Department of Communications Engineering, College of Engineering, University of Diyala

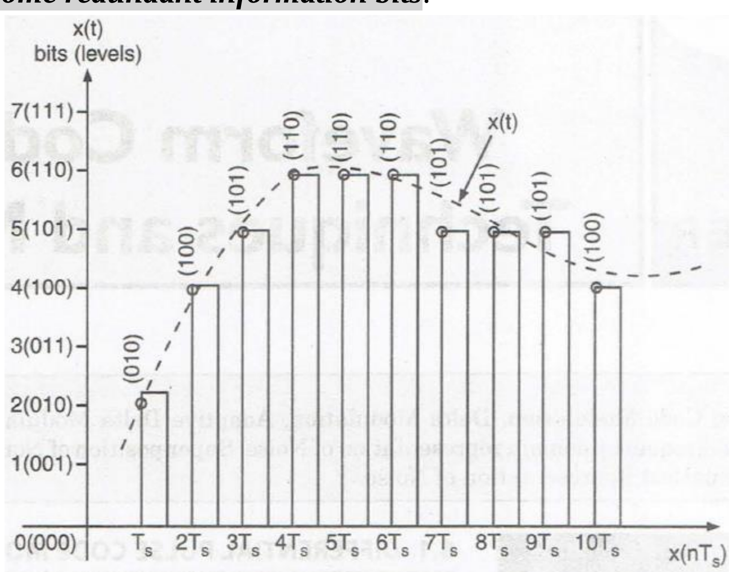
Digital Communication I

Lecture #4

Differential-PCM and Delta Modulation (DM)

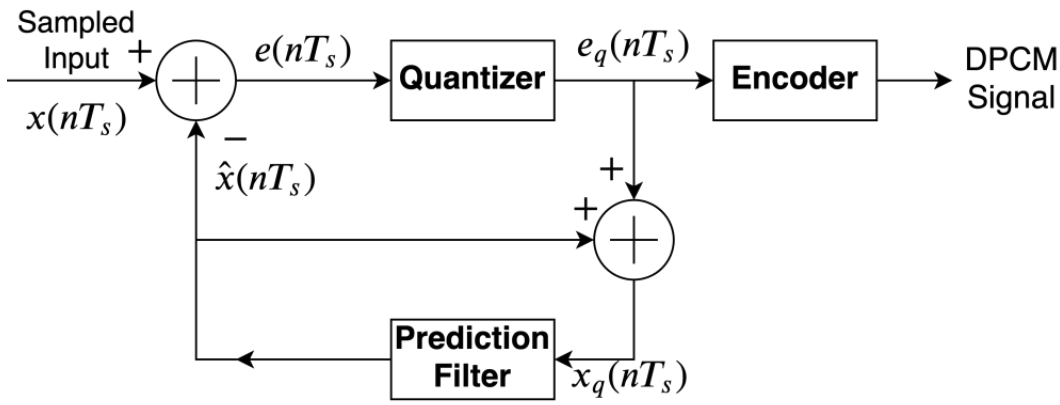
Differential-PCM (DPCM), Delta Modulation (DM), Adaptive-DM

- Differential pulse code modulation is a technique of analog to digital signal conversion.
- This technique samples the analog signal and then *quantizes the difference between the sampled value and its predicted value*,
- Then encodes the signal to form a digital value.
- We have to know the demerits of PCM.
    - The samples of a signal are ***highly correlated with each other***.
    - The signal's value from the present sample to next sample ***does not differ by a large amount.***
    - The *adjacent samples* of the signal ***carry the same information with a small difference***.
    - When these samples are encoded by the standard PCM system, the resulting encoded signal ***contains some redundant information bits***.
- The aside figure illustrates this.



- ✓ The figure shows a continuing time signal $x(t)$ denoted by a dotted line.
- ✓ This signal is sampled by *flat top sampling* at intervals $T_s, 2T_s, 3T_s \dots nT_s$. $F_s > 2F_{max}$
- ✓ Just observe the above figure at samples taken at $4T_s, 5T_s, and\ 6T_s$ are encoded to the same value of (110).
- ✓ *This information can be carried only by one sample value*.
- ✓ However, ***three samples are carrying the same information means redundant***.

- ✓ Now let consider the samples at $9T_s$ and $10T_s$,
- ✓ The difference between these samples only due to the ***last bit*** while ***first two bits are redundant since they do not change***.
- ✓ So in order to make the process this redundant information and to have a better output, it is an intelligent decision to *take a predicted sampled value*, assumed from its previous output and summaries them with the quantized values.
- ✓ Such a process is called as **Differential PCM (DPCM)** technique.
- ✓ If the redundancy is reduced, then the overall bit-rate will decrease and the number of bits required to transmit one sample will reduce.



- In fact, the differential pulse code modulation works on the principle of prediction.
- The value of the present sample is predicted from the past samples.
- The prediction may not be exact but it is very close to the actual sample value.
- The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicated sample value $\hat{x}(nT_s)$.
- This is known as Prediction error and it is denoted by $e(nT_s)$.

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \tag{8.1}$$

The quantizer output signal gap $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. ***This makes the prediction more and more close to the actual sampled signal***.

We can observe that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus, number of bits per sample are reduced in DPCM.

Reconstruction of DPCM Signal:

- The decoder first reconstructs the quantized error signal from incoming binary signal.

