Department of Communications Engineering, College of Engineering, University of Diyala

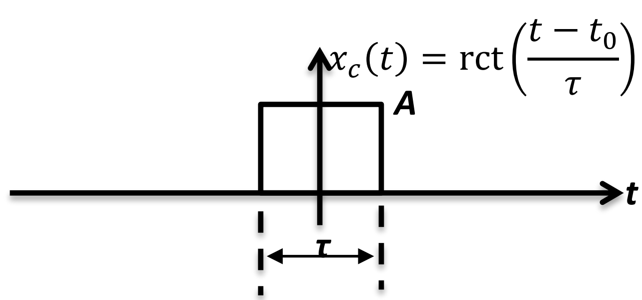
Digital Communication I

Lecture # 2

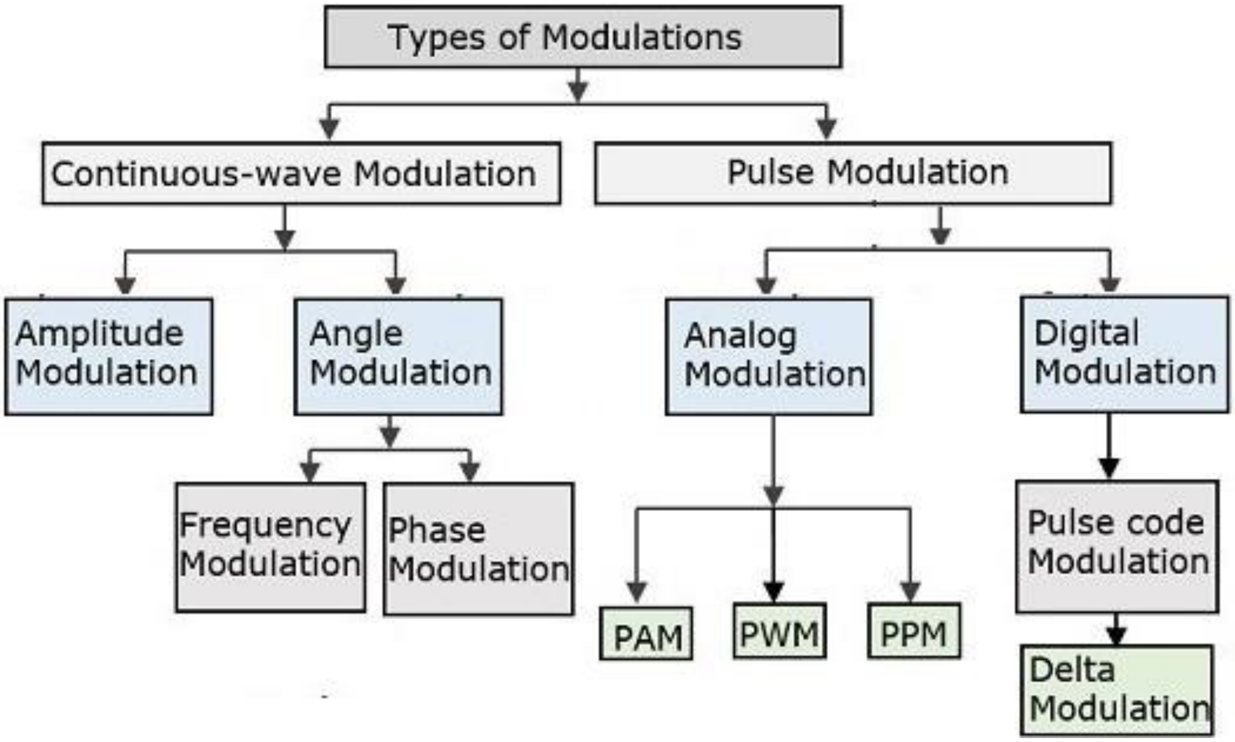
Pulse Modulation Techniques: PAM, PWM, PPM, Noise in Pulse modulations

Introduction:

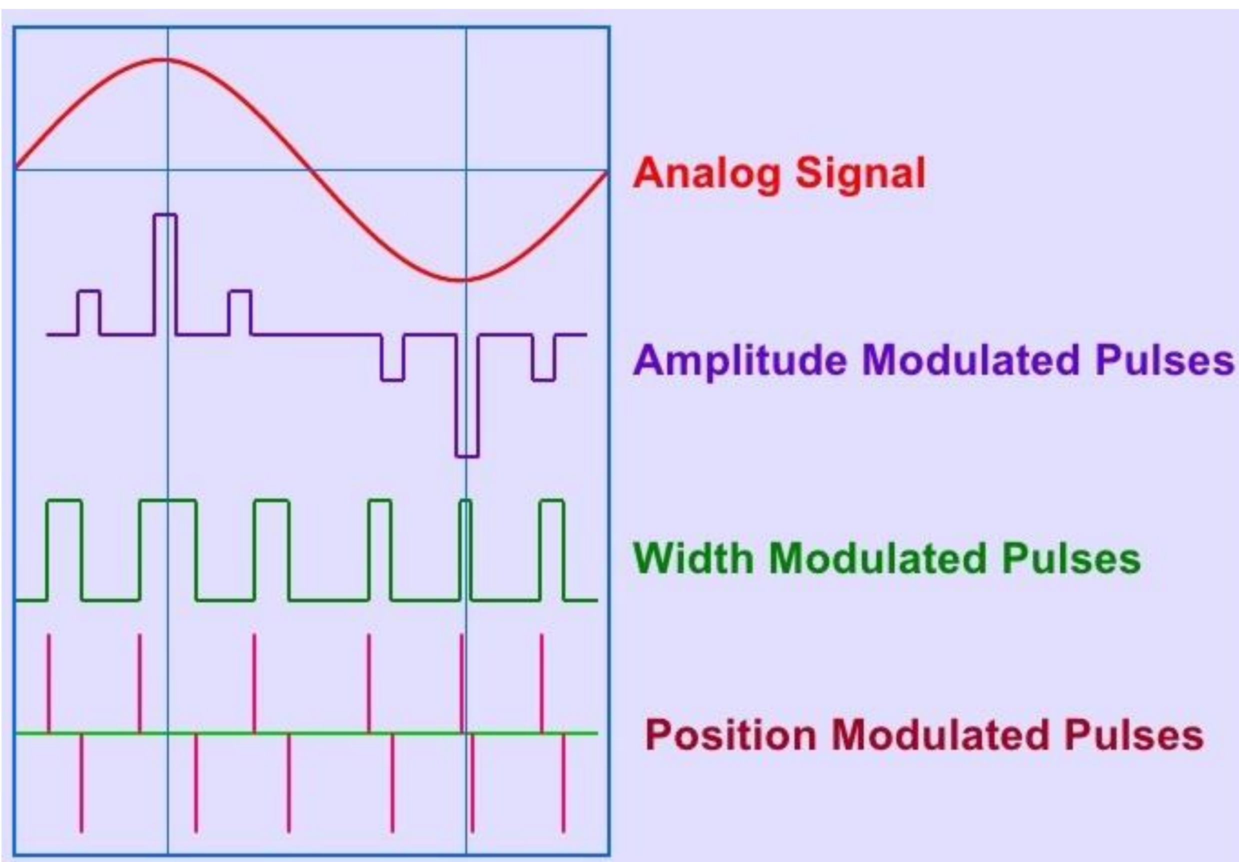
- We know that in analog modulation systems, some parameters of a sinusoidal carrier is varied according to the instantaneous value of the modulating signal.
- In pulse modulation methods, the carrier is no longer a continuous time signal.
- In pulse modulation, the carrier consists of a pulse train.
- Some parameters of the pulse are varied according to the instantaneous value of the modulating signal.



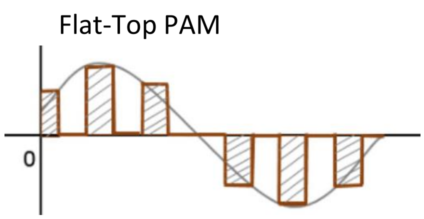
- There are three parameters can be changed in the pulse signal:
  1. The amplitude of the pulse A, this gives **P**ulse **A**mplitude **M**odulation (**PAM**).
  2. The width/duration of the pulse $\boldsymbol{\tau}$, this gives **P**ulse **W**idth/**D**uration **M**odulation (**PWM** or **PDM**).
  3. The position of the pulse $\boldsymbol{t_0}$, this gives **P**ulse **P**osition **M**odulation (**PPM**).
- In general, Pulse Modulation consists of two categories, analog modulation and digital modulation

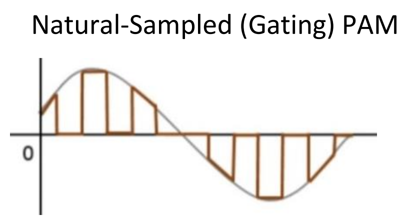


- In this lecture, we will study the Analog Pulse Modulation types.



1. Pulse Amplitude Modulation





- Flat-Top PAM is most popular and is widely used.
- The reason for using Flat-Top PAM is that during the transmission, the noise interferes with the top of the transmitted pulses and this noise can be easily removed if the PAM pulse has Flat Top.
- In case of natural samples PAM signal, the pulse has varying top in accordance with the signal variation. Then it becomes quite difficult to determine the shape of the top of the pulse and thus amplitude detection of the pulse is not exact due to noise.
- Due to this, errors are introduced in the received signal. Therefore, flat top sampled PAM is widely used.

Pulse Modulation

- Recall Fourier Series,
- Recall Fourier transform of periodic signal,
- Recall Sampling theorem.

However: Discrete time systems are:

i) Inexpensive ii) Light weight

iii) programmable iv) easily re-producible.

* Sampling Operation is the process to convert the continuous time signal to discrete time signal.

$\left\{ \begin{array}{l} \text{A band-Limited signal (message) of bandwidth } W \text{ Hz sampled} \\ \text{at } f_s \text{ can be reconstructed if } f_s \geq 2W \text{ Hz} \end{array} \right\}$

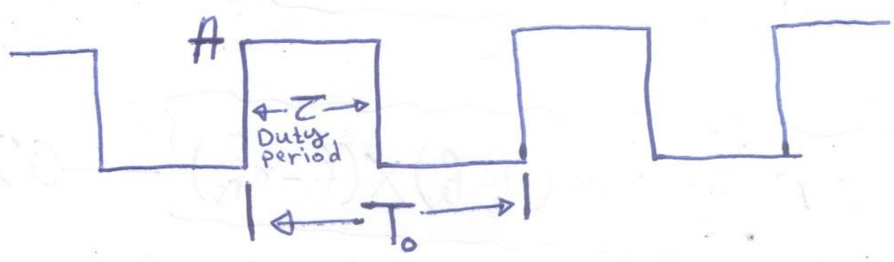
* The reconstruction filter must be of bandwidth B Hz in the range

$$W < B < (f_s - W)$$

* The $2W$ Hz is called the Nyquist frequency.

