Department of Communications Engineering, College of Engineering, University of Diyala

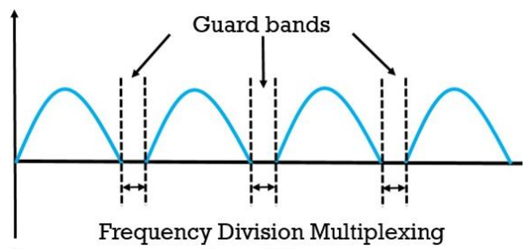
Digital Communication I

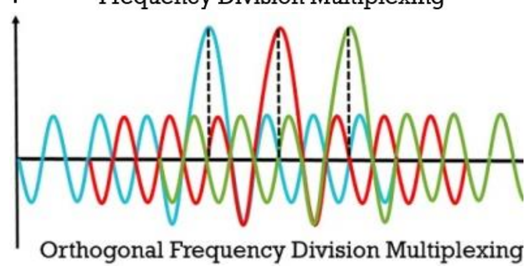
Lecture # 10

Orthogonal Frequency Division Multiplexing (OFDM)

Introduction:

- **OFDM** is a special case of **FDM**!
- The channels are separated with each other by unused bandwidth called as **guard bands** to prevent the inter-channel crosstalk and overlapping of the channel.





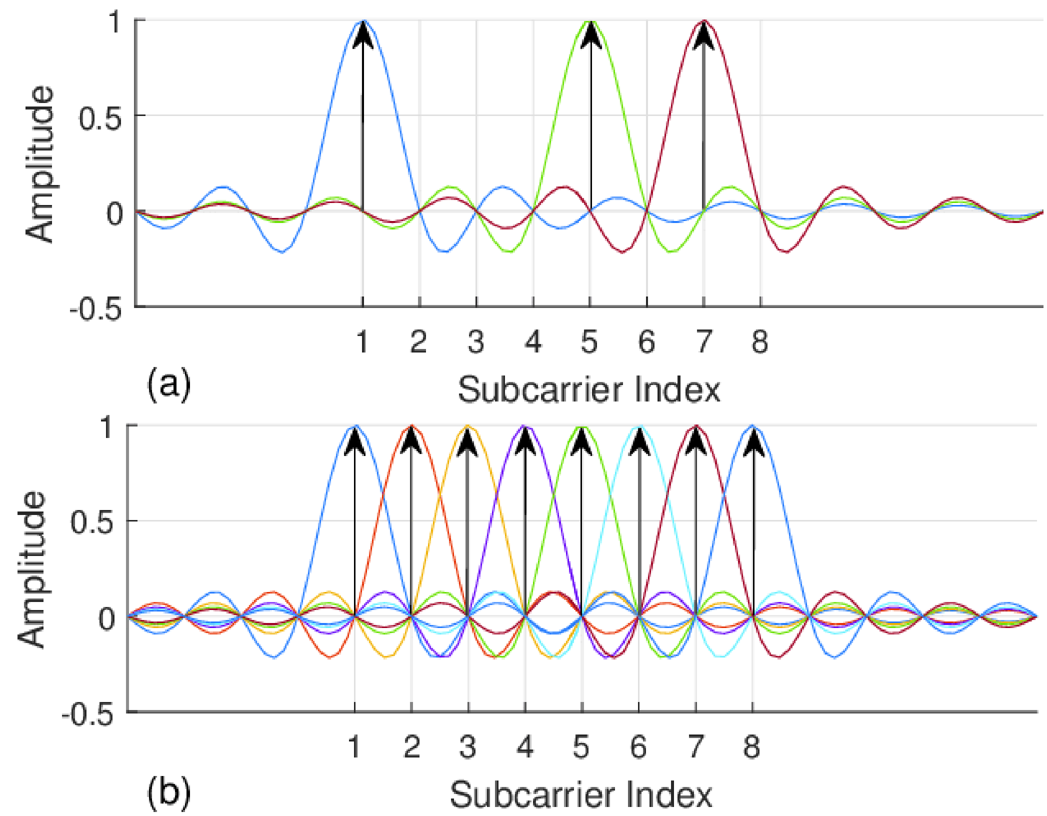
- The subchannels are closely spaced and ***overlap*** each other.

Why subchannels (subcarriers) do not interfere with each other?