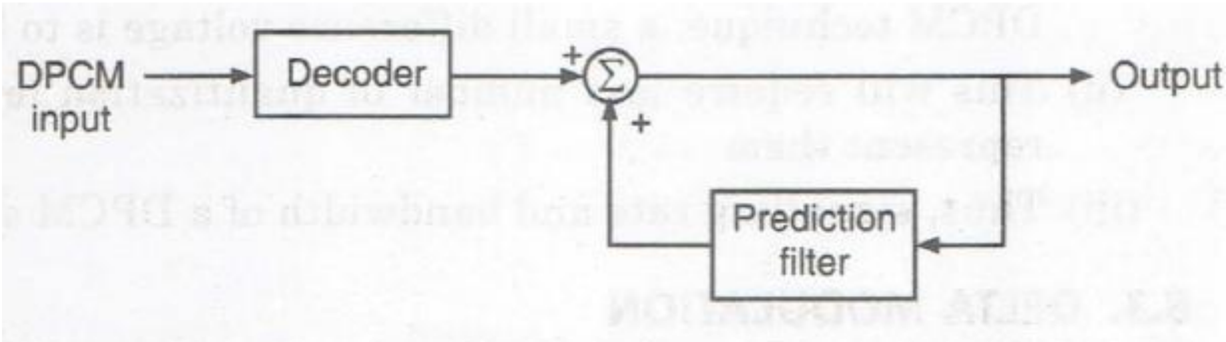


Fig. 8.3. DPCM receiver

DPCM Signal to Noise (S/N) Ratio

$$SNR = \frac{Mean\ Square\ Value\ of\ Signal}{Mean\ Square\ Value\ of\ Quantization\ Noise}$$

$$Mean\ Square\ Value = variance = \sigma^2$$

$$SNR = \frac{\sigma_X^2}{\sigma_Q^2}$$

where, σ_X^2 is the variance of original input signal $x(nT_s)$ and σ_Q^2 is the variance of the quantization error $q(nT_s)$, then

$$SNR = \frac{\sigma_X^2}{\sigma_E^2} \times \frac{\sigma_E^2}{\sigma_Q^2}$$

where σ_E^2 is the variance of the prediction error $e(nT_s)$, therefore,

$$SNR = G_p (SNR)_p$$

Where $G_p = \frac{\sigma_X^2}{\sigma_E^2}$ is the prediction gain and $(SNR)_p = \frac{\sigma_E^2}{\sigma_Q^2}$ is the prediction error-to-quantization noise ratio.

The prediction gain must be as high as possible. For a given baseband signal, the variance σ_X^2 is fixed. Hence to maximize G_p, we have to minimize the variance σ_E^2 of the prediction error $e(nT_s)$. The predictor must be designed accordingly.

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Quiz 7 – March 17, 2015

Open Book/Open Notes/10 minutes

You must circle or box your answers for full credit.

PROBLEM 1 (10 points): Consider the signals

$$f(t) = 2000sinc(2000\pi t)$$

and

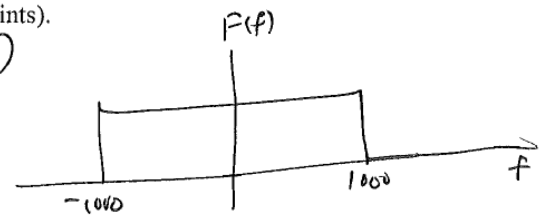
$$g(t) = 3000sinc^2(3000\pi t)$$

(a) Find the Nyquist sampling rate f_s for $f(t)$) (3 points).

Pair 18: $2B \operatorname{sinc}(2\pi B t) \Leftrightarrow \Pi\left(\frac{f}{2B}\right)$

$2000 \operatorname{sinc}(2000\pi t) \Leftrightarrow \Pi\left(\frac{f}{2000}\right)$

$B = 1000\,Hz$



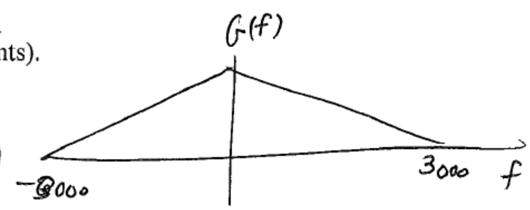
$f_s \geq 2B = 2(1000) = 2000\ Hz$

(b) Find the Nyquist sampling rate f_s for $g(t)$ (3 points).

Pair 20: $B \operatorname{sinc}^2(\pi B t) \Leftrightarrow \Delta\left(\frac{f}{2B}\right)$

$3000 \operatorname{sinc}^2(3000\pi t) \Leftrightarrow \Delta\left(\frac{f}{6000}\right)$

$B = 3000\,Hz$



$f_s \geq 2B = 2(3000) = 6000\ Hz$

(c) What is the Nyquist sampling rate for $f(t) + g(t)$ (use the bandwidth rule for the sum of two functions to solve)? (2 points)

$B = \max(B_f, B_g) = 3000\ Hz$

$f_s \geq 2B = 2(3000) = 6000\ Hz$

(d) What is the Nyquist sampling rate for $f(t)g(t)$ (use the bandwidth rule for the time product of two functions to solve)? (2 points)

$B = B_f + B_g = 1000 + 3000 = 4000$

$f_s \geq 2B = 2(4000) = 8000\ Hz$

Name key

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Quiz 8 – March 24, 2015

Open Book/Open Notes/10 minutes

You must circle or box your answers for full credit.

PROBLEM 1 (10 points): A signal has a bandwidth of 10 MHz. The signal is sampled, quantized, and binary coded to obtain a pulse-code modulated (PCM) signal. The signal is sampled at the Nyquist rate.

(a) What is the Nyquist rate (2 points)?

$$f_s = 2B = 2(10\times10^6) = 20\text{ MHz}$$

(b) If the samples are to be encoded into 128 levels, what is the number of binary pulses (bits) required to encode each sample (2 points)?

$$2^n = 128$$
$$n = \log_2(128) = \frac{\ln(128)}{\ln(2)} = 7\text{ bits}$$

(c) Based on your answers to parts (a) and (b), what is the minimum binary pulse rate (bits per second) of the binary-coded signal (3 points)?

$$7(20\times10^6) = 140\ \frac{\text{M bits}}{\text{second}}$$

(d) Using the knowledge that 2 bits can be transmitted per second over a 1 Hz bandwidth, determine the minimum transmission bandwidth B_T that can be used to successfully transmit this signal (3 points).

$$B_T = nB = 7(10\times10^6) = 70\text{ MHz}$$

Name KEY

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Quiz 9 – March 31, 2015

Open Book/Open Notes/10 minutes

You must circle or box your answers for full credit.

PROBLEM 1 (10 points): Differential pulse code modulation (DPCM) can be used to encode the message

$$m(t) = 4\cos(1000\pi t)$$

Sampling is performed with $T_s = 1 \times 10^{-4}$ seconds. For all parts of this problem, if the sampled DPCM analog value exceeds d_p or is below $-d_p$, place it in the extreme bin closest to the value.

