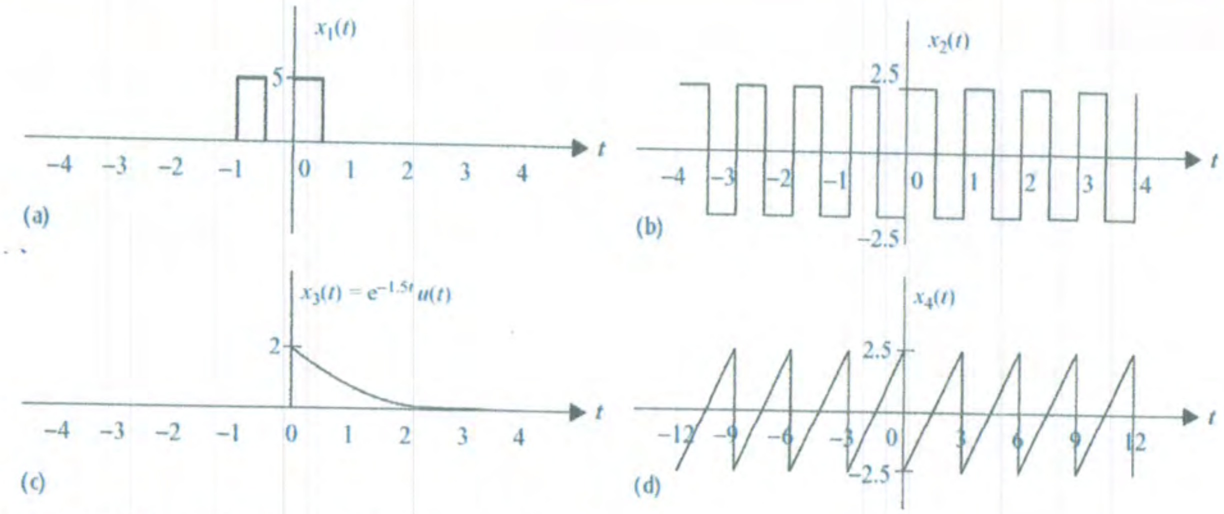

Fig. P1.17. Waveforms for Problem 1.17.

1.18 Sketch the following CT signals:

(i) $x1(t) = u(t) + 2u(t-3) - 2u(t-6) - u(t-9)$;
(ii) $x2(t) = u(\sin(\pi t))$;
(iii) $x3(t) = rect(t/6) + rect(t/4) + rect(t/2)$;
(iv) $x4(t) = r(t) - r(t-2) - 2u(t-4)$;
(v) $x5(t) = (exp(-t) - exp(-3t))u(t)$;
(vi) $x6(t) = 3\,sgn(t) \cdot rect(t/4) + 2\delta(t+1) - 3\delta(t-3)$.

1.19 Sketch the following functions with respect to the time variable (if a function is complex, sketch the real and imaginary components separately).

(i) $x1(t) = e^{j2\pi t+3}$;
(ii) $x2(t) = e^{j2\pi t+3t}$;
(iii) $x3(t) = e^{-j2\pi t+j3t}$;
(iv) $x4(t) = \cos(2\pi t + 3)$;
(v) $x5(t) = \cos(2\pi t + 3) + \sin(3\pi t + 2)$;
(vi) $x6(t) = 2 + 4\cos(2\pi t + 3) - 7\sin(5\pi t + 2)$.

1.20 Sketch the following DT signals:

(i) $x1[k] = u[k] + u[k-3] - u[k-5] - u[k-7]$;
(ii) $x2[k] = \sum_{m=0}^{\infty} \delta[k-m]$;
(iii) $x3[k] = (3^k - 2^k)u[k]$;
(iv) $x4[k] = u[\cos(\pi k/8)]$;
(v) $x5[k] = ku[k]$;
(vi) $x6[k] = |k|\,(u[k+4] - u[k-4])$.

1.21 Evaluate the following expressions:

(i) $\dfrac{5+2t+t^2}{7+t^2+t^4}\delta(t-1)$;

(ii) $\dfrac{\sin(t)}{2t}\delta(t)$;

(iii) $\dfrac{\omega^3-1}{\omega^2+2}\delta(\omega-5)$.

1.22 Evaluate the following integrals:

(i) $\displaystyle\int_{-\infty}^{\infty}(t-1)\delta(t-5)\mathrm{d}t$;

(ii) $\displaystyle\int_{-\infty}^{6}(t-1)\delta(t-5)\mathrm{d}t$;

(iii) $\displaystyle\int_{6}^{\infty}(t-1)\delta(t-5)\mathrm{d}t$;

(iv) $\displaystyle\int_{-\infty}^{\infty}(2t/3-5)\delta(3t/4-5/6)\mathrm{d}t$;

(v) $\displaystyle\int_{-\infty}^{\infty}\exp(t-1)\sin(\pi(t+5)/4)\delta(1-t)\mathrm{d}t$;

(vi) $\displaystyle\int_{-\infty}^{\infty}[\sin(3\pi t/4)+\exp(-2t+1)]\delta(-t-1)\mathrm{d}t$;

(vii) $\displaystyle\int_{-\infty}^{\infty}[u(t-6)-u(t-10)]\sin(3\pi t/4)\delta(t-5)\mathrm{d}t$;

(viii) $\displaystyle\int_{-21}^{21}\left(\sum_{m=-\infty}^{\infty}t\delta(t-5m)\right)\mathrm{d}t$.