- When a *signal reaching its peak* its two *neighboring signals are **at null** or **zero***.

$$\int_0^T \cos(n\omega t)\cos(m\omega t)\,dt = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$$

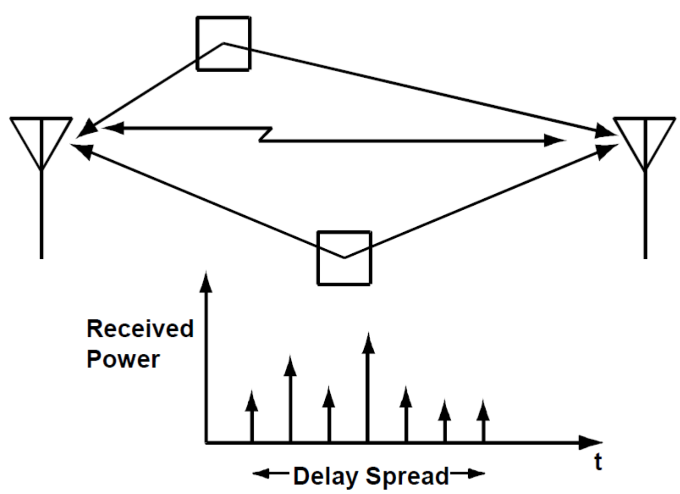
$$\int_0^T \cos(n\omega t)\sin(m\omega t)\,dt = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$$



Motivation

- High-bit-rate wireless applications.
- Limitations caused by the radio environment.
- OFDM can overcome these inherent bit rate limitations.

Multipath



$$h(t) = \sum_i a_i e^{j\theta_i} \delta(t - t_i)$$

Doppler Spread

- A measure of the spectral broadening caused by the channel time variation.

$$f_D \le \frac{c_{device}}{\lambda}$$

c_{device} is the speed of mobile device.

Implications (problems due to Doppler frequency shift)

- ***Signal amplitude and phase*** decorrelate after a time period $T_D = 1/f_D$
- **In other words, the signal will loss orthogonality.**

Example 1: A car moving at speed of 96Km/h. The user which riding the car has a 4G cellular phone transmits at carrier frequency of 900MHz, calculate the Doppler frequency.

Solution: The speed in m/s is $\frac{96\times10^3}{3600} = 26.6667 m/s$,

$$\lambda = \frac{c}{f_c} = \frac{3\times10^8}{900\times10^6} = \frac{1}{3} m$$

$$f_D = \frac{c_{device}}{\lambda} = \frac{26.66667}{\frac{1}{3}} = 80Hz$$

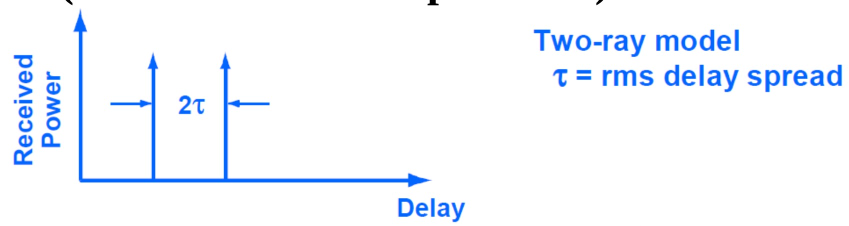
Example 2: A subscriber doing a call while he driving a car moving at speed of 8Km/h. If the transmission frequency 5GHz, what will be the Doppler frequency?

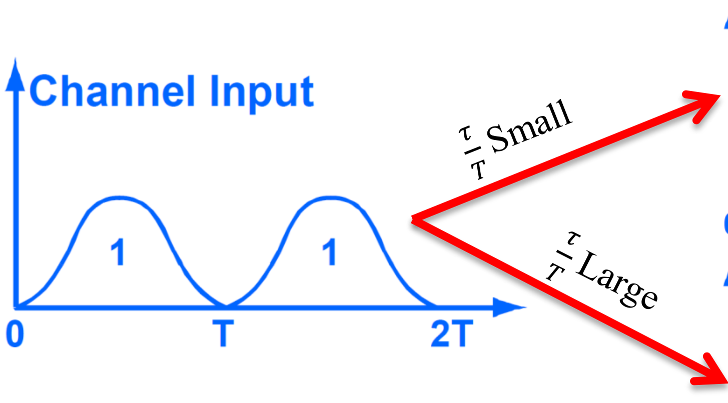
Solution: The speed in m/s is $\frac{8\times10^3}{3600} = 2.2222 m/s$,