(a) (2 points) What are the first four analog sample values $m[1], m[2], m[3]$, and $m[4]$?

$m[1] = m(1\times10^{-4}) = 3.80423$
$m[2] = m(2\times10^{-4}) = 3.23607$
$m[3] = m(3\times10^{-4}) = 2.35114$
$m[4] = m(4\times10^{-4}) = 1.23607$

(b) (4 points) If DPCM is used with 2-bit quantization, $d_p = 2$, and the bin ranges being inclusive upward, find the bitstream representing the first four samples. Use the 2-bit PCM quantization of the actual value with $m_p = 4$ for the first two-bit word, followed by the DPCM representations to find the second, third, and fourth samples. Use the predictor $\hat{m}[k] = m[k-1]$.

— 4
11 — 3
— 2
— 1
— 0
— -1
— -2
— -3
— -4

$m_q[1] = 3 \Rightarrow \boxed{11}$

$d[2] = m[2] - m[1]$
$= 3.23607 - 3.80423$
$\simeq -0.56816 \Rightarrow \boxed{01}$

$d[3] = m[3] - m[2]$
$= 2.35114 - 3.23607$
$\simeq -0.88493 \Rightarrow \boxed{01}$

$d[4] = m[4] - m[3]$
$= 1.23607 - 2.35114$
$\simeq -1.11507 \Rightarrow \boxed{00}$

— 2
11
— 1
10
— 0
01
— -1
00
— -2

11010100

(continued on next page)

(c) (4 points) For the bitstream you obtained in part (b), perform decoding of the this bitstream at the receiver. Provide the four quantized signal values $m_q[1], m_q[2], m_q[3], m_q[4]$ in decimal representation that would be obtained by using DPCM and the predictor $\widehat{m_q}[k] = m_q[k-1]$. For decoding assume that the first word (2 bits) is PCM representation of the first value with $m_p = 4$, and that the following words are DPCM with $d_p = 2$. Find the values of the quantization error at each sample: $q[1], q[2], q[3], q[4]$.

11 01 01 00

$m_q[1] = 3$

$d_q[2] = -0.5$

$d_q[3] = -0.5$

$d_q[4] = -1.5$

$m_q[k] = m_q[k-1] + d_q[k]$

$m_q[2] = m_q[1] + d_q[2] = 3 - 0.5 = 2.5$

$m_q[3] = m_q[2] + d_q[3] = 2.5 - 0.5 = 2.0$

$m_q[4] = m_q[3] + d_q[4] = 2.0 - 1.5 = 0.5$

$q[1] = m_q[1] - m[1] = 3 - 3.80423 = -0.80423$

$q[2] = m_q[2] - m[2] = 2.5 - 3.23607 = -0.73607$

$q[3] = m_q[3] - m[3] = 2.0 - 2.35114 = -0.35114$

$q[4] = m_q[4] - m[4] = 0.5 - 1.23607 = -1.28393$

Name KEY

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Quiz 10 – April 9, 2015

Open Book/Open Notes/10 minutes

You must circle or box your answers for full credit.

PROBLEM 1 (10 points): Consider a pulse shape

$$p(t) = \Delta\left(\frac{t}{0.25T_b}\right)$$

(a) (7 points) Find an expression for the power spectral density (PSD) $S_y(f)$ of the line coded waveform in terms of T_b if polar signaling is used. Hint: $S_x(f)$ has been derived in your book already for polar signaling. Start with this expression; you do not need to derive it again.

$$\Delta\left(\frac{t}{\tau}\right) \Leftrightarrow \frac{\tau}{2}\,\mathrm{sinc}^2\left(\frac{\pi f \tau}{2}\right)$$ Pair 19 of Table 3.1

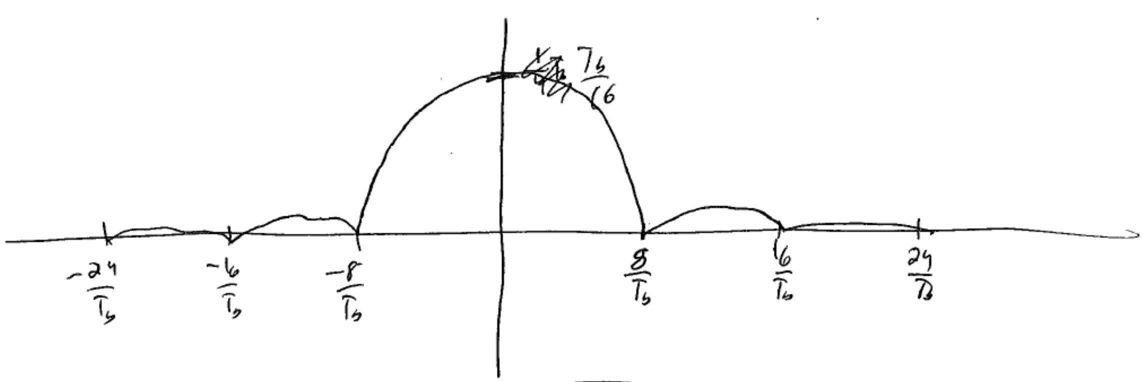
$$\Delta\left(\frac{t}{0.25T_b}\right) \Leftrightarrow 0.25T_b\,\mathrm{sinc}^2\left[\frac{\pi f 0.25T_b}{2}\right]$$

$$P(f) = 0.25\,T_b\,\mathrm{sinc}^2\left[\frac{\pi T_b}{8} f\right]$$

$$S_y(f) = \frac{|P(f)|^2}{T_b} = \left(\frac{1}{16}T_b^2\right)\left(\frac{1}{T_b}\right)\mathrm{sinc}^4\left[\frac{\pi T_b}{8} f\right]$$

$$S_y(f) = \frac{T_b}{16}\,\mathrm{sinc}^4\left[\frac{\pi T_b}{8} f\right]$$

(b) (3 points) Sketch roughly the PSD in terms of T_b and find the bandwidth in terms of T_b (if bandwidth is defined as the frequency width from 0 Hz to the first null in the PSD).



$$B = \frac{8}{T_b}\ \text{Hz}$$

Name____

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Quiz 11 – April 21, 2015

Open Book/Open Notes/10 minutes

You must circle or box your answers for full credit.