* Nyquist frequency is $f_s = f_N = 2W$.

* Nyquist period is $T_N = 1/f_N$.



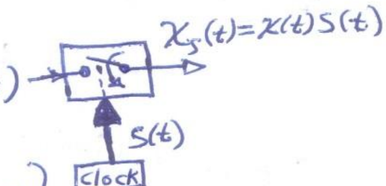
Pulse Amplitude Modulation (PAM):

* Is an analog modulating scheme in which the amplitude of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

→ There are Natural Sampling PAM (Gating)

→ and there is Flat-Top Sampling PAM.

1) Gating (Natural Sampling)



* Let $x(t)$ be a baseband bandlimited signal to B Hz.

$$x_s(t) = x(t)s(t) \quad \text{——} (1) \quad \left\{ \text{Gated, Naturally sampled} \right\}$$

where:
$$s(t) = \sum_{k=-\infty}^{\infty} rect\left(\frac{t-kT_s}{\tau}\right) \quad \text{——} (2)$$

$$f_s = \frac{1}{T_s} \geqslant 2B \ \text{Hz}$$

* The spectrum of the naturally sampled PAM is:

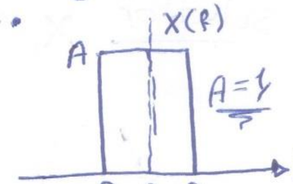
$$\boxed{X_s(f) = \frac{\tau}{T_s}\sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n\tau}{T_s}\right) X(f-nf_s)} \quad \text{——} (3)$$

OR

$$\boxed{X_s(f) = \tau f_s \sum_{n=-\infty}^{\infty} \text{sinc}(n\tau f_s) X(f-nf_s)} \quad \text{——} (3')$$

EX. 1 For an input waveform of rectangular spectrum of bandwidth B Hz, draw the natural sampled version of this input signal and calculate the PAM bandwidth if you know that the sampling frequency is 4B Hz and the duty cycle of the switching waveform is $d = \tau / T_s = 1/3$.

Solution: The i/p signal is $X(f) = A\,\text{rect}\left(\frac{f}{2B}\right)$



$$X_s(f) = \tau f_s \sum_{n=-\infty}^{\infty} \text{sinc}(n\tau f_s)\, X(f - f_s n)$$

We have $d = \frac{\tau}{T_s} = \tau f_s = \frac{1}{3}$

$$X_s(f) = \frac{1}{3} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n}{3}\right) X(f - n f_s)$$

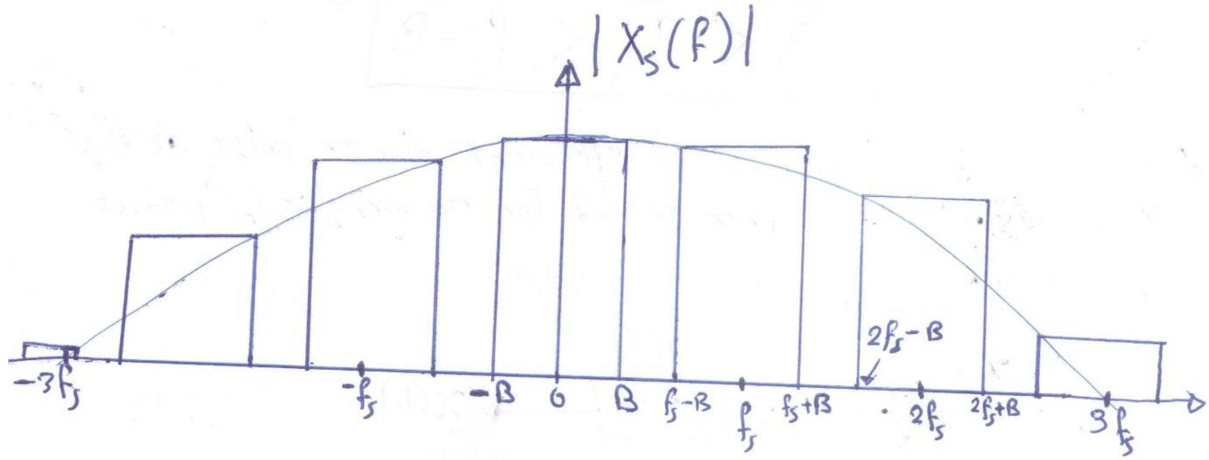
$$|X_s(f)| = \sum_{n=-\infty}^{\infty} \left\{ \left(d\, |\text{sinc}(nd)| \right) |X(f - nf_s)| \right\}$$

when it becomes (1)?

$\therefore\ BW = 3f_s = 3 \times 4B = 12B$ Hz

it becomes (1) when n=3

$\therefore\ BW = 3f_s$.



<u>EX.2</u> Repeat EX.1 but use $d = 0.5$ and $f_s = 4B$ Hz.
Not need to draw, calculate the Bandwidth only.