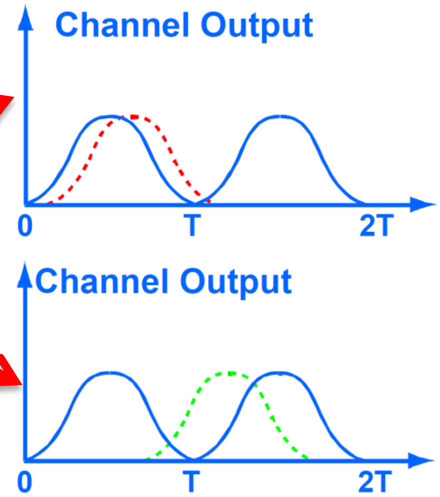
$$\lambda = \frac{c}{f_c} = \frac{3\times10^8}{5\times10^9} = 0.06m$$

$$f_D = \frac{c_{device}}{\lambda} = \frac{2.2222}{0.06} = 37Hz$$

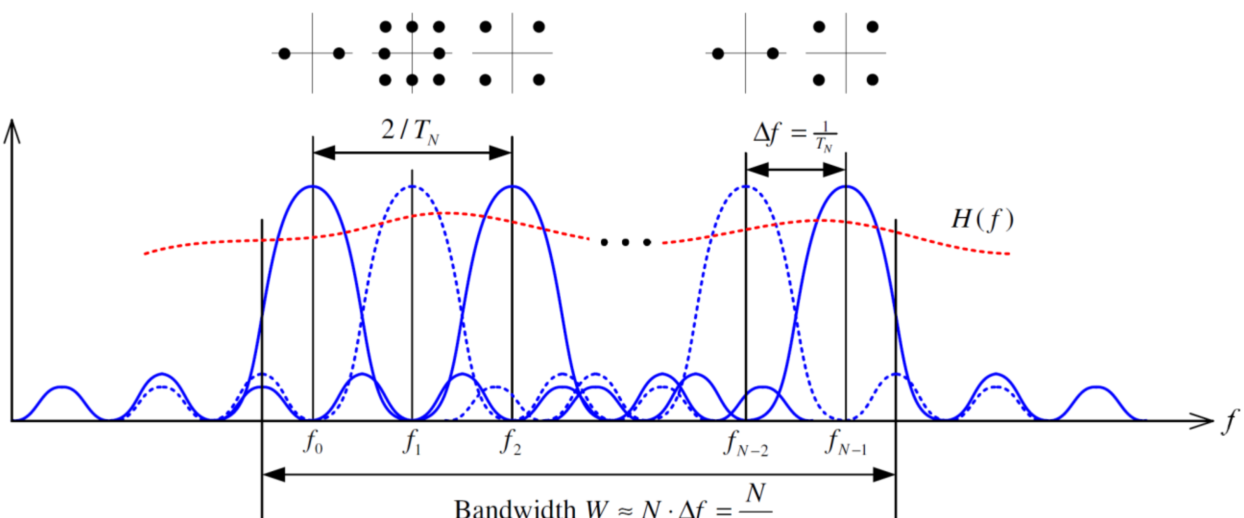
Delay Spread (Time Domain Interpretation):







- When $\frac{\tau}{T}$ **Small** → Negligible ISI
- When $\frac{\tau}{T}$ **Large** → Significant ISI, causes irreducible error floor



Red: Channel frequency response, $H(f)$
Blue: Subchannel spectrum in OFDM

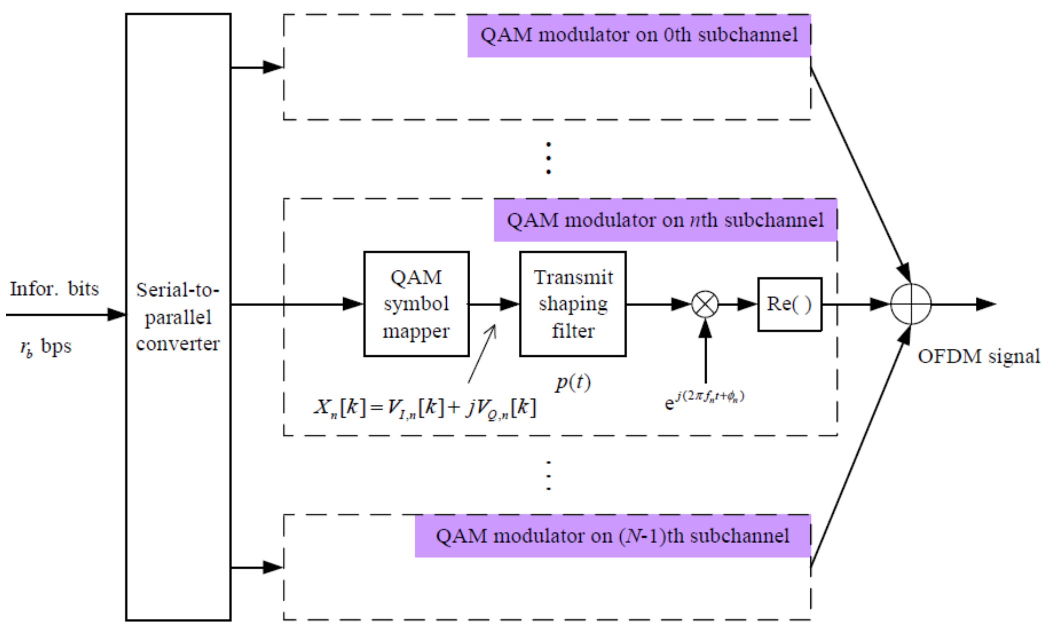
- We know that **QAM** is a combination of **ASK** and **PSK**.
- Also, we know that he symbol duration $T_s = vT_b$.
- **OFDM** is a combination of QAM and FSK.