PROBLEM 1 (10 points): Determine the PSD of quarternary (4-ary) baseband signaling when the message bits **1** and **0** are equally likely. Assume that the mapping is as follows:

Message bits **00** → $a_k = -6$

Message bits **01** → $a_k = -2$

Message bits **10** → $a_k = 2$

Message bits **11** → $a_k = 6$

Give the power spectral density $S_y(f)$ of this signaling in terms of $P(f)$, the Fourier Transform of the pulse shape, and T_s, the transmission interval.

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_n a_n^2 = \frac{1}{4}\left[(-6)^2+(-2)^2+(2)^2+(6)^2\right] = 20$$

$$R_n = \lim_{N\to\infty} \sum_n a_n a_{n+n} = \frac{2}{16}(36) + \frac{4}{16}(12) + \frac{4}{16}(-12) + \frac{2}{16}(-36) + \frac{2}{16}(4) + \frac{2}{16}(-4)$$

$$= 0$$

	$\cancel{-3}$ -6	$\cancel{-1}$ -2	$\cancel{+1}$ 2	$\cancel{+3}$ 6
$\cancel{-3}$ -6	$\cancel{9}$ 36	$\cancel{3}$ 12	$\cancel{-3}$ -12	$\cancel{-9}$ -36
$\cancel{-1}$ -2	12	4	-4	-12
$\cancel{+1}$ 2	-12	-4	4	12
$\cancel{+3}$ 6	-36	-12	12	36

$$S_y(f) = \frac{|P(f)|^2}{T_s}\left[R_0 + 2\sum_{n=1}^{\infty} \underbrace{R_n}_{0} \cos 2\pi f n T_b\right]$$

$$= \frac{|P(f)|^2}{T_s}[20]$$

Delta Modulation (DM):

- We have observed in PCM that it transmits all the bits, which are used to code a sample.
- Hence, signaling rate and transmission channel bandwidth are quite large in PCM.
- To overcome this problem, Delta Modulation is used.
- DM transmits only one bit for one sample, therefore, the signaling rate and transmission channel bandwidth is quite small for DM.
- The transmitter and receiver implementation is very much simple for DM.
- There is no analog to digital converter required in DM

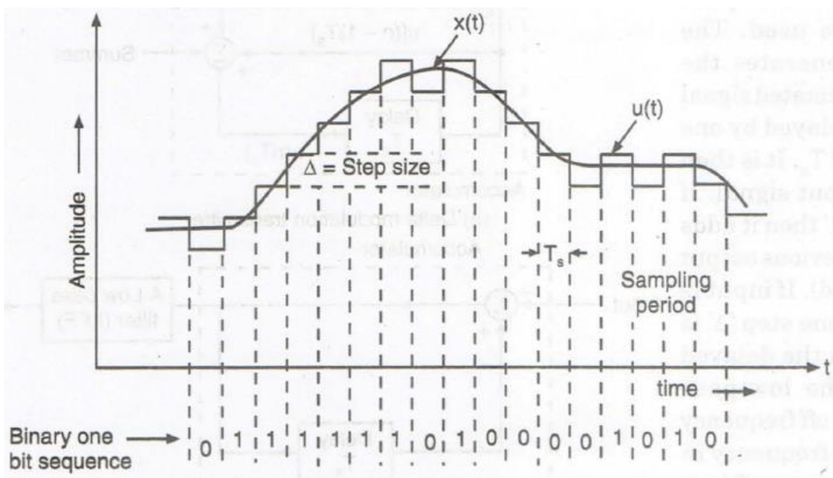


Fig. 8.4. Delta modulation waveform.

1) Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted.
2) Input signal $x(t)$ is approximated to step size signal by the delta modulator. This step size is kept fixed.
3) The difference between the input signal $x(t)$ and staircase approximated signal is confined to two levels, i.e., $+\Delta$ and $-\Delta$.
4) Now, if the difference is ***positive***, then approximated signal is increased by Δ.
5) If the difference is ***negative,*** then approximated signal is reduced by Δ.

In other words: When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted. Hence, for each sample, only one binary bit is transmitted.

The error between the sampled value of $x(t)$ and last approximated sample is,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

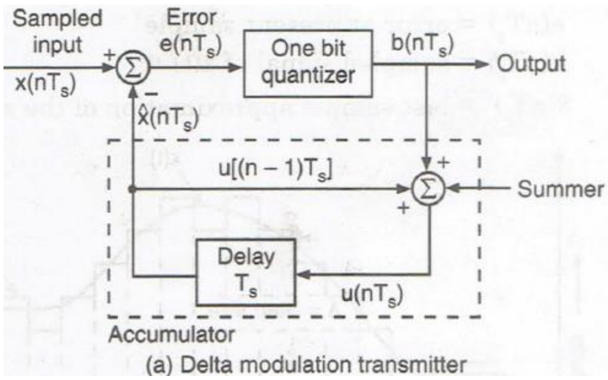
Remember that T_s is the sampling interval.
Where $e(nT_s)$ = Error at present sample
$x(nT_s)$ =Sampled signal of $x(t)$
$\hat{x}(nT_s)$ =Last sample approximation of the staircase waveform.