<u>solution</u> $X_s(f) = \tau f_s \sum_{n=-\infty}^{\infty} \text{sinc}(n\tau f_s)\, X(f - nf_s)$

$$d = \frac{\tau}{T_s} = \tau f_s = 0.5$$

$$\therefore X_s(f) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{n}{2}\right) X(f - nf_s)$$

to get first zero, $n = 2$

$$\therefore BW = 2f_s = 2 * 4B = 8B \text{ Hz}$$

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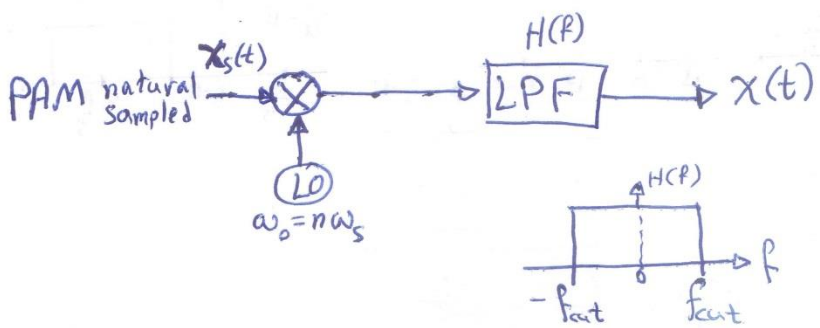
* Recovery of natural sampled PAM is by Low pass filter with cut-off bandwidth of

$$\boxed{B < f_{cut} < f_s - B}$$

* However, just LPF may not efficient due to noise at high frequency. Thus, another method for recovery is the product detection.



$B < f_{cut} < f_s - B$

2) Flat-Top Sampling PAM :

- $x(t)$ is an analog bandlimited to B Hz
- The Flat-Top PAM signal will be

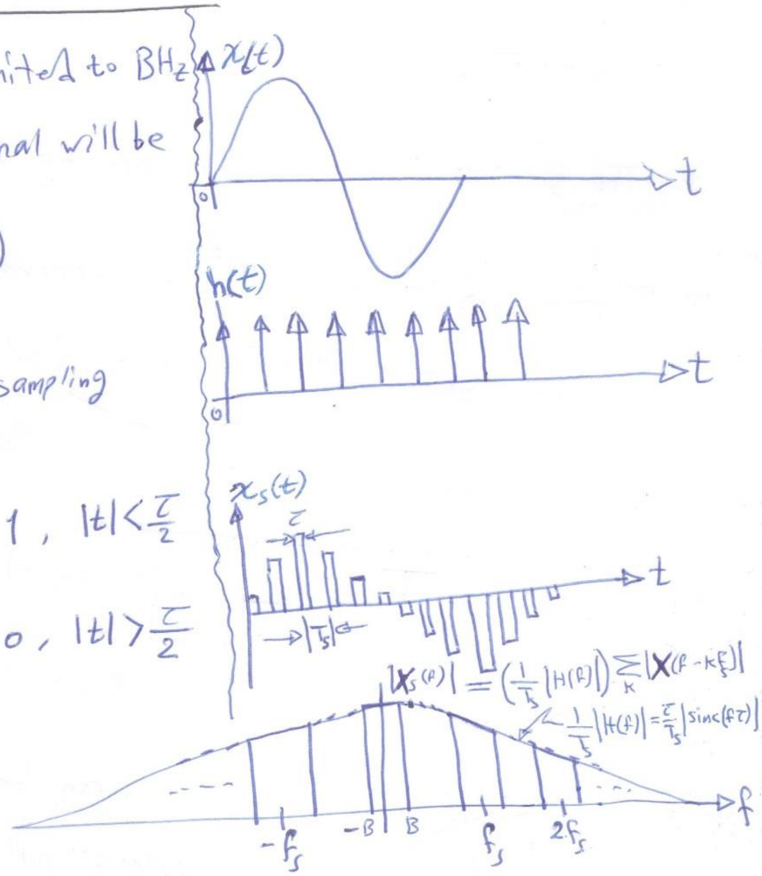
$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, h(t - kT_s)$$

$h(t)$ is the shape of the sampling waveform,

$$h(t) = \text{rect}\left(\frac{t}{\tau}\right) = \Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$

where $\tau \leq T_s = \frac{1}{F_s}$

$F_s \geq 2B$



Hence : $X_s(f) = \frac{1}{T_s} H(f) \sum_{k=-\infty}^{\infty} X(f - kf_s)$

where $H(f) = \tau \,\text{sinc}(\tau f)$

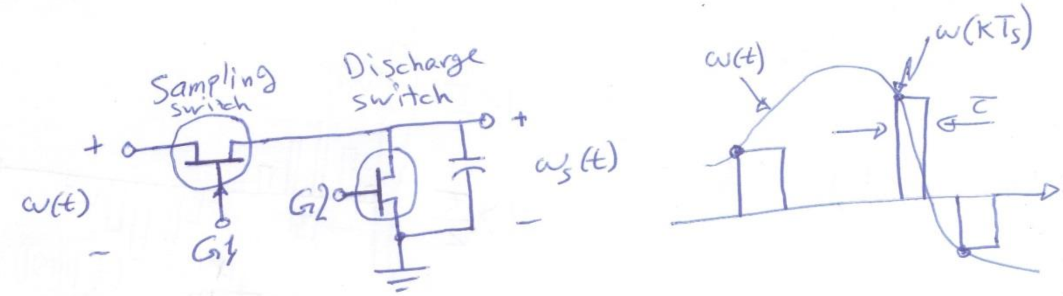
$$\therefore X_s(f) = \frac{\tau}{T_s}\, \text{sinc}(\tau f) \sum_{k=-\infty}^{\infty} X(f - kf_s)$$

$$X_s(f) = \tau F_s\, \text{sinc}(\tau F) \sum_{k=-\infty}^{\infty} X(f - kf_s)$$

* Flat-Top PAM consists of instantaneous samples at $t = kT_s$,

* The sample values are determined by $w(kT_s)$

* Flat-top PAM can be generated using a circuit called Sample-and-Hold (S/H).



