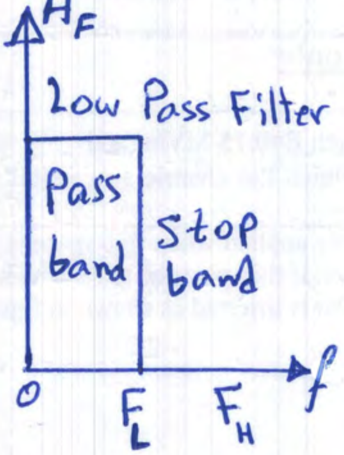
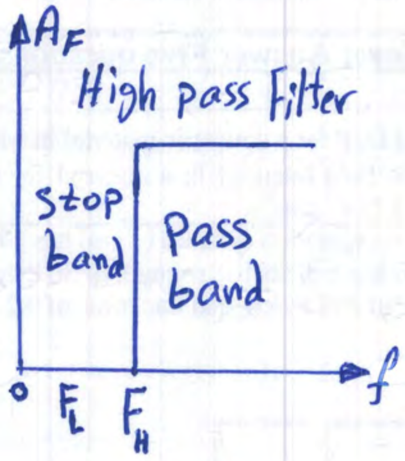
Lecture # 10

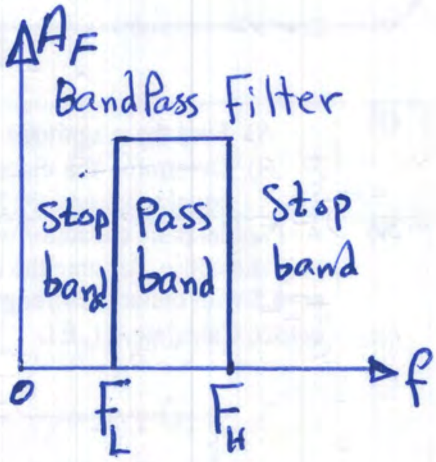
Filters



(LPF)



(HPF)



(BPF)

* Filter passes certain band of frequency and rejects (stops) the other band.

* The basic types of filters are three:-

(1) Low Pass Filter (LPF), passes Low frequencies starting at $f = 0$ Hz until the cut-off frequency (F_L).

(2) High Pass Filter (HPF), passes high frequencies starting at the cut-off frequency f_H and extending to ∞ Hz.

(3) Band Pass Filter (BPF), passes a band of frequencies only starting at F_L and ends at F_H.

* Filter is a circuit consists of Resistors and Reactive components.
* Filters are either passive (NO amplifier) or active (with amplifier to give gain to the output signal).
* Reactive components are capacitors and inductors.
* Filter can be constructed from RC or RL components.
* As the number of reactive components increases, the Filter's order increases. For instance, one capacitor exists in the circuits stands for one order (first order) passive filter. Two capacitors in the circuit stands for 2^{nd} order filter and so on.
* The ideal Filter should has unity gain across the pass band, otherwise it attenuates the other bands (stop bands).

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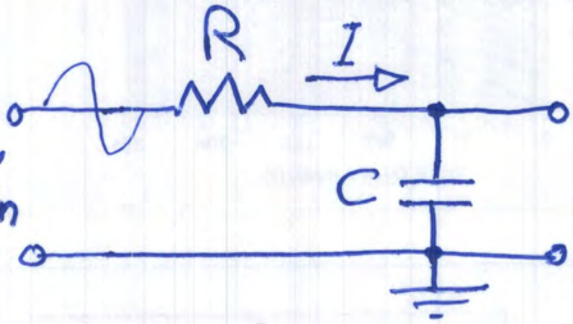
The Low Pass Filter (LPF):



$$V_{out} = V_{in} * \frac{X_c}{\sqrt{R^2 + X_c^2}} = V_{in} \frac{X_c}{Z}$$

<u>EX. 1</u> An RC LPF consists of $R = 4K7\,\Omega$ and $C = 47\,nF$. Calculate V_{out} if the input is $V_{in}(t) = 10 \sin(2\pi f t)$, when $f = 100\,Hz$ and when $F = 10\,KHz$.

<u>Solution</u> $X_c = \frac{1}{2\pi FC} = \frac{1}{2\pi\, 100 \times 47 \times 10^{-9}} = 33.863\,K\Omega$.

when $F = 100\,Hz$.

