Dr. Montadar Abas Taher 2019–2020

Lecture # 8 Fourier Series

The periodic function which satisfys Dirichlet conditions can be expressed as

$$f(t) = a_0 + \sum_{n=0}^{\infty} \left\{ a_n \cos(n\omega t) + b_n \sin(n\omega t) \right\}$$

where $a_0 = \frac{1}{T} \int_0^T f(t)\,dt$ = D.C. or Average Value

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t)\,dt$$

a_0, a_n, b_n are called Fourier series coefficients

Dirichlet Conditions are

① The signal has finite number of discontinuities.

② The signal has finite number of maxima or minima.

③ The integral $\int_0^T |f(t)|\,dt < \infty$

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NOTE :-

→ If $f(t)$ is even, then $b_n = 0$

→ If $f(t)$ is odd, then $a_n = 0$

→ $\sin(n\pi) = 0$ for all n

→ $\cos(n\pi) = (-1)^n$

Exponential Fourier Series

A simple formulation of Fourier series is the exponential.

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\omega t}}$$

where

$$\boxed{D_n = \frac{1}{T} \int_T f(t) e^{-jn\omega t} \, dt}$$

$\boxed{D_n = |D_n| e^{j\theta_n}}$ is the polar form used to plot the Line or amplitude and phase spectra.

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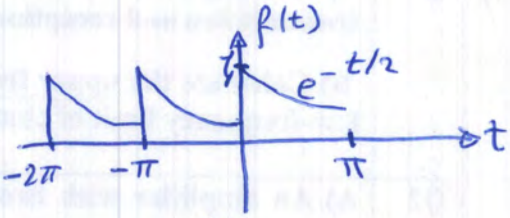
EX. 1 Find the fourier series for the signal $f(t)=e^{-t/2}$.

Sketch $f(t)$ and the amplitude and phase spectrums.

Note that $0 \leq t \leq \pi$

Solution

$$T=\pi \Rightarrow \omega=\frac{2\pi}{T}=2$$



$$\therefore f(t)=\sum_{n=-\infty}^{\infty} D_n e^{j2nt}$$

$$D_n=\frac{1}{T}\int_0^T f(t)\, e^{-j\omega t}\, dt=\frac{1}{\pi}\int_0^{\pi} e^{-t/2} e^{-j2nt}\, dt=\frac{1}{\pi}\int_0^{\pi} e^{-\left(\frac{1}{2}+j2n\right)t}\, dt$$

$$D_n=\frac{-1}{\pi\left(\frac{1}{2}+j2n\right)} e^{-\left(\frac{1}{2}+j2n\right)t}\Big|_0^{\pi}=\frac{0.504}{1+j4n}.$$

$$\therefore \text{so} \quad \boxed{f(t)=\sum_{n=-\infty}^{\infty}\frac{0.504}{1+j4n}e^{j2nt}}$$

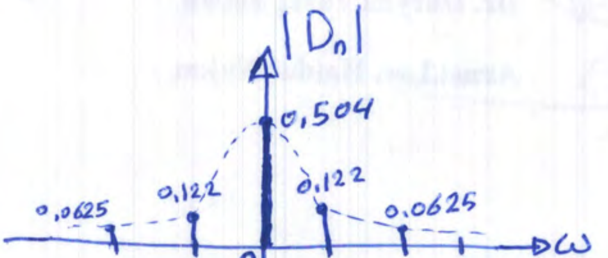
$D_0=0.504$

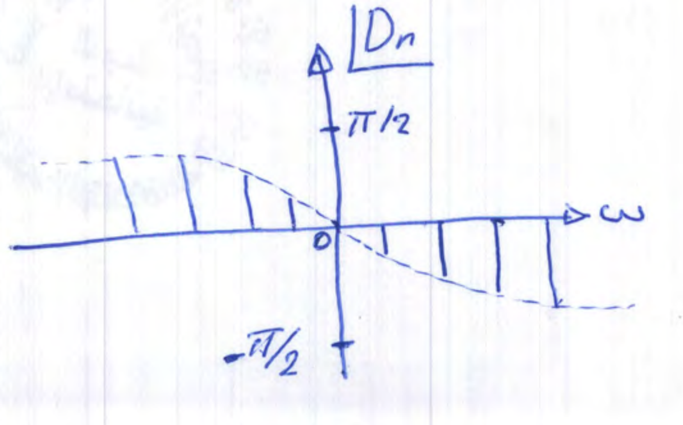
$D_1=\frac{0.504}{1+j4}=0.122e^{-j(75.96^\circ)} \longrightarrow |D_1|=0.122,\ \angle D_1=-75.96^\circ$

