Department of Communications Engineering, College of Engineering, University of Diyala

Digital Communication I

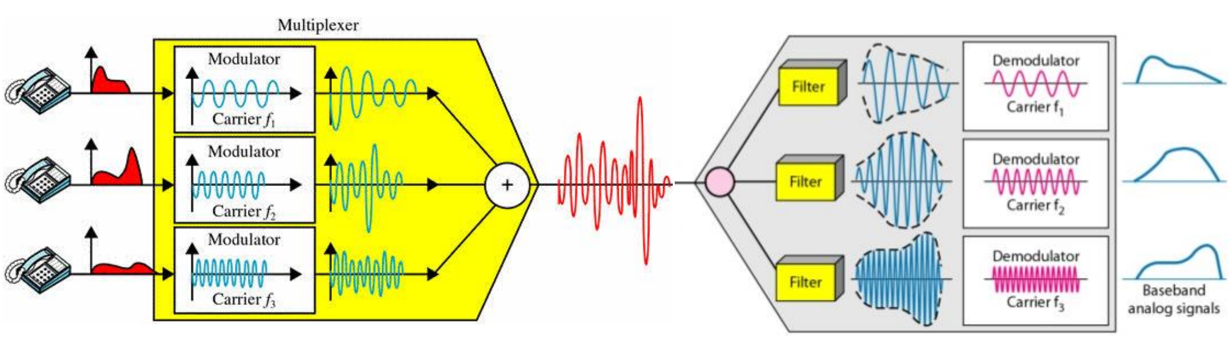
Lecture # 5

Multiplexing Techniques
FDM, TDM, Line Coding, Applications

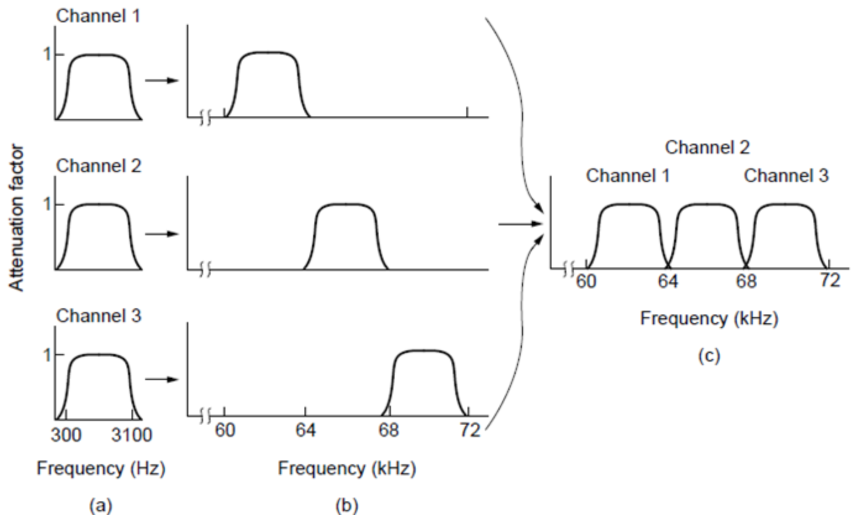
Multiplexing:

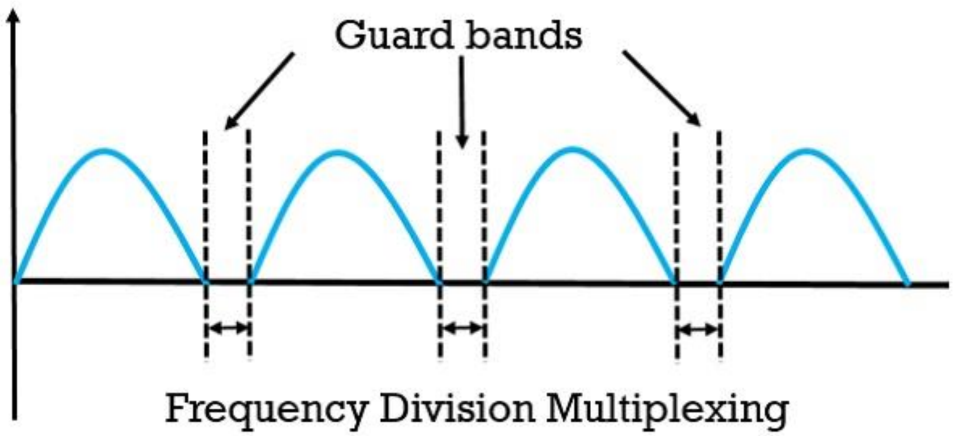
- Multiplexing is a technique which allows many users to *share a common communication channel simultaneously*.
- Thus, Multiplexing is the process of **transmitting multiple signals over a single communication channel**
- The major types of multiplexing techniques are:
    1) **F**requency **D**ivision **M**ultiplexing (**FDM**),
    2) **T**ime **D**ivision **M**ultiplexing (**TDM**).
    3) **C**ode **D**ivision **M**ultiplexing (**CDM**)
    4) **S**patial **M**ultiplexing (**SM**)
        - ✓ Spatial Multiplexing is a scheme of multiplexing used widely in wireless systems.
        - ✓ Spatial multiplexing can be achieved using different parameters such as the antenna polarization.

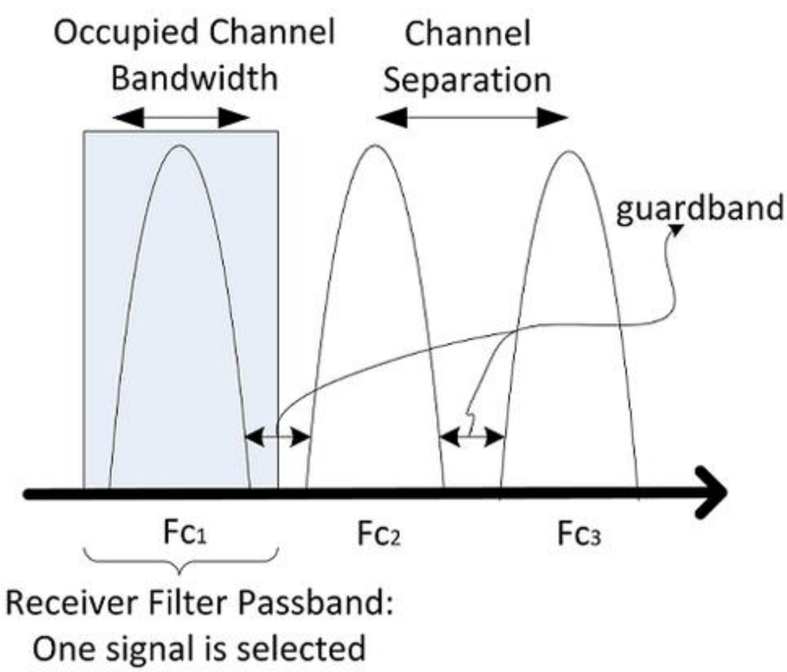
Frequency Division Multiplexing:



Example:


Channel 1
Channel 2
Channel 3
Attenuation factor
1
300
3100
Frequency (Hz)
(a)
60
64
68
72
Frequency (kHz)
(b)
Channel 2
Channel 1
Channel 3
60
64
68
72
Frequency (kHz)
(c)


Guard bands
Frequency Division Multiplexing


Occupied Channel Bandwidth
Channel Separation
guardband
Fc1
Fc2
Fc3
Receiver Filter Passband:
One signal is selected

- Each sub-band is called frequency channel ***slot***.
- Major problem in FDM is the **cross talk** due to the nonlinearity of the amplifier, thus, cross talk (**intermodulation**) appears.
- There are *intelligible* cross talk (cross modulation) and *unintelligible* cross talk.
  - ✓ Unintelligible cross talk is due to the effects of the nonlinearity of the filters.
  - ✓ Guard bands are the solution to the unintelligible cross talk problem.
- Total occupied bandwidth, hence, is the summation of the frequency slots and the guard bands.