Assume $u(nT_s)$ as the present sample approximation of staircase output, then,

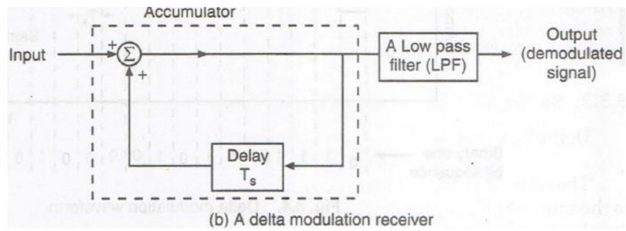
$$u[(n-1)T_s] = \hat{x}(nT_s) = \text{Last sample approximation of the staircase}$$

Let us define a quantity $b(nT_s)$ in such a way that, $b(nT_s) = \Delta\,\mathrm{sgn}[e(nT_s)]$, This means that depending on the sign of error $e(nT_s)$, the sign of step size Δ is decided.

$$b(nT_s) = \begin{cases} +\Delta & if\ x(nT_s) \geq \hat{x}(nT_s) \rightarrow binary\ \mathbf{1}\ is\ transmitted \\ -\Delta & if\ x(nT_s) < \hat{x}(nT_s) \rightarrow binary\ \mathbf{0}\ is\ transmitted \end{cases}$$



(a) Delta modulation transmitter

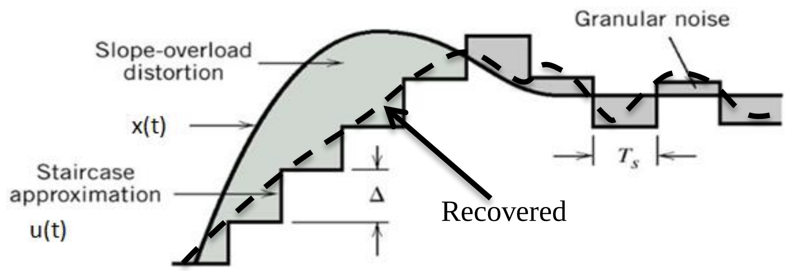


(b) A delta modulation receiver

The delta modulation has two major drawbacks

(i) **Slope overload distortion**,

(ii) **Granular or idle noise**



Slope overload**:** the rate of rise of input signal $x(t)$ is so high that the staircase signal can not approximate it, the step size 'Δ' becomes too small for staircase signal $u(t)$ to follow the step segment of $x(t)$.

Hence, there is a large error between the staircase approximated signal and the original input signal $x(t)$.

Granular or Idle noise**:** occurs when the *step size is too large compared to small variation in the input signal*. This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size.

It is shown that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by ±Δ around the signal.

Therefore, a large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Infact, Adaptive delta modulation is the modification to overcome these errors.

Delta modulation bit rate: $r = Number\ of\ bits\ transmitted/second,$

$$r = Number\ of\ samples/sec \times Number\ of\ bits/sample = F_s \times 1 = F_s$$

$$r = F_s\ bps$$

- Therefore, the delta modulation bit rate is $(1/N)$ times the bit rate of a PCM system, where N is the number of bits per transmitted PCM codeword.
- Hence, we can say that the channel bandwidth for the Delta modulation system is reduced to a great extent as compared to that for the PCM system.

The Maximum Output SNR in DM is:

$$SNR_q = \frac{3}{3\pi^2 f_m^2 F_M T_s^3}$$

Where F_M is the cut-off frequency of the LPF at the receiver. **[Proof: see Example 8.3 in the BOOK]**

Example 8.1: Given a sine wave of frequency f_m and amplitude A_m applied to a delta modulator having step size Δ. Show that the slope overload distortion will occur if $A_m > \frac{\Delta}{2\pi f_m T_s}$ where T_s is the sampling period.

Solution: $x(t) = A_m \sin(2\pi f_m t)$, It is known that the slope of $x(t)$ *will be maximum* when *the derivative of $x(t)$ is maximum*. The maximum slope of delta modulator is,

$$Maximum\ Slope = \frac{Step\ Size}{Sampling\ Period} = \frac{\Delta}{T_s}$$

We know that, slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator, i.e.,

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\Delta}{T_s}$$

$$\max\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| > \frac{\Delta}{T_s}$$

$$\max|A_m 2\pi f_m \cos(2\pi f_m t)| > \frac{\Delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\Delta}{T_s}$$

Therefore;

$$A_m > \frac{\Delta}{2\pi f_m T_s}$$

End of Solution

EXAMPLE 8.2. A delta modulator system is designed to operate at five times the Nyquist rate for a signal having a bandwidth equal to 3 kHz bandwidth. Calculate the maximum amplitude of a 2 kHz input sinusoid for which the delta modulator does not have slope over load. Given that the quantizing step size is 250 mV. Also, derive the formula that you use.

(U.P.S.C. I.E.S. Engg. Examination-1999)

Solution: In last example, we have derived the relation for slope overload distortion which will occur if,

$$A_m > \frac{\Delta}{2\pi f_m T_s} \quad \text{...(i)}$$

Thus, slope overload will not occur if,

$$A_m \le \frac{\Delta}{2\pi f_m T_s} \quad \text{...(ii)}$$

The maximum frequency in the signal is,

$$f_m = 3 \text{ kHz}$$

Hence, Nyquist rate $= 2 f_m = 2 \times 3 \text{ kHz} = 6 \text{ kHz}$

Sampling frequency = 5 times Nyquist rate

or $f_s = 5 \times 6 \text{ kHz} = 30 \text{ kHz}$

Hence, Sampling interval,

$$T_s = \frac{1}{f_s} = \frac{1}{30 \times 10^3} \text{ seconds.}$$

Given that step size $\Delta = 250 \text{ mV} = 250 \times 10^{-3} \text{ V} = 0.25 \text{ V}$

Again, Given that $f_m = 2 \text{ kHz} = 2 \times 10^3 \text{ Hz}$

Substituting all these values in equation (ii), we get

$$A_m \le \frac{0.25}{2\pi \times 2 \times 10^3 \times \frac{1}{30 \times 10^3}}$$

Simplifying, we obtain

$$A_m \le 0.6 \text{ volts} \qquad \textbf{Ans.}$$

EXAMPLE 8.4. A sinusoidal voice signal x(t) = cos (6000 πt) is to be transmitted using either PCM or DM. The sampling rate for PCM system is 8 kHz and for the transmission with DM, the step size Δ is decided to be of 31.25 mV. The slope overload distortion is to be avoided. Assume that the number of quantization levels for a PCM system is 64. Determine the signalling rates of both these systems and also comment on the result.