* To recover the signal:- LPF can be used.

* Using LPF to demodulate the PAM signal will cause losses in the high frequency because of the LPF $H(f)$.

* To reduce the effect of $H(f)$, τ can be reduced, or by using an equalization filter of transfer function $\frac{1}{H(f)}$

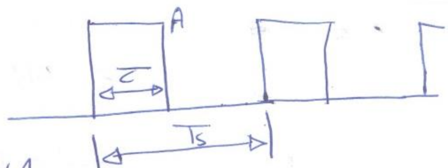
* Another method to recover $w(t)$ from $w_s(t)$ (demodulation) is by using the product detection.

* Because of a rectangular basis of the PAM, the bandwidth is very large.

* PAM can not be used for long distances.

Example: Proof that the bandwidth of any type of PAM is $BW \leq \frac{1}{2\tau}$ where $\boldsymbol{\tau}$ is the duty cycle of the pulse.

Solution



a signal to be sampled

is $x(t)$ with bandwidth B Hz

$\therefore f_s = 2B$ Hz

* every T_s second only τ is selected from the analog continuous time signal

$\therefore T_s = \frac{1}{2B}$.

since $\tau \leq T_s$

$\therefore \tau \leqslant \frac{1}{2B}$

$\therefore \boxed{BW \leqslant \frac{1}{2\tau}}$

Q / For an input waveform of rectangular spectrum with bandwidth B Hz. Calculate the PAM bandwidth if the sampling frequency f_s is 4B Hz and $\tau = T_s/3$.

Solution

$\tau = \frac{T_s}{3}$, $f_s = 4B$ Hz

$T_s = \frac{1}{f_s} = \frac{1}{4B}$ seconds

$\tau = \frac{T_s}{3} = \frac{1/4B}{3} = \frac{1}{12B}$

$\therefore \boxed{BW = \frac{1}{12\tau} \text{ Hz}}$

Q/ consider PAM transmission of a voice signal with $W \cong 3$ kHz. calculate total bandwidth if $F_s = 8$ kHz and $\tau = 0.1 T_s$.

<u>Solution</u> Given $W = 3$ kHz, $F_s = 8$ kHz, $\tau = 0.1 T_s$ sec.

we know $\tau \leq T_s$

and we know $T_s = \frac{1}{F_s}$

$$F_s = 2B$$

$$\therefore T_s = \frac{1}{2B}$$

$$\therefore \tau \leq \frac{1}{2B}$$

$$\therefore \boxed{B = \frac{1}{2\tau}}$$

since $\tau = 0.1 T_s$

$$\tau = \frac{0.1}{F_s}$$

$$\therefore B = \frac{1}{2\frac{0.1}{F_s}} = \frac{F_s}{0.2}$$

$$\therefore B = \frac{F_s}{0.2} = \frac{8000}{0.2}$$

$$\boxed{B = 40 \text{ kHz}}$$

Q/ pulse-amplitude modulated waveform $g(t) = \sin(2\pi \frac{1}{T_m} t)$, where $T_m = 0.2$ seconds, sampled at $f_s = 100$ Hz, calculate the total transmission bandwidth if the duty cycle was $0.25 T_s$.

Solution Given $\tau = 0.25 T_s$ seconds

$$F_s = 100 \text{ Hz}$$

$$B \leqslant \frac{1}{2\tau} = \frac{1}{2 \times 0.25 T_s} = \frac{F_s}{0.5} = \frac{100}{0.5} = 200 \text{ Hz}.$$

Q/ A rectangular PAM signal $g(t) = 2\,\text{rect}(t)$, sampling frequency was $F_s = 150$ Hz. The sampling duty cycle was 10% of the sampling period, what is the transmission bandwidth?

solution

Given $F_s = 150$ Hz, $\tau = 0.1 T_s$

$$B \leqslant \frac{1}{2\tau} = \frac{1}{2 \times 0.1 T_s} = \frac{F_s}{0.2} = \frac{150}{0.2}$$