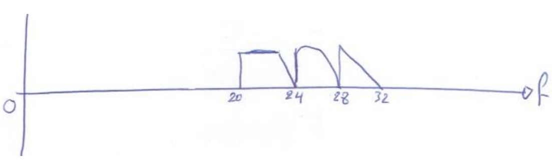
$$B_{FDM} = \sum_{n=1}^{N-1} G_n + \sum_{n=1}^{N} S_n$$

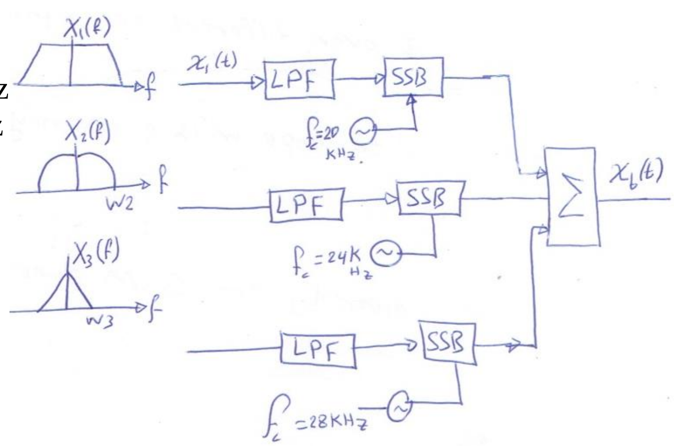
Where: $N = number\ of\ channels\ (slots)$
$G = guard\ band\ bandwidth$
$S = channel\ slot\ bandwidth$

The commercial AM and FM broadcasting stations are examples of FDM.

Example 1: assume that a voice channel occupies a bandwidth of 4KHz. We need to combine three voice channels into a link with a bandwidth of 12KHz, from (20 to 32)KHz. Show the configuration, using frequency domain. Assume there are no guard bands (or assume that the guard bands width is 0Hz).

Solution: *first channel*: 20 – 24 KHz
Second channel: 24 – 28 KHz
Third channel: 28 – 32 KHz





Example 2: Fifteen channels, each with a 200KHz bandwidth, are to be multiplexed together. What is the minimum bandwidth of the link if there is a need for a guard band of 15KHz between the channels to prevent interference?

Solution: Number of Channels $N = 15$
Slot bandwidth $S = 200KHz$
Guard bandwidth $G = 15KHz$
$Number\ of\ guard\ bands = Number\ of\ slots - 1 = 14$

Then, total bandwidth $B_{FDM} = 14 \times 15KHz + 15 \times 200KHz$
$= 210KHz + 3000KHz = 3210KHz$

Example 3: A cable TV service uses a single coaxial cable with a bandwidth of 860MHZ to transmit multiple TV signals to subscribers. Each TV signal is 6MHz wide. How many channels can be carried? Assume no guard bands.

Solution: Total Bandwidth $B_{FDM} = 860MHz$

Channel Bandwidth $S = 6MHz$

Then, Number of channels $N = \frac{B_{FDM}}{S} = \frac{860MHz}{6MHz} = 143.3 \cong 143$ channels.

Example 4: Advanced Mobile Phone System (AMPS) was an analog mobile phone system standard developed by Bell Labs. AMPS uses two bands. The first band is used for uplink (from mobile station to the system) from 824MHz to 849MHz, and the second band for downlink (from the system to the mobile station) from 869MHz to 894MHz. Each user has a bandwidth of 30KHz in each direction. How many users can use their Cellular Phones simultaneously if you know that there are 42 channels reserved for control and signaling?