- Instead of activating only one out of ***N*** carriers as in **FSK**, in **OFDM,** all the carriers can be activated all the time and **QAM** symbols are sent over each carrier.
- The data rate on each subchannel is much less than the total data rate,
- Then the corresponding subchannel bandwidth is much less than the total system bandwidth.

The number of subchannels (N) can be chosen *so that each subchannel has a bandwidth small enough* **so that the frequency response over each subchannel's frequency range is approximately constant**. **This ensures that ISI on each subchannel is small**.

- The subchannels in OFDM need not be **contiguous** (neighboring), so a large continuous block of spectrum is not needed for high rate transmission.

- The main technical issues that impair performance of OFDM are **frequency offset** and **timing jitter**, which degrade the orthogonality of the subchannels



- OFDM signal is obtained as a sum of ***N*** **QAM** signals, one is centered at its own carrier frequency
- Let $\boldsymbol{T_N}$ be the duration of each OFDM symbol.

Then the relationship between $\boldsymbol{T_N}$ and the information bit rate R_b (or the bit duration $T_b = \frac{1}{R_b}$) can be found as follows.

- Let $M = 2^v$ be the size of the QAM constellation used on carrier f_n,
- It means that each QAM symbol sent over the n^{th} channel can carry $\boldsymbol{v}\ bits$.

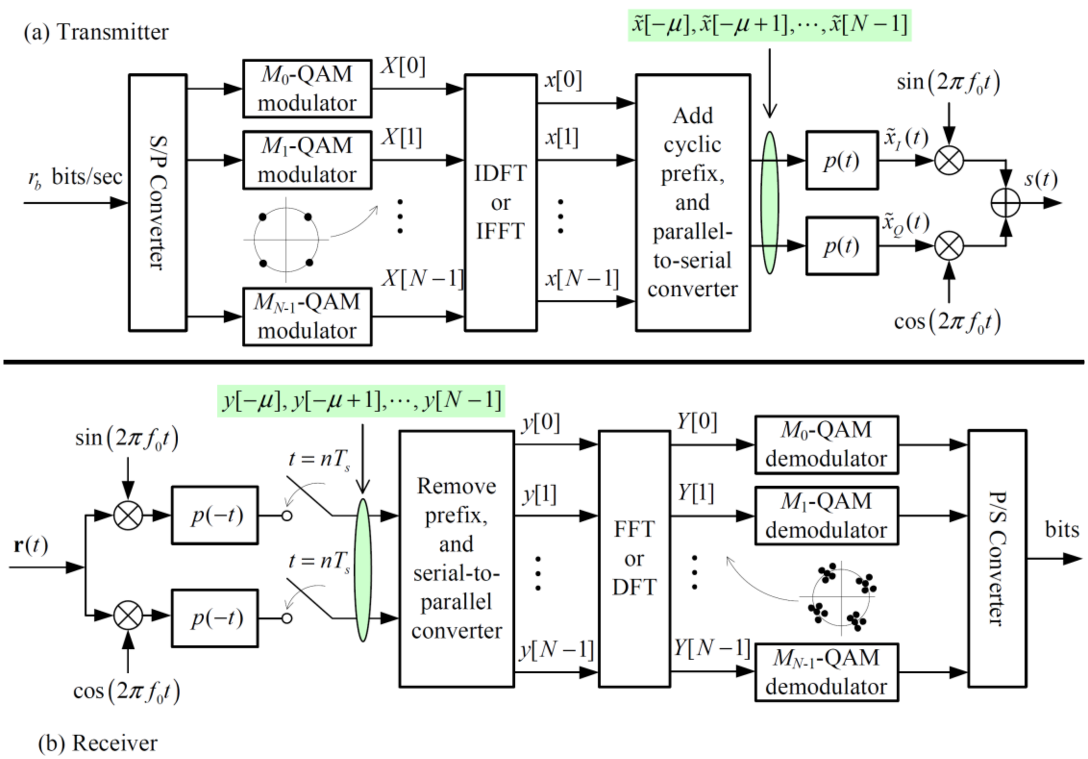
- The total number of bits sent over all channels in each OFDM symbol is therefore equal to $V = vN$
- That is, $\boldsymbol{T_N = VT_b = \frac{vN}{R_b} = vNT_b}$.