$$B = 750 \text{ Hz}.$$

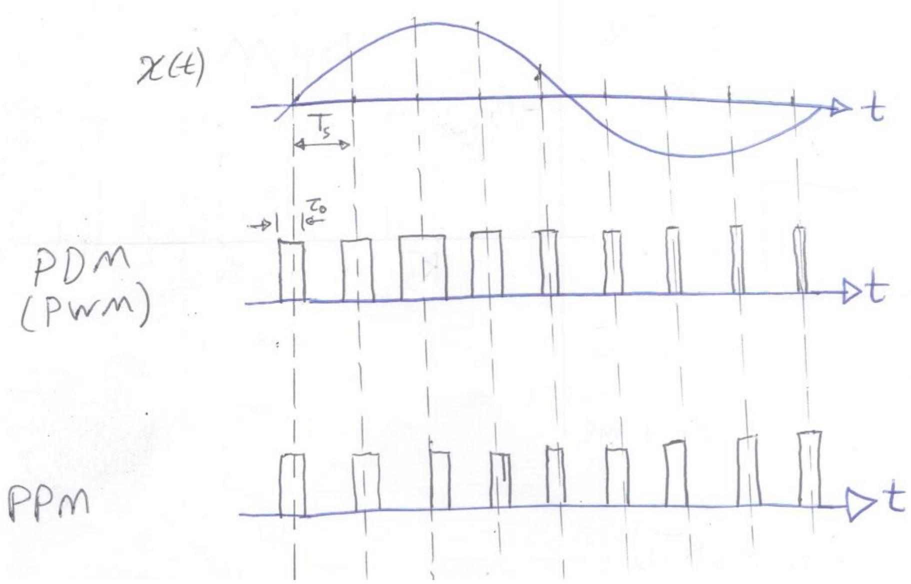
Pulse-Time Modulation

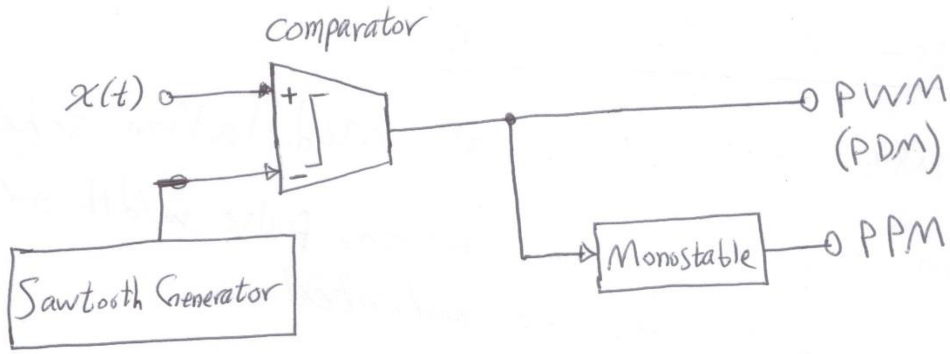
* Just like traditional modulation schemes; in pulse-time modulation, pulse width and pulse position can also be modulated.

* Two types of pulse-time modulation:

① pulse width modulation (PWM) also called pulse duration modulation (PDM)

② Pulse-Position Modulation (PPM)





* The output of the comparator is zero except when the message waveform $x(t) >$ the sawtooth waveform.

Thus the output is constant positive A

* Assuming uniform sampling: The pulse duration in PWM is

$$\boxed{\tau_K = \tau_0\left[1 + \mu\, x(KT_s)\right]}$$

*In PPM, pulse width and amplitude are constants.

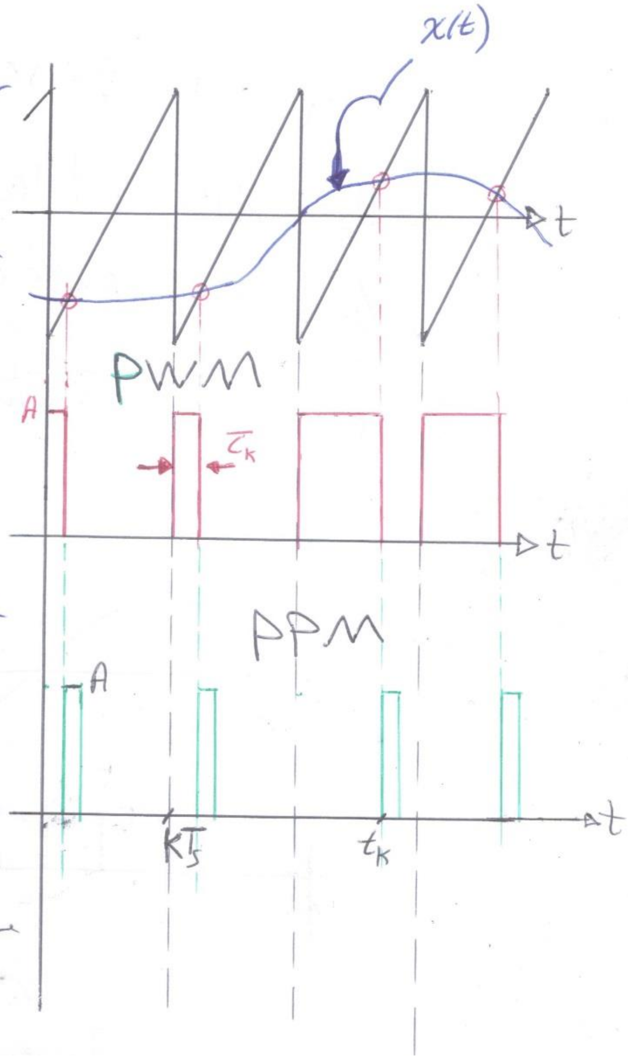
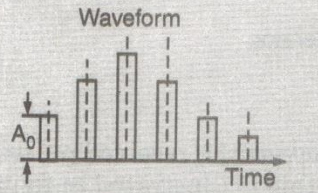
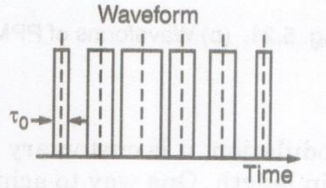
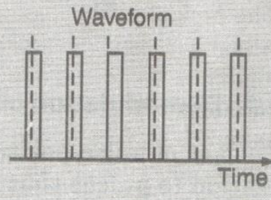


TABLE 5.1. Performance Comparison of PAM, PPM and PDM

S. No.	Pulse Amplitude Modulation (PAM)	Pulse Width/Duration Modulation (PWM) or (PDM)	Pulse Position Modulation (PPM)
1.			
2.	Amplitude of the pulse is proportional to amplitude of modulating signal.	Width of the pulse is proportional to amplitude of modulating signal.	The relative position of the pulse is proportional to the amplitude of modulating signal.
3.	The bandwidth of the transmission channel depends on width of the pulse.	Bandwidth of transmission channel depends on rise time of the pulse.	Bandwidth of transmission channel depends on rising time of the pulse.
4.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter remains constant.
5.	Noise interference is high. System is complex	Noise, interference is minimum.	Noise, interference is minimum
6.	Similar to amplitude modulation.	Simple to implement similar to frequency modulation.	Simple to implement similar to phase modulation.