Solution: Uplink band $= 849MHz - 824MHz = 25\ MHz$

Downlink band $= 894MHz - 869MHz = 25\ MHz$

For each direction: each $S = 30\ KHz$

Number of channels $N = \frac{one\ direction\ bandwidth}{S} = \frac{25\ MHz}{30\ KHz} = 833.3 \cong 833$

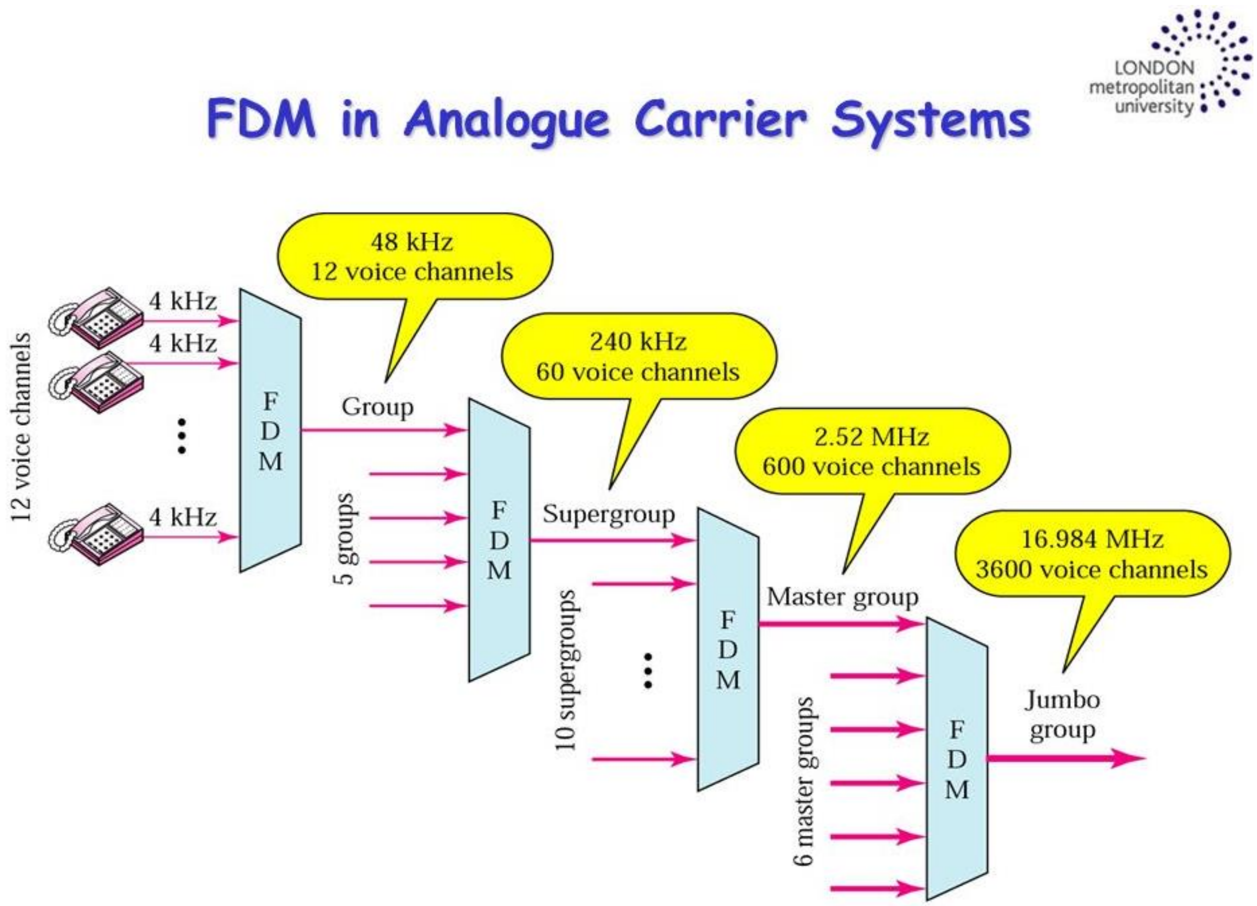
In real practice is $N = 832$ channels only.