1.24 Consider the following signal:

$$x(t)=\begin{cases} t+2 & -2\le t\le -1 \\ 1 & -1\le t\le 1 \\ -t+2 & 1<t\le 2 \\ 0 & \text{elsewhere.}\end{cases}$$

(a) Sketch the functions: (i) $x(t-3)$; (ii) $x(-2t-3)$; (iii) $x(-2t-3)$; (iv) $x(-0.75t-3)$.

(b) Determine the analytical expressions for each of the four functions.

1.25 Consider the function f (t) shown in Fig. P1.25.

(i) Sketch the function g(t) = f (−3t + 9).

(ii) Calculate the energy and power of the signal f (t). Is it a power signal or an energy signal?

(iii) Repeat (ii) for g(t).

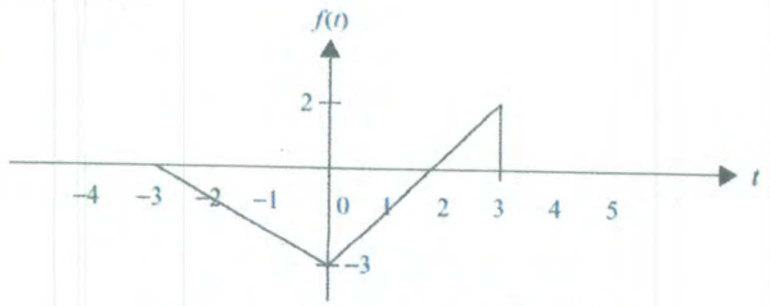

Fig. P1.25. Waveform for Problem 1.25.

1.26 Consider the function f (t) shown in Fig. P1.26.

(i) Sketch the function g(t) = f (−2t + 6).

(ii) Represent the function f (t) as a summation of an even and an odd signal. Sketch the even and odd parts.

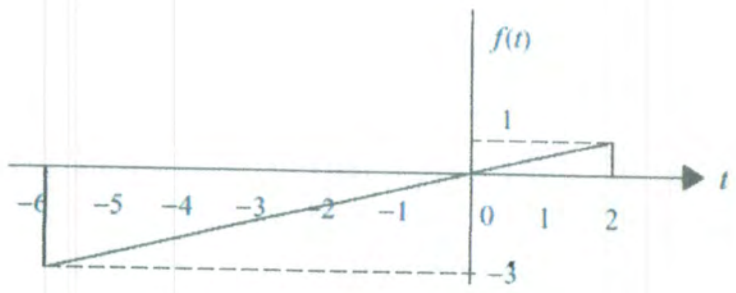

Fig. P1.26. Waveform for Problem 1.26.

1.27 Consider the function $f(t)$ shown in Fig. P1.27.

(i) Sketch the function $g(t) = t\,f\,(t+2) - t\,f\,(t-2)$.

(ii) Sketch the function $g(2t)$.

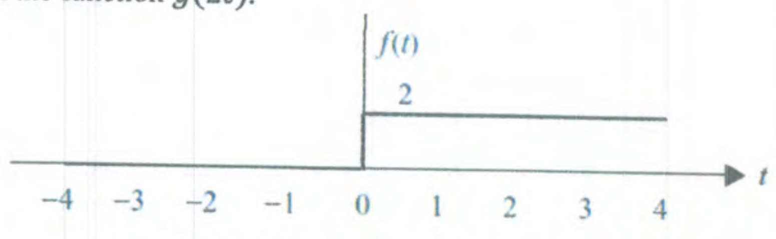

Fig. P1.27. Waveform for Problem 1.27.

1.28 Consider the two DT signals

$$x_1[k] = |k|(u[k+4] - u[k-4])$$

and

$$x_2[k] = k(u[k+5] - u[k-5]).$$

Sketch the following signals expressed as a function of $x_1[k]$ and $x_2[k]$:

(i) $x_1[k]$;
(ii) $x_2[k]$;
(iii) $x_1[3-k]$;
(iv) $x_1[6-2k]$;
(v) $x_1[2k]$;
(vi) $x_2[3k]$;
(vii) $x_1[k/2]$;
(viii) $x_1[2k] + x_2[3k]$;
(ix) $x_1[3-k]x_2[6-2k]$;
(x) $x_1[2k]x_2[-k]$.

1.29 In most parts of the human body, a small electrical current is often produced by movement of different ions. For example, in cardiac cells the electric current is produced by the movement of *sodium* (Na^+) and *potassium* (K^+) ions (during different phases of the heart beat, these ions enter or leave cells). The electric potential created by these ions is known as an ECG signal, and is used by doctors to analyze heart conditions. A typical ECG pattern is shown in Fig. P1.29.

Assume a hypothetical case in which the ECG signal corresponding to a normal human is available from birth to death (assume a longevity of 80 years). Classify such a signal with respect to the six criteria mentioned in Section 1.1. Justify your answer for each criterion.

(i) continuous-time and discrete-time signals;
(ii) analog and digital signals;
(iii) periodic and aperiodic (or nonperiodic) signals;
(iv) energy and power signals;
(v) deterministic and probabilistic signals;
(vi) even and odd signals.

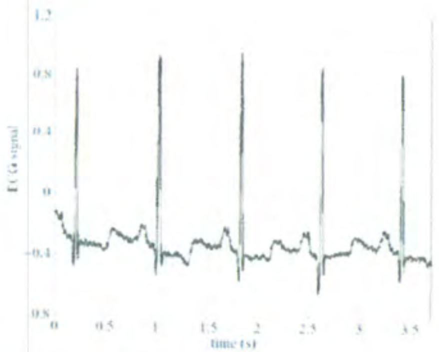

Fig. P1.29. ECG pattern for Problem 1.29.