Solution : We know that the signalling rate of a PCM system is given by

$$r = v f_s$$

But

$$q = 2^v$$

$$v = \log_2 q = \log_2 64 = 6$$

Therefore, signalling rate of PCM $= r = 6 \times 8$ kHz = 48 kHz. **Ans.**

Now, let us calculate the signalling rate of DM system as under :

The signalling rate of a delta modulation (DM) system is equal to its sampling rate f_s because in DM only one bit is transmitted per sample. We know that the condition to avoid the slope overload distortion is given by,

$$A \le \frac{\Delta}{\omega_m T_s} \quad \text{or} \quad A \le \frac{\Delta f_s}{2\pi f_m}$$

Now, let us calculate f_s.

We know that

$$f_s \ge \frac{2\pi f_m A}{\Delta}$$

Substituting all the values, we obtain

$$f_s \ge \frac{2\pi \times 3 \times 10^3 \times 1}{31.25 \times 10^{-3}} \quad \text{or} \quad f_s \ge 603.18 \text{ kHz.}$$

Therefore, signalling rate of DM ≥ 603.18 kHz. **Ans.**

Comment: To transmit the same voice signal, the DM needs a very large signaling rate as that of PCM. This is the biggest drawback of DM, which makes it an impractical system.

EXAMPLE 8.5. Determine the output signal to noise ratio of a linear delta modulation system for a 2 kHz sinusoidal input signal sampled at 64 kHz. Slope overload distortion is not present and the post reconstruction filter has a bandwidth of 4 kHz.

Solution : We know that

$$(\text{SNR})_0 = \frac{3f_s^3}{8\pi^2 f_m^2 f_M}$$

Here, $f_s = 64$ kHz, $f_m = 2$ kHz and $f_M = 4$ kHz

Therefore,

$$(\text{SNR})_0 = \frac{3 \times (64 \times 10^3)^3}{8\pi^2 \times (2 \times 10^3)^2 \times 4 \times 10^3}$$

$$(\text{SNR})_0 = 622.51 = 27.94 \text{ dB} \qquad \textbf{Ans.}$$

EXAMPLE 8.6. For the same sinusoidal input of Example 8.5, calculate the signal to quantization noise ratio of a PCM system which has the same data rate of 64 Kbits/s. The sampling frequency is 8 kHz and the number of bits per sample is N = 8. Comment on the result.

Solution : The signal to noise ratio of a PCM system is given by,

$$(\text{SNR})_q = (1.8 + 6\text{ N}) \text{ dB}$$

$$= 1.8 + (6 \times 8) = 49.8 \text{ dB} \qquad \textbf{Ans.}$$

Comment: The SNR of a DM system is 27.94 dB which is too poor as compared to 49.8 dB of an 8-bit PCM system. Thus, for all the simplicity of DM, it cannot perform as well as an 8-bit PCM.

Delta-Sigma Modulation (The ΔΣ modulator or DS modulator)

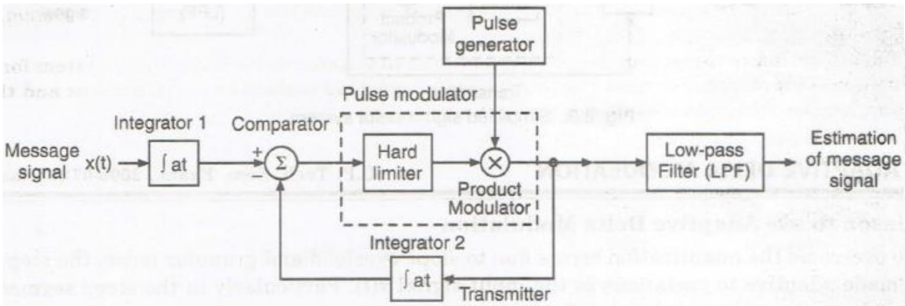
- The quantizer input in the conventional delta modulator can be considered as an approximation to the derivative of the input message signal.
- Hence, the noise results in an ***accumulated error*** in the demodulated signal.
- This is the drawback of the conventional delta modulator.
- This drawback can be overcome by using the ***delta sigma modulator***.
- In the delta sigma modulator, *the input sign is passed through an integrator before applying it to the delta modulator* circuit.

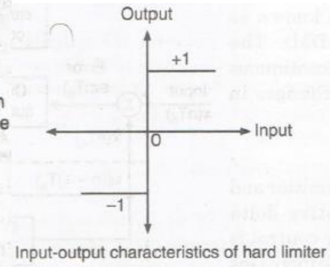
The use of integrator has the following advantages:

(i) The low frequency components in the input signal are boosted (pre-emphasized).

(ii) The correlation between the adjacent samples of delta modulator is increased. This improves the overall system performance, due to reduction in the variance of the error signal at the quantizer input.

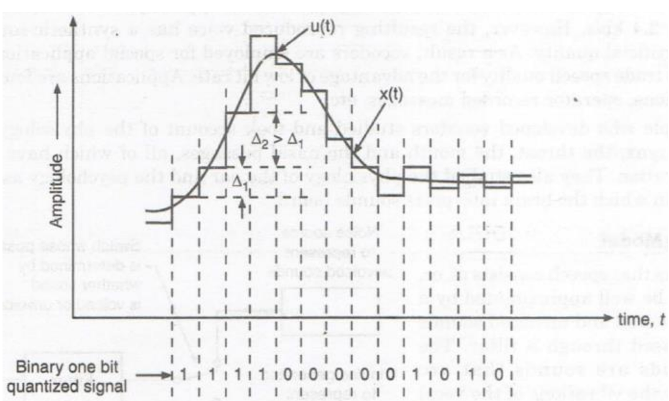
(iii) It simplifies the receiver design.