Since there are 42 channels are reserved for control and signaling,

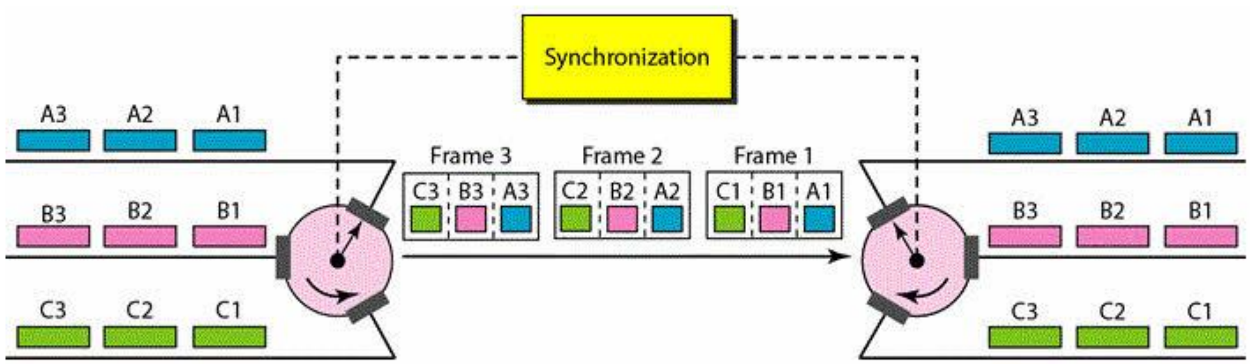
The Channels for users are $833 - 42 = 791$ users can use the cellular phones.

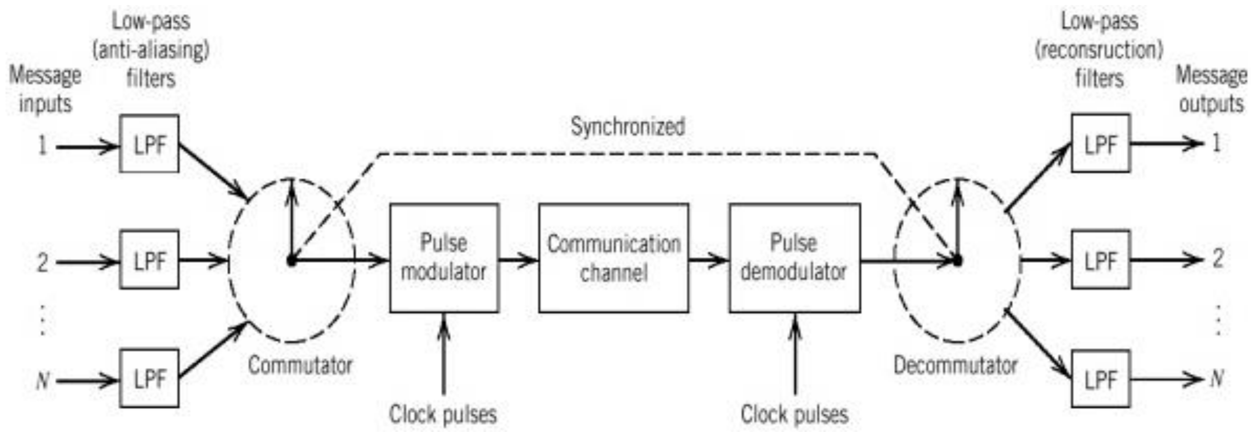
Analog Carrier Systems:

- Carrier systems are designed for *long-distance services lines* such as Coaxial cables and microwave systems.
- Long distance carrier systems carry ***voicebands***.
- A very common technique for utilizing high-capacity links is FDM.
- ITU-T institute has put an international standards as follows:
  - Each $\boldsymbol{voicechannel} = 4KHz$
  - Each 12 $\boldsymbol{voicechannel}$ are combined to $\boldsymbol{Group} = 12 \times 4 = 48KHz$ ranging from $60KHz - to - 108KHz$.
  - Each 5 $\boldsymbol{Groups}$ are combined to $\boldsymbol{Super - Group}$
    $\boldsymbol{Super - Group} = 12\ \boldsymbol{voicechannel} \times 5 = 60\ voicechannels$
    $\boldsymbol{Super - Group} = 48KHz \times 5 = 240KHz$ in the range $312KHz - 552KHz$
  - Each 10 $\boldsymbol{Super - Group}$ combined in $\boldsymbol{Mastergroup}$
  - Thus, 1 $\boldsymbol{Mastergroup = 10 \times 60\ voicechannel = 600\ voicechannels}$
  - Then, 1 $\boldsymbol{Mastergroup = 2.520\ MHz}$
  - Each 6 $\boldsymbol{Mastergroups}$ multiplexed to $\boldsymbol{Jumbogroup}$.
  - Then 1 $\boldsymbol{Jumbogroup = 6 \times 600\ voicechannel = 3600\ voicechannels.}$
  - 1 $\boldsymbol{Jumbogroup = 16.984\ MHz}$

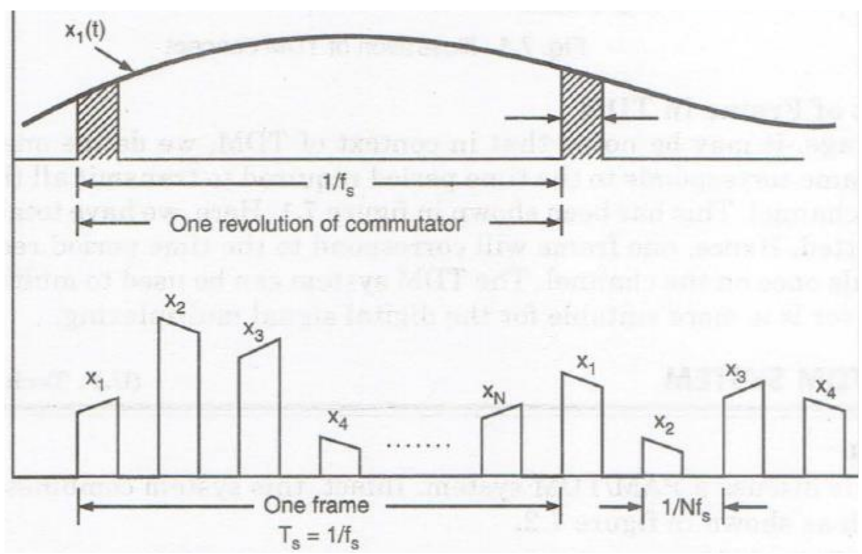


Time Division Multiplexing (TDM)