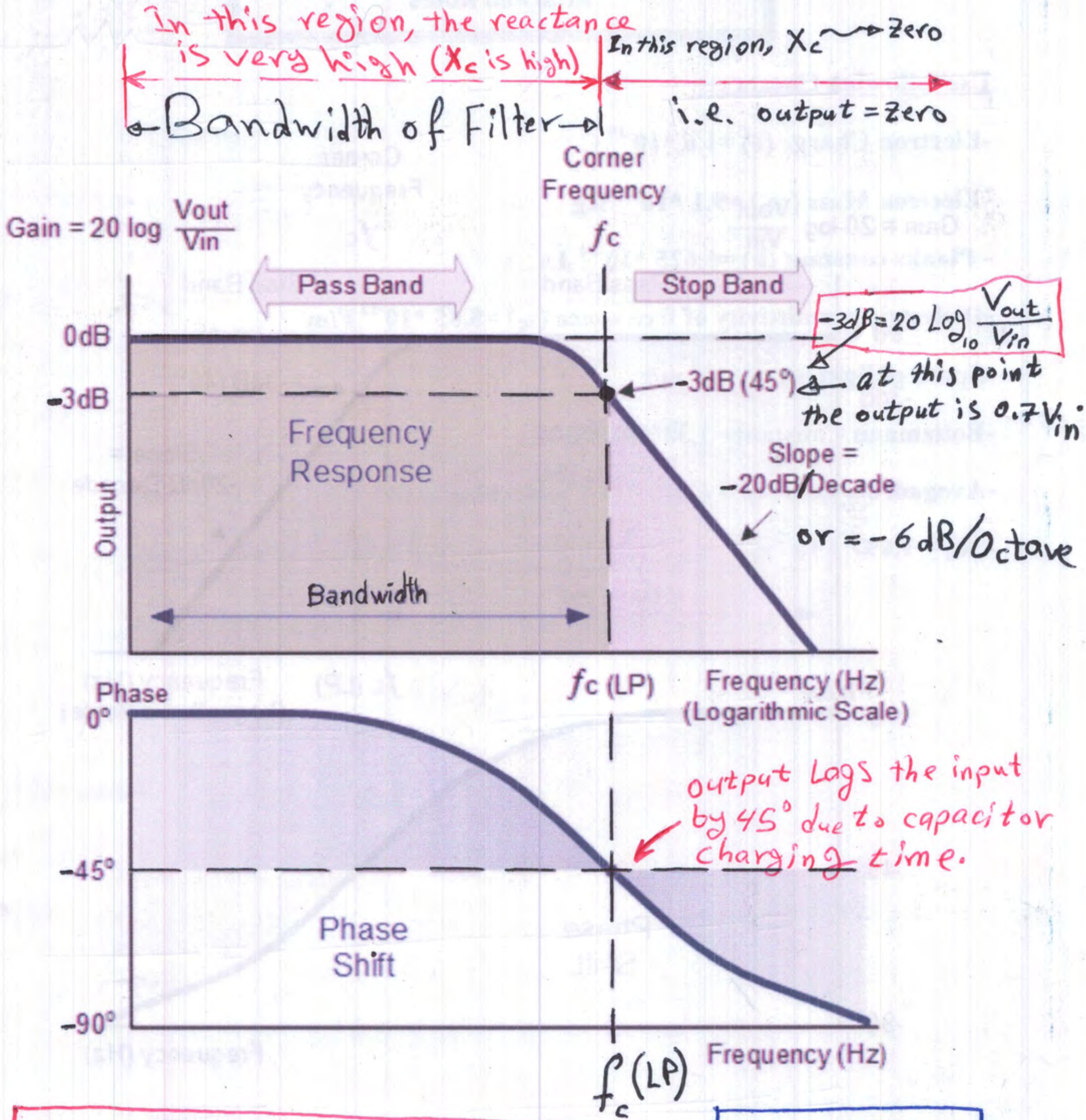
$$V_{out} = V_{in} \times \frac{X_c}{\sqrt{R^2 + X_c^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9\,V.$$

when $F = 10{,}000\,Hz$

$$X_c = \frac{1}{2\pi FC} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\,\Omega$$

$$V_{out} = V_{in} \times \frac{X_c}{\sqrt{X_c^2 + R^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6}} = 0.718\,V.$$