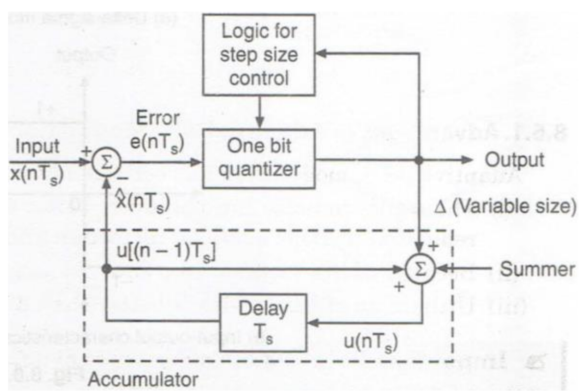




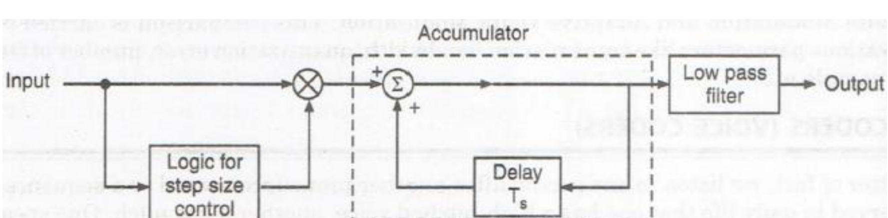
- Here, $x(t)$ is the message signal which is a continuous time signal.
- It is first passed through an ***integrator*** and a ***comparator***.
- The comparator output is applied to the ***pulse modulator*** block. It consists of a hard limiter, and a product modulator.
- The hard limiter output (± 1) is applied to a multiplier (i.e., product modulator).
- The other input to the amplifier is the clock pulses produced by the external pulse generator.
- The frequency of clock pulses should be higher than the Nyquist rate.
- At the output of the product modulator, we get the sampled version of limiter output. It is transmitted over the communication channel.
- Thus, transmit a one bit encoded signal. The same output signal is applied to the second integrator.
- The output of integrator 2 is compared with the output of integrator 1 with the help of the comparator.

Adaptive Delta Modulation (ADM)





Transmitter



Receiver

- To overcome the quantization errors due to ***slope overload and granular noise***, the step size (Δ) is made adaptive to variations in the input signal $x(t)$.
- Particularly in the ***steep*** segment of $x(t)$, the step size is increased.
- Also, if the input is ***varying slowly***, the step size is reduced.
- Then, this method is known as ***A***daptive ***D***elta ***M***odulation (**ADM**).

Adaptive delta modulation has certain advantages over delta modulation:

(i) The signal to noise ratio becomes better than ordinary delta modulation because of the reduction in slope overload distortion and idle noise.

(ii) Because of the variable step size, the dynamic range of ADM is wider than simple DM.

(iii) Utilization of bandwidth is better than delta modulation.

Comparison between PCM, Delta Modulation, Adaptive Delta Modulation and Differential Pulse Code Modulation

#	Parameter	PCM	DM	ADM	DPCM
1	Number of Bits	It can use 4, 8 or 16 bits per sample	It uses only one bit for one sample	It uses only one bit for one sample	Bits can be more than one but less than PCM
2	Levels and step size	The number of levels depend on number of bits. Level size is kept fixed	Step size is kept fixed and cannot be varied	According to the signal variation, step size varies (Adaptive)	Fixed number of levels are used
3	Quantization error and distortion	Quantization error depends on number of levels used	Slope overload distortion and granular noise are present	Quantization noise is present but other errors are absent	Slope overload distortion and quantization noise is present
4	Transmission bandwidth	Highest bandwidth is required since number of bits are high	Lowest bandwidth is required	Lowest bandwidth is required	Bandwidth required is lower than PCM
5	Feedback	There is no feedback in transmitter or receiver	Feedback exists in transmitter	Feedback exists	Feedback exists
6	Complexity of implementation	System is complex	Simple	Simple	Simple

Additional Solved Examples: (see the book)

EXAMPLE 8.12. A DM system is designed to operate at 3 times the Nyquist rate for a signal with a 3 kHz bandwidth. The quantizing step size is 250 mV.

(i) Determine the maximum amplitude of a 1-kHz input sinusoid for which the delta modulator does not show slope overload.

(ii) Determine the post filtered output signal-to-quantizing-noise ratio for the signal of part (i)

Solution: We have

$$m(t) = A\cos\omega_m t = A\cos 2\pi(10^3)t$$

$$\left|\frac{dm(t)}{dt}\right|_{max} = A(2\pi)(10^3)$$

The maximum allowable amplitude of the input sinusoid is

$$A_{max} = \frac{\Delta}{\omega_m T_s} = \frac{\Delta}{\omega_m}$$

$$f_s = \frac{250}{2\pi(10^3)}3(2)(3)(10^3) = 71.2 \text{ mV} \quad \textbf{Ans.}$$

(ii) Assuming that the cut off frequency of the low-pass filter is f_m, we have

$$(SNR)_0 = \left(\frac{S}{N_q}\right)_0 = \frac{3[(3)(6)(10^3)]^3}{8\pi^2(10^3)^3} = 221.6 = 23.5 \text{ dB} \quad \textbf{Ans.}$$

EXAMPLE 8.13. The pulse rate in a DM system is 56,000 per sec. The input signal is 5 cos (2π 1000 t) + 2 cos (2π 2000 t) V, with t in sec. Find the minimum value of step size which will avoid slope overload distortion. What would be the disadvantages of choosing a value of larger than the minimum? (GATE Examination-1998)

Solution: Input signal,

$$m(t) = 5\cos(2000\,\pi t) + 2\cos(4000\,\pi t) = m_1(t) + m_2(t)$$

To avoid slope overloading, we have

$$\left|\frac{dm_1(t)}{dt}\right|_{max} \le \Delta_1 f_s,$$

where Δ is step size and f_s is sampling rate.

or $$\Delta_1 f_s \ge \left|\frac{dm_1(t)}{dt}\right|_{max}$$

$$\Delta_{1min} \text{ (minimum value of step size)} = \left|\frac{dm_1(t)}{dt}\right|_{max} = \frac{5.(2000\pi)}{56000} = \frac{10\pi}{56} = 0.56 \text{ V}$$

Also, $$\Delta_2 f_s \ge \left|\frac{dm_2(t)}{dt}\right|_{max} \quad \text{or} \quad \Delta_{2\,min} = \frac{8000\pi}{56000} = \frac{\pi}{7} = 0.45 \text{ V}$$

Hence, larger step size out of two will be the required step size, i.e., = 0.56 V.