SUMMARY

- There are two types of signals, continuous time signal and discrete-time signals.
- Due to some recent advance development in digital technology over the past few decades, the inexpensive, light weight, programmable and easily reproducible discrete-time systems are available. Therefore, the processing of discrete-time signals is more flexible and is also preferable to processing of continuous-time signals.
- The sampling theorem is extremely important and useful in signal processing.
- With the help of sampling theorem, a continuous-time signal may be completely represented and recovered from the knowledge of samples taken uniformly.
- The concept of sampling provides a widely used method for using discrete-time system technology to implement continuous-time systems and process the continuous-time signals.
- Sampling of the signals is the fundamental operation in signal-processing. A continuous time signal is first converted to discrete-time signal by sampling process.
- The sufficient number of samples of the signal must be taken so that the original signal is represented in its samples completely. Also, it should be possible to recover or reconstruct the original signal completely from its samples. The number of samples to be taken depends on maximum signal frequency present in the signal.
- A continuous-time signal may be completely represented in its samples and recovered back if the sampling frequency is $f_s \geq 2 f_m$. Here f_s is the sampling frequency and f_m is the maximum frequency present in the 'signal.

- When the sampling rate becomes exactly equal to 2 f_m samples per second, then it is called Nyquist rate. Nyquist rate is also called the minimum sampling rate. It is given by

$$f_s = 2 f_m$$

- Maximum sampling interval is called Nyquist interval. It is given by

$$\text{Nyquist Interval} \qquad T_s = \frac{1}{2f_m} \text{ seconds.}$$

- The low pass filter is used to recover original signal from its samples. This is also known as interpolation filter.
- A low-pass filter is that type of filter which passes only low-frequencies upto a specified cut-off frequency and rejects all other frequencies above cut-off frequency.
- The process of reconstructing a continuous-time signal x(t) from its samples is called as interpolation.
- Aliasing is the phenomenon in which a high frequency component in the frequency-spectrum of the signal takes identity of a lower-frequency component in the spectrum of the sampled signal.
- Because of the overlap due to aliasing phenomenon, it is not possible to recover original signal x(t) from sampled signal g(t) by low-pass filtering since the spectral components in the overlap regions add and hence the signal is distorted.
- Since any information signal contains a large number of frequencies, so, to decide a sampling frequency is always a problem. Therefore, a signal is first passed through a low-pass filter. This low-pass filter blocks all the frequencies which are above f_mHz. This process is known as band limiting of the original signal x(t). This low-pass filter is called prealias filter because it is used to prevent aliasing effect.
- To avoid aliasing, we must have :
  - (i) Prealias filter must be used to limit band of frequencies of the signal to f_mHz.
  - (ii) Sampling frequency 'f_s' must be selected such that

$$f_s > 2 f_m.$$

- The bandpass signal x(t) whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth. Here, f_m is the maximum frequency component present in the signal.
- In analog modulation systems, some parameter of a sinusoidal carrier is varied according to the instantaneous value of the modulating signal. In pulse modulation methods, the carrier is no longer a continuous signal but consists of a pulse train. Some parameter of which is varied according to the instantaneous value of the modulating signal.
- There are two types of pulse modulation systems as under :
  - (i) Pulse Amplitude Modulation (PAM)
  - (ii) Pulse Time Modulation (PTM)
- In pulse amplitude modulation (PAM), the amplitude of the pulses of the carrier pulse train is varied in accordance with the modulating signal whereas in Pulse time modulation (PTM), the timing of the pulses of the carrier pulse train is varied.
- There are two types of PTM :
  - (i) Pulse width modulation (PWM)
  - (ii) Pulse position modulation (PPM).
- In Pulse width modulation, the width of the pulses of the carrier pulse train is varied in accordance with the modulating signal whereas in Pulse position modulation (PPM), the position of pulses of the carrier pulse train is varied.
- It may be noted that all the above pulse modulation methods (i.e., PAM, PWM and PPM) are called analog Pulse modulation methods because the modulating signal is analog in nature in PAM, PWM and PPM.

GLOSSARY

1. **Aliasing :** Distortion created by using too low a sampling rate when coding an analog signal for digital transmission.
2. **Digital Signal Processing (DSP) :** Filtering of signals by converting them to digital form, performing arithmetic operations on the data bits, then converting back to analog form.
3. **Flat-topped Sampling :** Sampling of an analog signal using a sample-and-hold circuit, such that the sample has the same amplitude for its whole duration.

4. **Foldover Distortion :** See aliasing
5. **Pulse-Amplitude Modulation (PAM) :** A series of pulses in which the amplitude of each pulse represents the amplitude of the information signal at a given time.
6. **Pulse-Duration Modulation (PDM) :** A series of pulses, in which the duration of each pulse represents the amplitude of the information signal at a given time.
7. **Pulse-Position Modulation (PPM) :** A series of pulses, in which the timing of each pulse represents the amplitude of the information signal at a given time.
8. **Sample and Hold Circuit :** A device that detects the amplitude of an input signal at a particular time called the sampling time and maintains its output at or near that amplitude until the next sampling time.