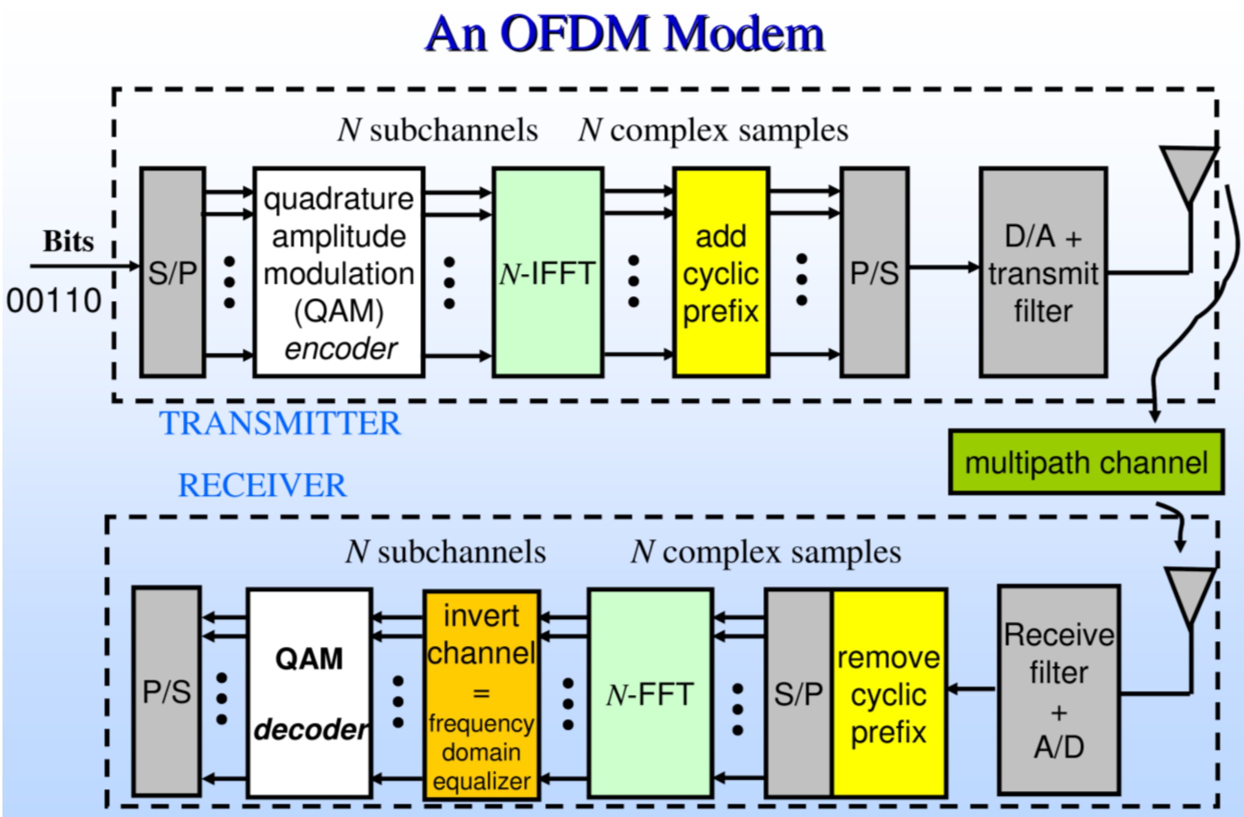
- Since each QAM symbol lasts over $\boldsymbol{T_N}$ seconds,
- all the carriers $\{\cos(2\pi f_n t), \sin(2\pi f_n t)\}_{n=0}^{N-1}$, can be made noncoherently orthogonal over the duration of $\boldsymbol{T_N}$ if the minimum carrier spacing is $\Delta f = \frac{1}{T_n} = f_n$,
- i.e., $f_n = f_0 + \frac{n}{T_N}$, $\boldsymbol{n} = 0, 1, \ldots, \boldsymbol{N}-1$.
- Now, let the bandwidth of each subchannel is approximated as $\frac{1}{T_n}$. Then, the bandwidth of the OFDM signal is

$$B = \frac{1}{T_N} + (N-1)\Delta f = \frac{1}{T_N} + (N-1)\frac{1}{T_N} = \frac{N}{T_N}$$