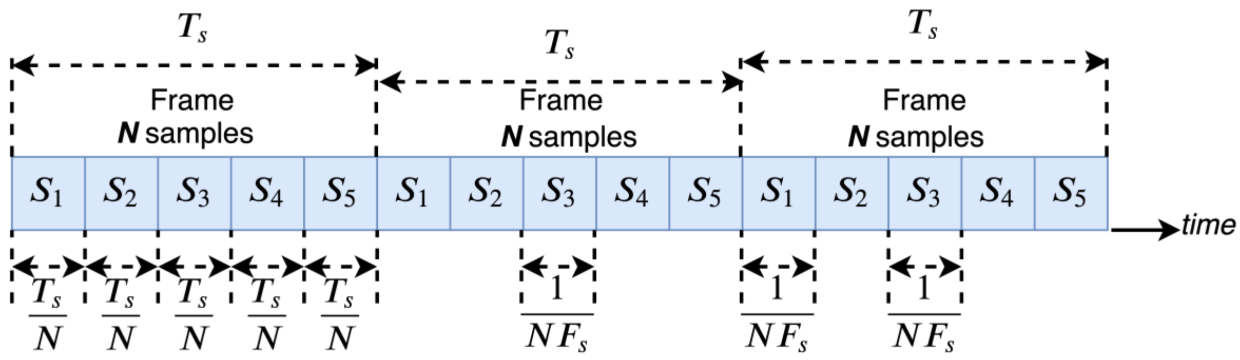




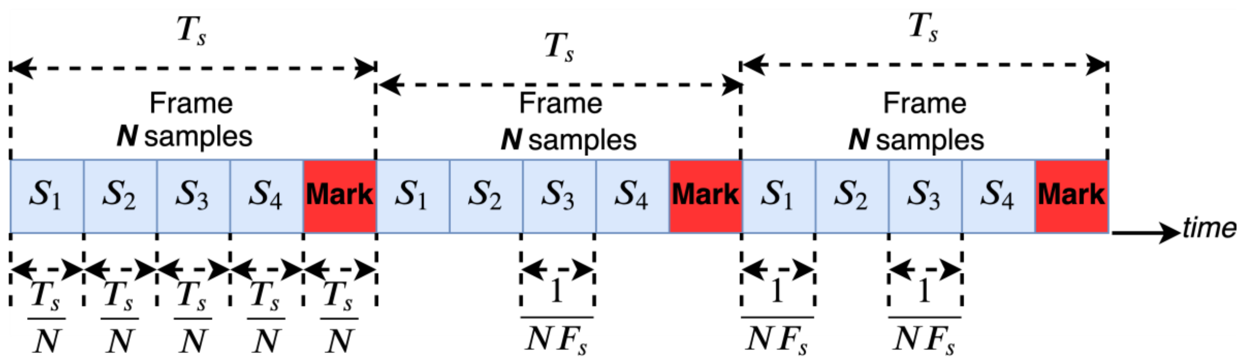
- A sampled waveform is **off** most of the time, leaving the time between samples available for other purposes.
- In particular, sample values from several different signals can be ***interleaved*** into a single waveform.
- This is the principle of **T**ime **D**ivision **M**ultiplexing (**TDM**)



- If all inputs have the same bandwidth $= f_m$ then the ***commutator*** rotates at a rate $F_s \geq 2f_m$.
- Successive samples from any one input are spaced by T_s
- **The time interval T_s** contains one sample from each input, this interval is called **frame**.
- If there are N input channels, the pulse to pulse spacing within a frame is $\frac{T_s}{N} = \frac{1}{NF_s}$
- Total number of pulses per second is: $r = NF_s \geq 2Nf_m$ *pulse per second*
- Here r is called the signaling rate of the TDM system.
- Minimum transmission bandwidth of a PAM-TDM channel is $B_{TDM} = \frac{1}{2}r = \frac{NF_s}{2} = Nf_m$ this is also the bandwidth of the LPF



- In practice, the commutator is not mechanical, instead, it is electronic.
- Synchronization between commutator and decommutator is very important to deliver the correct signal's pulse to the assigned branch.
- One synchronization technique is by using ***marker*** pulse or **non-pulse** time-slot to help the receiver in the synchronization operation.
- Therefore, because of the synchronization marker, the number of signal channels (time-slots) reduced by one.



- If sources are **not equal bandwidth**!!! The source of larger band will be sampled more often than smaller band sources.
- If the sources are **digital**, ***pulse-stuffing*** must be used *to make all sources have the same data rate of the largest source*.

Example 7.1: Two analog signals $x_1(t)$ and $x_2(t)$ are to be transmitted over a common channel by means of time division multiplexing (TDM). The highest frequency of $x_1(t)$ is 4 kHz and that of $x_2(t)$ is 4.5 kHz. What will be the minimum value of permissible sampling rate?