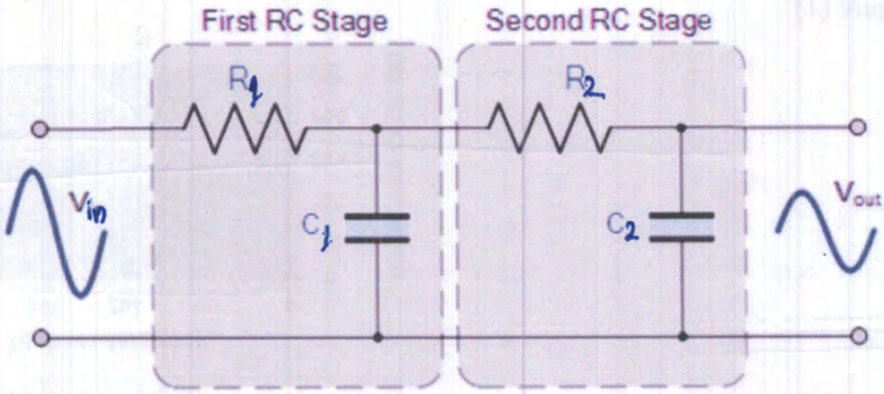
Frequency Response:



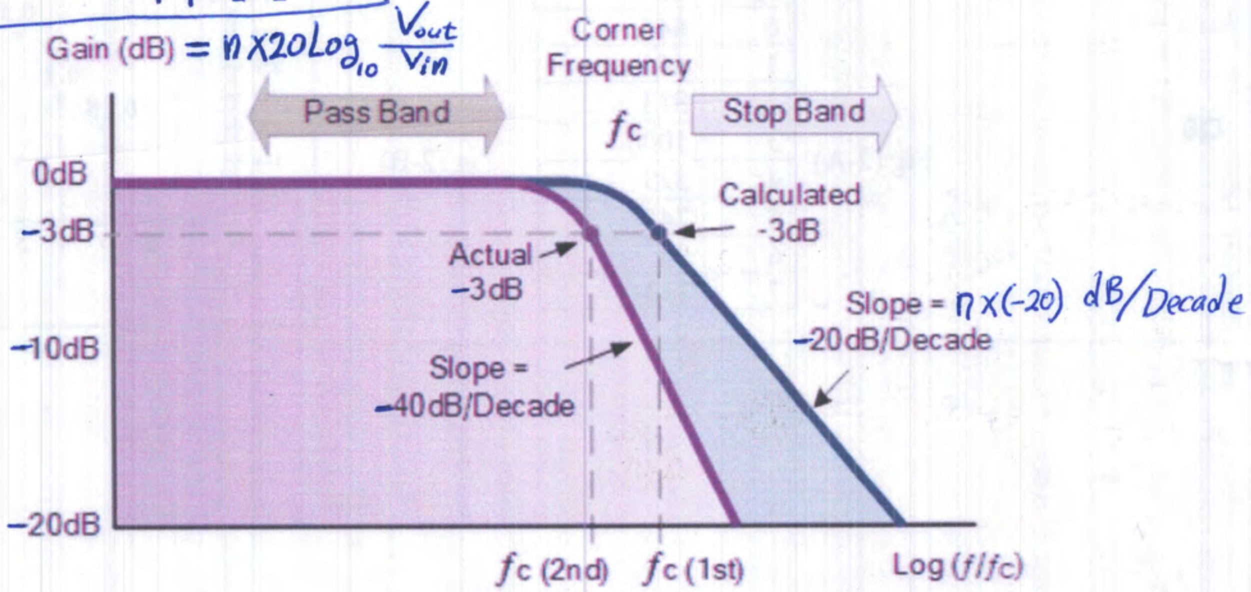
$$\boxed{\text{Phase shift } \phi = -\tan^{-1}(2\pi f RC)}$$