Implementation:

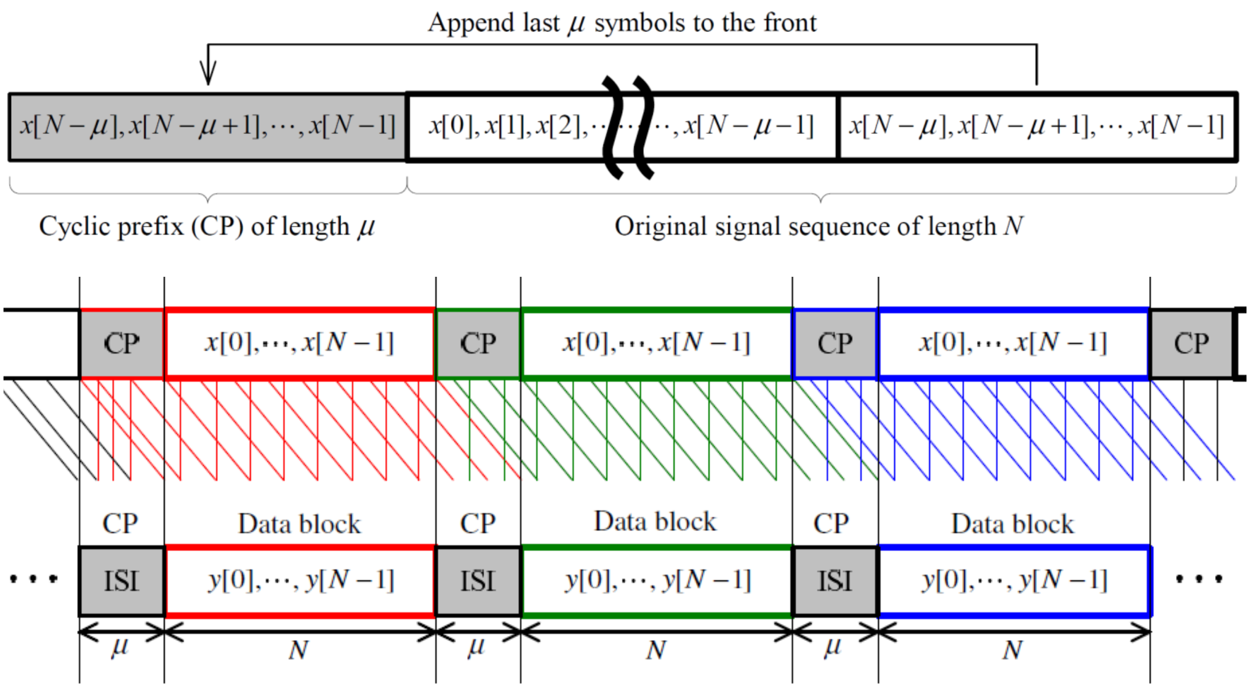
- OFDM Modulation/Demodulation technique can be implemented using Digital Signal Processor (**DSP**),
- **Modulation:** is using **I**nverse **D**iscrete **F**ourier **T**ransform (**IDFT**),
- **Demodulation:** is using the **D**screte **F**ourier **T**ransform (**DFT**).





Cyclic Prefix (CP)

A very important operation called ***cyclic prefix*** (**CP**) extension is performed on the OFDM symbol to eliminate the ISI.



- Let T_N be the duration of one OFDM symbol,
- Then the distance between two adjacent time samples is $\boldsymbol{T_s} = \frac{\boldsymbol{T_N}}{\boldsymbol{N}}$.
- With $\boldsymbol{\mu}$ being the number of time samples in the cyclic prefix,
- The length of the cyclic prefix is $\mu T_s = \frac{\mu}{N} T_N$
- Thus, $\mu = \left\lceil \frac{\tau}{T_s} \right\rceil$ to prevent ISI.

➢ The benefits of adding a cyclic prefix come at a cost!!!
  - Since $\boldsymbol{\mu}$ symbols are added to the input data blocks, there is an overhead of $\boldsymbol{\mu/N}$ and a resulting data-rate reduction of $\mathbf{1} - \boldsymbol{N/(\mu + N)}$.
  - The transmit power associated with sending the cyclic prefix is also wasted since this prefix consists of redundant data.