SHORT QUESTIONS WITH ANSWERS

Q. 1. What is sampling ?

Ans. The process of converting an analog signal into a discrete signal or making an analog or continuous signal to occur at a particular interval of time is known as sampling.

Q. 2. What do you mean Nyquist rate ?

Ans. $f_s \geq 2f_m$, i.e., sampling rate greater than or equal to twice of the message bandwidth or highest frequency component of the message signal (f_m) is called Nyquist rate.

Q. 3. What is Guard band ?

Ans. When the sampling rate is chosen much higher than the Nyquist rate then a small space occurs between the samples. This space is said to be Guard band. This is the desired one for sampling the signals.

Q. 4. What is Sampling Theorem ?

Ans. Let m(t) be a signal which is bandlimited such that its highest frequency spectral component is f_M. Let the values of m(t) be determined at regular intervals separated by times $T_s \leq 1/2f_M$, i.e., the signal is periodically sampled every T_s seconds. Then there samples $m(nT_s)$ where n is an integer, uniquely determine the signal, and the signal may be reconstructed from these samples with no distortion.

Q. 5. What do you mean by aperture effect ?

Ans. During flat top sampling, to convert varying amplitudes of pulse to flat top pulses we use a sin c function. Because of this, there would be decrease in the amplitude. This distortion is named as Aperture effect.

This may be eliminated by using an equalizer in cascade with the output low pass filter.

Q. 6. What is Pulse Amplitude Modulation (PAM) ?

Ans. The process in which amplitudes of regularly spaced rectangular pulses vary with the instantaneous sample values of a continuous message signal in a one to one fashion is known as pulse amplitude modulation.

Q. 7. What do you mean by Pulse-time Modulation ?

Ans. The modulation technique in which the time (or) duration of the pulses is varied in accordance with the amplitude of the message signal keeping the amplitude of the pluses constant is referred to as pulse-time modulation.

Q. 8. What are the different types of PTM systems ?

Ans. There are two kinds of Pulse-time modulation schemes. They are :

(i) Pulse duration (or) pulse width (or) pulse length modulation (PDM (or) PWM (or) PLM).

(ii) Pulse position modulation.

Q. 9. What is Pulse duration modulation (PDM)?

Ans. The method in which the samples of the message signal are used to vary the duration (or) width of the individual pulses. This is referred to as pulse duration modulation.

Q. 10. What is pulse position modulation (PPM)?

Ans. In PPM, the position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal.

REVIEW QUESTIONS

1. State and prove sampling theorem in time domain.
2. What is Nyquist rate and Nyquist interval ?
3. A bandlimited signal x(t) is sampled by a train of rectangular pulses of width τ and period T.
   (i) Find an expression for the sampled signal.
   (ii) Determine the spectrum of the sampled signal and sketch it.
4. What is aliasing and how it is reduced ?

NUMERICAL PROBLEMS

1. Determine the Nyquist sampling rate and the Nyquist sampling interval for the following signals:
   (a) $\sin c(100\,\pi t)$
   (b) $\sin c^2(100\pi t)$
   (c) $\sin c(100\pi t) + \sin c\,(50\pi t)$
   (d) $\sin c(100\pi t) + 3\sin c^2\,(60\pi t)$
   (e) $\sin c(100\pi t)\sin c\,(100\pi t)$
2. A signal g(t) band-limited to B Hz is sampling by a periodic pulse train $pT_s(t)$ made up of a rectangular pulse of with $\frac{1}{8}B$ seconds (centred at the origin) repeating at the Nyquist rate (2B pulses per second).

   Show that the sampled signal $\bar{g}(t)$ is given by

$$\bar{g}(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty}\frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right)g(t)\cos n u_s t$$

$$\omega_s = 4\pi B$$

   Show that the signal g(t) can recovered by passing $\bar{g}(t)$ through an ideal low-pass filter of bandwidth B Hz and a gain of 4.
3. Signal $g_1(t) = 104\ \text{rect}\ (104t)$ and $g_2(t) = \delta(t)$ are applied at the inputs of ideal low-pass filters $H_1(\omega) = \text{rect}\ (\omega/40,000\pi)$ and $H_2(\omega) = \text{rect}\ (\omega/20,000\pi)$ (Figure 5.24). The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)\,y_2(t)$. Find the Nyquist rate of $y_1(t)$, $y_2(t)$, and $y(t)$.
   (GATE Examination, 1998)

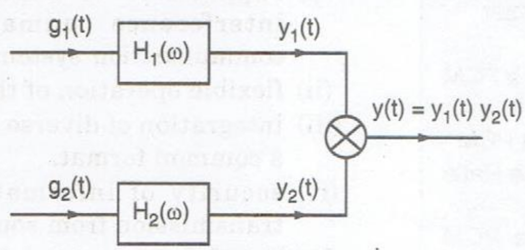


Fig. 5.24.

4. A signal g(t) $\sin c^2\,(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of : (i) 5 Hz ; (ii) 10 Hz ; (iii) 20 Hz. Now, for each of the three cases :
   (a) Sketch the sampled signal.
   (b) Sketch the spectrum of the sampled signal.
   (c) Explain whether you can recover the signal g(t) from the sampled signal.
   (d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5Hz, sketch the spectrum of the output signal.