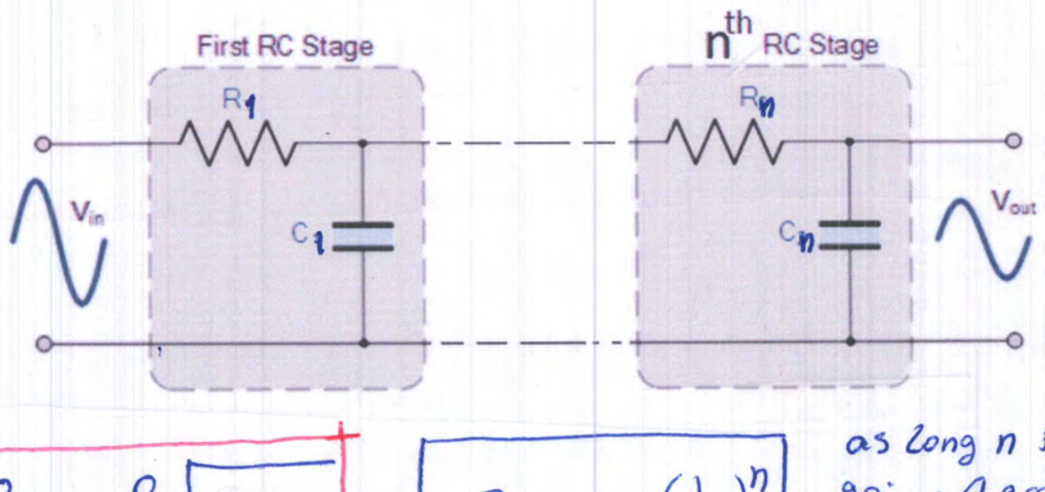
$$\boxed{\text{time constant } \tau = RC}$$

Second- and higher-order Low Pass Filter:-



$$\boxed{f_c = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}\ Hz}$$





$$\boxed{f_{-3dB} = f_c\sqrt{2^{1/n} - 1}}$$

$$\boxed{Gain_{f_c} = \left(\frac{1}{\sqrt{2}}\right)^n}$$

as long n increases, the gain and accuracy of the filter become worse

EX.1 Passive LPF consists of two identical RC-LPF. The cut-off frequency $f_{c1} = 20 KHz$, calculate the -3dB frequency of the final stage.

Solution $n = 2$, $f_c = 20 KHz$ then

$$f_{-3dB} = 20 \times 10^3 \sqrt{2^{1/2} - 1} = 20{,}000 \sqrt{\sqrt{2} - 1} = 12.87 \text{ KHz}.$$

EX.2 If the gain at -3dB frequency was 0.125, where the calculated -3dB frequency (f_{c1}) was 10 KHz. Find the actual -3dB cut-off frequency and the capacitor value of each stage of the n^{th} order LPF if the resistance was 3 KΩ.

solution G_{ain} at -3dB Frequency $= 0.125 = \left(\frac{1}{\sqrt{2}}\right)^n$

$$\ln(0.125) = \ln\left(\frac{1}{\sqrt{2}}\right)^n = n \ln\left(\frac{1}{\sqrt{2}}\right) = -2.079442 \Rightarrow n = \frac{-2.079442}{\ln\left(\frac{1}{\sqrt{2}}\right)}$$

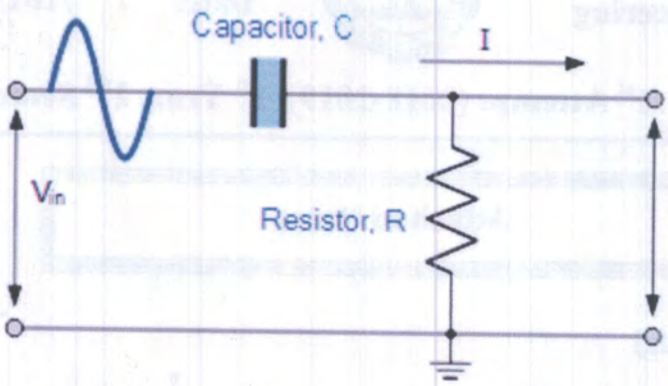
$$\boxed{n = 6} \Rightarrow f_{-3dB} = f_c \sqrt{2^{1/n} - 1} = 10000 \sqrt{2^{1/6} - 1} = 3.5 \text{ KHz}$$

$$f_c = \frac{1}{2\pi RC} \rightarrow C = \frac{1}{2\pi f_c R} = \frac{1}{2\pi \times 3000 \times 10000}$$

$$\therefore C = 5.3 \text{ nF}$$

The High Pass Filter (HPF):-

$$\boxed{A_v = \frac{V_{out}}{V_{in}}}$$



$$V_{out} = V_{in} \times \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{Z}$$