Example 1: In 802.11a standard, N = 64 subcarriers are generated. However, only 48 carriers are actually used for data transmission, the outer 12 carriers are zeroed in order to reduce adjacent channel interference, and 4 carriers used as pilot symbols for channel estimation and synchronization. The cyclic prefix consists of μ = 16 samples. The modulation types that can be used on the subchannels are BPSK, QPSK, 16-QAM, or 64-QAM. The bandwidth B (and sampling rate $1/T_s$) is 20 MHz, and there are 64 subcarriers evenly spaced over that bandwidth. Determine:

1. The total number of samples associated with each OFDM symbol, including both data samples and the cyclic prefix,
2. The subcarrier bandwidth,
3. The maximum delay spread for which ISI is removed,
4. The symbol time per subchannel,
5. The data rate for this system, corresponding to BPSK, QPSK, 16-QAM, and 64-QAM,
6. Find the Date Rate Reduction percentage.

Solution:

(1) The cyclic prefix consists of $\boldsymbol{\mu}$ = 16 samples, so the total number of samples associated with each OFDM symbol, including both data samples and the cyclic prefix, is 64+16=80.

(2) The subcarrier bandwidth is:

$$B_N = \frac{B}{N} = \frac{20 \times 10^6}{64} = 312.5KHz$$

(3) The maximum delay spread ($\boldsymbol{\tau}$) for which ISI is removed is:

$$\tau < \mu T_s = \mu \frac{1}{B} = \frac{\mu}{B} = \frac{16}{20 \times 10^6} = 0.8 \times 10^{-6} sec = 0.8 \mu sec$$

(4) The symbol time per subchannel is: We have both the data and CP, there are 80=64+16 samples per OFDM symbol, Thus the symbol time per subchannel:

$$T_{total} = T_N + \mu T_s = (N + \mu) T_s = 80 T_s = \frac{80}{20 \times 10^6} = 4 \mu sec$$

(5) The data rate for this system, corresponding to **BPSK**, **QPSK**, **16-QAM**, and **64-QAM**:

The data rate per subchannel is $\frac{\log_2(M)}{T_{total}}$. Remember there are only **48** subcarriers carry data, then:

BPSK:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(2)}{4 \times 10^{-6}} = \frac{48}{4 \times 10^{-6}} = 12 \times 10^6 bps = 12 Mbps$

QPSK:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(4)}{4 \times 10^{-6}} = 48 \times \frac{2}{4 \times 10^{-6}} = 24 \times 10^6 bps = 24 Mbps$

16-QAM:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(16)}{4 \times 10^{-6}} = 48 \times \frac{4}{4 \times 10^{-6}} = 48 \times 10^6 bps = 48 Mbps$

64-QAM:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(64)}{4 \times 10^{-6}} = 48 \times \frac{6}{4 \times 10^{-6}} = 72 \times 10^6 bps = 72 Mbps$

(6) The percentage data rate reduction is:

$$DRR\% = \left(1 - \frac{N}{\mu + N}\right) \times \% = \left(1 - \frac{64}{16 + 64}\right) \times \frac{100}{100} = 20\%$$

Example 2: In an OFDM system, the maximum delay spread was 40μsec uses QPSK as baseband modulation (constellation mapping). It is required to transmit at overall OFDM period of 200μsec. If the Bandwidth of the system was 800KHz, and 8-subcarriers are reserved for zeros and synchronization, what will be the subcarrier spacing, transmitted rate, and the percentage data rate reduction?

Solution:

$$\tau = \frac{\mu}{B} \quad \rightarrow \quad \mu = \tau B = 40 \times 10^{-6} \times 800 \times 10^3 = 32\ subcarriers\ as\ CP$$

$$T_{total} = T_N + \mu T_s = T_N + \frac{\mu}{B}$$

$$200 \times 10^{-6} = T_N + \frac{32}{800 \times 10^3}$$

$$T_N = 0.00016 sec = 160 \mu sec$$

$$B = \frac{N}{T_N} \quad \rightarrow \quad N = T_N B = 800 \times 10^3 \times 160 \times 10^{-6}$$

$$N = 128 \; subcarriers \; in \; this \; OFDM \; system$$

$$subcarrier \; spacing \; \Delta f = \frac{B}{N} = \frac{800 \times 10^3}{128} = 6250Hz = 6.25KHz$$

Since there are 8-subcarriers reserved for zeros and synchronization, then there are

$$N_{carry \; data} = N - 8 = 128 - 8 = 120 \; carrier \; holds \; data$$

QPSK:- $R_b = 120 \; sub. \times \frac{\log_2(M)}{T_{total}} = 120 \times \frac{\log_2(4)}{200 \times 10^{-6}} = \frac{120 \times 2}{200 \times 10^{-6}} = 1.2 \times 10^6 bps = 1.2Mbps$