$D_{-1}=\frac{0.504}{1-j4}=0.122e^{j(75.96^\circ)} \longrightarrow |D_{-1}|=0.122,\ \angle D_{-1}=75.96^\circ$

$D_2 \Rightarrow |D_2|=0.0625,\ \angle D_2=-82.87^\circ$ ⊛ $D_{-2} \Rightarrow |D_{-2}|=0.0625,\ \angle D_{-2}=82.87^\circ$

and so on...

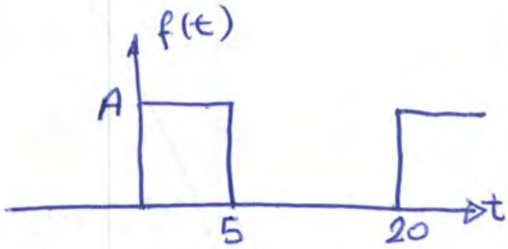




EX. Find the Fourier series coefficients for the signal $f(t) = A$ when $0 \leq t < 5$ and zero in $5 \leq t < 20$.

solution

$$f(t) = \begin{cases} A & 0 < t \leq 5 \\ 0 & 5 < t \leq 20 \end{cases}$$



Period $T = 20$

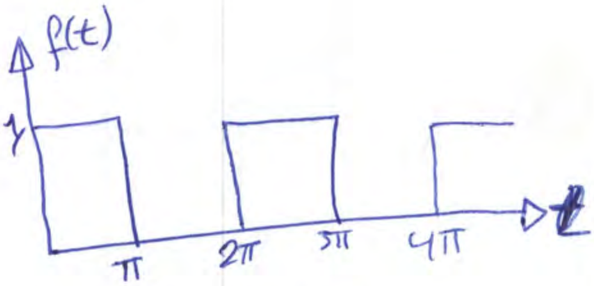
$$D_n = \frac{1}{T}\int_T f(t)\, e^{-jn\omega t}\, dt = \frac{1}{T}\int_0^5 A\, e^{-jn\omega t}\, dt = \frac{A}{T}\int_0^5 e^{-j2\pi n f t}\, dt$$

$$= \frac{A}{T(-j2\pi\frac{n}{T})}\left[e^{-j2\pi\frac{n}{T}t}\right]_0^5 = \frac{-A}{j2\pi n}\left[e^{-j2\pi n f5} - 1\right] = \frac{A}{j2\pi n}\left[1 - e^{-j2\pi n f5}\right]$$

$$= \frac{A}{j2\pi n}\, e^{-j\pi n5f}\left[e^{j\pi n5f} - e^{-j\pi n5f}\right] = \frac{A\, e^{-jn\pi 5f}}{n\pi}\sin\left(\frac{n\pi}{4}\right)$$

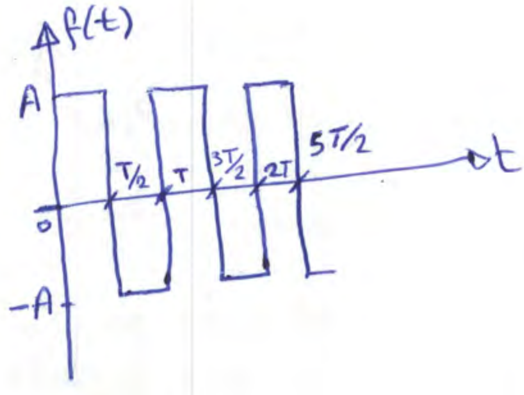
$$= \frac{8}{n\pi}\sin\left(\frac{n\pi}{4}\right) e^{-j\frac{n\pi}{4}}$$

Q1/ Determine the Fourier series for the waveform shown below



Ans: $f(t) = \frac{1}{2} + \frac{2}{\pi}\left(\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \dots\right)$