If a value larger than the minimum will be choosen, then granular noise will occur.

EXAMPLE 8.14. Determine the output SNR in a DM system for 1 kHz sinusoid, sampled at 32 kHz without slope overload and followed by a 4 kHz post construction filter. Derive the formula used.

Solution : Given that, $f_m = 1$ kHz, $f_s = 32$ kHz, BW = 4 kHz

It is given that there is no slope overload.

The output signal to noise ratio in a DM system is expressed as

$$(SNR)_0 = \frac{3f_s^3}{8\pi^2 f_m^2 f_M}$$

Therefore, $$(SNR)_0 = \frac{3\times(32\times10^3)^3}{8\pi^2\times(1\times10^3)^2\times4\times10^3} \quad \text{or} \quad (SNR)_0 = 311.25 \quad \text{or} \quad 24.93 \text{ dB.} \quad \textbf{Ans.}$$

EXAMPLE 8.15. In a DM system, the voice signal is sampled at a rate of 64,000 samples/sec. The maximum signal amplitude $A_{max} = 1$.

(i) Determine minimum value of step size to avoid slope overload.

(ii) Determine quantization noise power if voice signal bandwidth is 3.5 kHz.

(iii) Assuming voice signal to be a sine wave, determine S_0 and the SNR.

Solution : Given that $f_s = 64{,}000$ samples/sec.

$A_{max} = 1.$

(i) First, we determine the minimum value of step size (Δ_{min}).

The condition to avoid slope overload error is expressed as

$$A \le \frac{\Delta}{2\pi f_m T_s} \quad \text{or} \quad A_{max} = \frac{\Delta f_s}{2\pi f_m}$$

Therefore, $$\Delta_{min} = \frac{2\pi f_m A_{max}}{f_s}$$

Since the input to DM system is a voice signal, therefore, let us assume $f_m = 3.5$ kHz.

Hence, $$\Delta_{min} = \frac{2\pi\times3.5\times10^3\times1}{64\times10^3} = 343.61 \text{ mV. } \textbf{Ans.}$$

(ii) Quantization noise power N_q' can be calculated as under :

The quantization noise power N_q for DM system is given by,

$$N_q = \frac{\Delta^2}{3} = \frac{(343.61\times10^{-3})^2}{3} = 39.35 \times 10^{-3}\ \text{W}$$

However, as the voice bandwidth is only 3.5 kHz, the normalized quantization noise power at the receiver output is given by,

$$N_q' = N_q \times \left(\frac{f_m}{f_s}\right) = 39.35 \times 10^{-3} \times \frac{3.5}{64}$$

or $\quad N_q' = 2.15\ \text{mW}$

(iii) Signal power S_0 and SNR can be calculated as under :

$$\text{Output signal power} = \frac{A_{max}^2}{2} = \frac{1}{2}. \qquad \textbf{Ans.}$$

$$\text{Output signal to noise ratio} = \frac{S_0}{N_q'} = \frac{1/2}{(2.15\times10^{-3})}$$

or $\quad (SNR)_0 = 232.3$ or 23.66 dB. **Ans.**

EXAMPLE 8.16. In a single integration DM scheme, the voice signal is sampled at a rate of 64 kHz. The maximum signal amplitude is 1 Volt.

(i) Determine the minimum value of step size to avoid slope overload.

(ii) Determine granular noise power N_0, if the voice signal bandwidth is 3.5 kHz.

(iii) Assuming signal to be sinusoidal, calculate signal power S_0 and signal to noise ratio (SNR).

(iv) Assuming that the voice signal amplitude is uniformly distributed in the range (– 1, 1), determine S_0 and SNR.

Solution : Given that $\quad f_s = 64$ kHz $\quad A = 1$ Volt

(i) Minimum step size to avoid slope overload is given by

$$A \le \frac{\Delta f_s}{2\pi f_m} \quad \text{or} \quad \Delta_{min} = \frac{2\pi f_m A}{f_s} = \frac{2\pi\times3.5\times10^3\times1}{64\times10^3}$$

or $\quad \Delta_{min} = 0.3436$ Volt. **Ans.**

(ii) Granular noise power is expressed as

$$N_q = \frac{\Delta^2}{3} \times \frac{f_m}{f_s} = \frac{(0.3436)^2}{3} \times \frac{3.5}{64}$$

Solving, we get

$$N_q = 2.15 \times 10^{-3}\ \text{W} \qquad \textbf{Ans.}$$

SHORT QUESTIONS WITH ANSWERS

Q.1. Explain the importance (or) use of prediction in Differential pulse code modulation (DPCM).

Ans. In standard PCM, each sample of the baseband signal is encoded independently of all others.

Sometimes, in some signals, when they are sampled at the Nyquist rate (or) faster exhibit significant correlation between successive samples, i.e., the relative change in amplitude between the successive samples is very small.

Under these circumstances, if this highly correlated signal is encoded using PCM, then the resultant signal consists of redundant information and results in a lower bit rate. This is because the symbols that are not essential to the transmission of information are generated as a result of the encoding process.

By removing this redundancy before encoding, we obtain a more efficient coded signal.

Thus, if a sufficient part of a redundant signal is known it is possible to make the most probable estimate of the rest, i.e., if a small part of the sample is known, the other half can be estimated from the knowledge of previous sample.

This process is known as prediction.

Q.2. What is Delta Modulation and give the comparison between DM and DPCM.

Ans. Delta Modulation is the one-bit (or) two-level version of DPCM.

They are similar except for two important differences namely, the use of a one-bit quantizer in delta modulator and the prediction filter is replaced by a single delay element.

Q.3. Discuss the Noise effects in Delta Modulation.

Ans. In Delta modulation we observe quantization noise. There are two major sources of quantizing error in DM systems. They are

(i) Slope overload distortion

(ii) Granular noise.

Q.4. Write short notes on Granular noise.

Ans. In contrast to the slope overload distortion, the granular noise occurs when the step size Δ is too large relative to the local slope characteristics of the input waveform, thereby causing the staircase approximation to hunt around a relatively flat segment of the input waveform.

This is also known as hunting process.