The percentage data rate reduction is:

$$DRR\% = \left(1 - \frac{N}{\mu + N}\right) \times \% = \left(1 - \frac{128}{32 + 128}\right) \times \frac{100}{100} = 25\%$$

Example 3: The Advanced Cellular Internet Service (**ACIS**) provides wide-area Internet service to mobile subscribers in **Europe**. This ACIS combines OFDM with multiple transmitter and receiver antennas and coding. The total working bandwidth is 800KHz, the tone spacing is 4.17KHz. If the guard interval was 48.5μsec, determine the total number of subcarriers, the OFDM symbol duration and the total system duration, and the data rate if 64-QAM mapping was employed assuming 64-tones reserved for channel estimation and other signaling operations.

Solution:

$$\Delta f = \frac{1}{T_N} \quad \rightarrow \quad T_N = \frac{1}{\Delta f} = \frac{1}{4.17 \times 10^3} = 240 \mu sec$$

$$B = \frac{N}{T_N} \quad \rightarrow \quad N = BT_N = 800 \times 10^3 \times 240 \times 10^{-6} = 192 \; subcarriers \; holding \; data$$

$$\tau = \frac{\mu}{B} \quad \rightarrow \quad \mu = \tau B = 4.85 \times 10^{-6} \times 800 \times 10^3 = 39 \; subcarriers \; as \; CP$$

$$B = \frac{1}{T_s} \quad \rightarrow T_s = \frac{1}{B} = \frac{1}{800 \times 10^3} = 1.25 \mu sec$$

$$T_{total} = T_N + \mu T_s = 240 \times 10^{-6} + 39 \times 1.25 \times 10^{-6} = 288.75 \mu sec$$

Example 4: The IEEE802.11(a) is the standard name of the wireless LAN (WLAN) for indoor applications. Its spectrum is around 5GHz. However, WLAN uses OFDM modulation scheme where the subcarrier modulation is either DBPSK or QPSK or 16-QAM. If there are 48 Subchannels out of 64 for data, and 16 samples as cyclic prefix, calculate:

1. The total number of samples associated with each OFDM symbol, including both data samples and the cyclic prefix,

2. The subcarrier bandwidth,
3. The maximum delay spread for which ISI is removed,
4. The symbol time per subchannel,
5. The data rate for this system, corresponding to DBPSK, QPSK, and 16-QAM,
6. Find the percentage Date Rate Reduction.

Solution:

(1) The cyclic prefix consists of $\boldsymbol{\mu}$ = 16 samples, so the total number of samples associated with each OFDM symbol, including both data samples and the cyclic prefix, is 64+16=80.

(2) The subcarrier bandwidth is:

$$B_N = \frac{B}{N} = \frac{5 \times 10^9}{64} = 78.125MHz$$

(3) The maximum delay spread ($\boldsymbol{\tau}$) for which ISI is removed is:

$$\tau < \mu T_s = \mu \frac{1}{B} = \frac{\mu}{B} = \frac{16}{5 \times 10^9} = 3.2 \times 10^{-9} sec = 3.2nsec$$

(4) The symbol time per subchannel is: We have both the data and CP, there are 80=64+16 samples per OFDM symbol, Thus the symbol time per subchannel:

$$T_{total} = T_N + \mu T_s = (N + \mu)T_s = 80T_s = \frac{80}{5 \times 10^9} = 16nsec$$

(5) The data rate for this system, corresponding to **DBPSK**, **QPSK**, and **16-QAM:**

The data rate per subchannel is $\frac{\log_2(M)}{T_{total}}$. Remember there are only **48** subcarriers carry data, then:

DBPSK:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(2)}{16 \times 10^{-9}} = \frac{48}{16 \times 10^{-9}} = 3 \times 10^9 bps = 3Gbps$

QPSK:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(4)}{16 \times 10^{-9}} = 48 \times \frac{2}{16 \times 10^{-9}} = 6 \times 10^9 bps = 6Gbps$

16-QAM:- $R_b = 48\ sub. \times \frac{\log_2(M)}{T_{total}} = 48 \times \frac{\log_2(16)}{16 \times 10^{-9}} = 48 \times \frac{4}{16 \times 10^{-9}} = 12 \times 10^9 bps = 12Gbps$

(6) The percentage data rate reduction is:

$$DRR\% = \left(1 - \frac{N}{\mu + N}\right) \times \% = \left(1 - \frac{64}{16 + 64}\right) \times \frac{100}{100} = 20\%$$