① at Low Frequency:

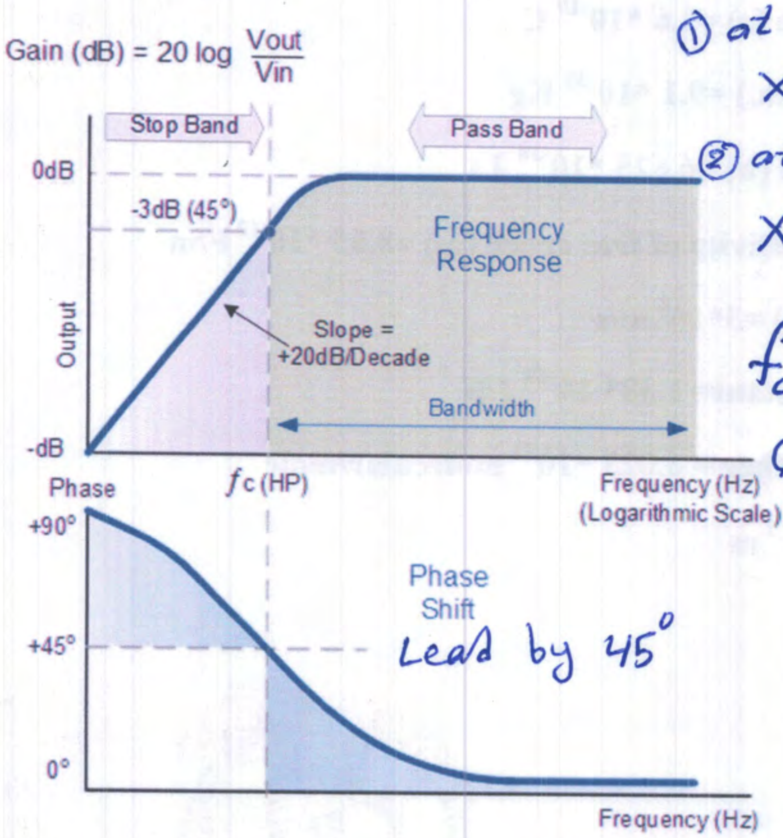
$X_C \to \infty$, $V_{out} = 0$

② at High Frequency:

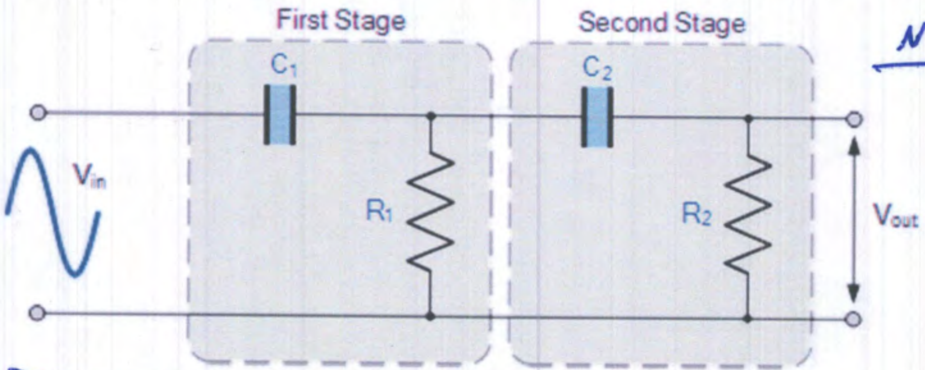
$X_C \to 0$, $V_{out} = V_{in}$

$$f_c = \frac{1}{2\pi RC} \text{ Hz}$$

$$\phi = \tan^{-1}\left[\frac{1}{2\pi f RC}\right]$$



Lead by 45°



Note:

$R_2 = 10R_1$

$C_2 = \frac{1}{10} C_1$

$R_3 = 10R_2$

$C_3 = \frac{1}{10} C_2$

and so on.

$$\boxed{f_c = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}} \text{ Hz}$$

Higher orders can not be implemented esily. due to Loading effect of each next stage.

Band Pass Filter (BPF):-