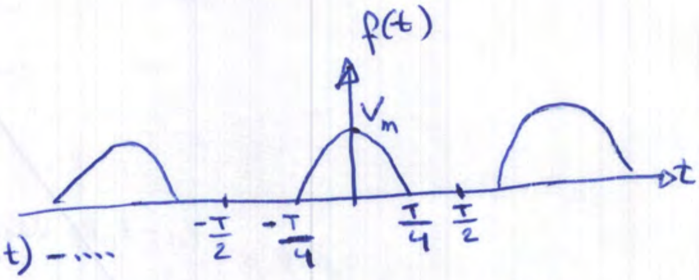
Q2/ Find the Fourier series of the waveform shown below



Ans: $f(t) = \frac{2A}{jn\pi}$ for $n = 1, 3, 5, 7 \dots$

$f(t) = 0$ for $n = 2, 4, 6, 8, \dots$

Q/ Obtain the trigonometric Forier series for the waveform shown below



Ans: $f(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\cos(\omega t) + \frac{2V_m}{3\pi}\cos(2\omega t)$

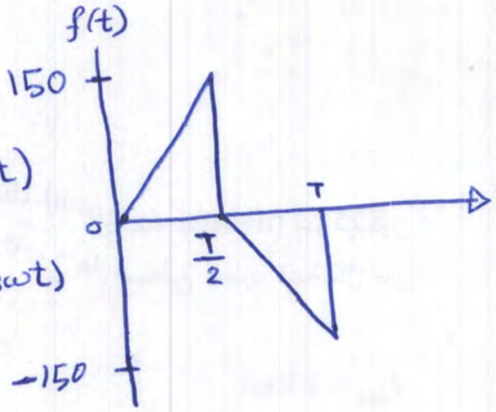
$-\frac{2V_m}{15\pi}\cos(4\omega t) + \frac{2V_m}{35\pi}\cos(6\omega t) - \dots$

Q/ The output of a rectifier is $\begin{cases} V_m\cos(\omega t) & 0 \leq \omega t \leq \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq \omega t \leq \frac{3\pi}{2} \\ V_m \cos\omega t & \frac{3\pi}{2} \leq \omega t \leq 2\pi \end{cases}$

find its Fourier series.

Ans: $f(t) = \frac{V_m}{2\pi} + \frac{V_m}{2\pi}\sin(\omega t) + \frac{V_m}{\pi}\sum_{n=2}^{\infty}\left(\frac{n}{n^2-1}\right)\sin(n\omega t)$

Q/ Determine the Fourier series of repetitive waveform of the figure below upto 7th harmonic when repetition time $T = 25\pi$ ms.

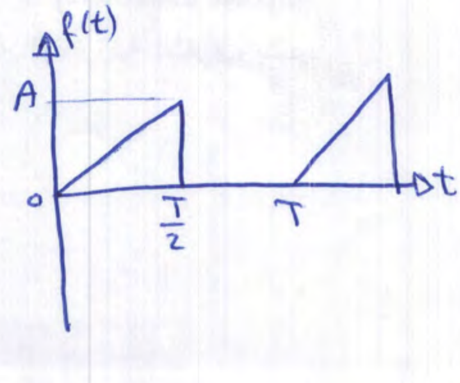


Ans: $f(t) = \frac{600}{\pi^2}\cos(\omega t) - \frac{600}{3^2\pi^2}\cos(3\omega t) - \frac{600}{5^2\pi^2}\cos(5\omega t)$

$-\frac{600}{7^2\pi^2}\cos(7\omega t) + \frac{300}{\pi}\sin(\omega t) + \frac{300}{3\pi}\sin(3\omega t)$

$+\frac{300}{5\pi}\sin(5\omega t) + \frac{300}{7\pi}\sin(7\omega t)$

Q/ Obtain the trigonometric Fourier series for the signal shown below



Ans: $f(t) = \frac{A}{4} - \frac{2A}{\pi^2}\left[\cos(\omega t) + \frac{1}{3^2}\cos(3\omega t) + \frac{1}{5^2}\cos(5\omega t)\right.$

$\left. + \dots\right] + \frac{A}{\pi}\left[\sin(\omega t) - \frac{1}{2}\cos(2\omega t) + \frac{1}{3}\cos(3\omega t)\right.$

$\left. - \frac{1}{4}\cos(4\omega t) + \dots\right]$.