Solution: The highest frequency component of the composite signal consisting of $x_1(t)$ and $x_2(t)$ is 4.5 kHz. Therefore, the minimum value of permissible sampling rate will be,

$$f_{s(min)} = 2 \times 4.5\, kHz = 9 kHz$$

EXAMPLE 7.2. A signal $x_1(t)$ is bandlimited to 3 kHz. There are three more signals $x_2(t)$, $x_3(t)$ and $x_4(t)$ which are bandlimited to 1 kHz each. These signals are to be transmitted by a TDM system.

(i) Design a TDM scheme where each signal is sampled at its Nyquist rate.

(ii) What must be the speed of the commutator?

(iii) Calculate the minimum transmission bandwidth of the channel.

Solution: (i) Table 7.1 shows different message signals with corresponding Nyquist rates.

TABLE 7.1

S.No.	Message signal	Bandwidth	Nyquist rate
1.	$x_1(t)$	3 kHz	6 kHz
2.	$x_2(t)$	1 kHz	2 kHz
3.	$x_3(t)$	1 kHz	2 kHz
4.	$x_4(t)$	1 kHz	2 kHz

If the sampling commutator rotates at the rate of 2000 rotations per second then the signals $x_2(t)$, $x_3(t)$ and $x_4(t)$ will be sampled at their Nyquist rate. But, we have to sample $x_1(t)$ also at its Nyquist rate which is three times higher than that of the other three. In order to achieve this, we should sample $x_1(t)$ three times in one rotation of the commutator. Therefore, the commutator must have atleast 6 poles connected to the signals as shown in figure 7.8.

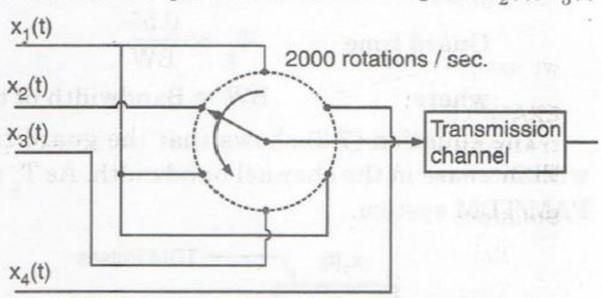


Fig. 7.8

(ii) The speed of rotation of the commutator is 2000 rotations/sec.

(iii) Number of samples produced per second is calculated as under:

$x_1(t)$ produces $3 \times 2000 = 6000$ samples/sec.

$x_2(t)$, $x_3(t)$ and $x_4(t)$ produce 2000 samples/sec. each.

Therefore, number of samples per second = $6000 + (3 \times 2000) = 12000$ samples/sec.

$\therefore$ Signaling rate = 12000 samples/sec.

(iv) The minimum channel bandwidth will be

$$BW = \frac{1}{2}(\text{signaling rate}) = 12000/2 = 6000 \text{ Hz} \quad \textbf{Ans.}$$

EXAMPLE 7.3. Twenty-four voice signals are sampled uniformly and then time division multiplexed. The sampling operation uses flat top samples with 1 μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of appropriate amplitude and 1 μs duration. The highest frequency component of each voice signal is 3.4 kHz.

(i) Assuming a sampling rate of 8 kHz, calculate the spacing between successive pulses of the multiplexed signal.

(ii) Repeat (i) assuming the use of Nyquist rate sampling.* **(GATE Examination 1997)**

Solution: (i) Given that

Sampling rate = 8 kHz = 8000 samples/sec.

There are 24 voice signals + 1 synchronizing pulse.

Pulse width of each voice channel and synchronizing pulse is 1 μs.

Now, time taken by the commutator for 1 rotation = $\frac{1}{8000}$ = 125 μ sec.

Number of pulses produced in 1 rotation = 24 + 1 = 25

Therefore, the leading edges of the pulses are at $\frac{125}{25}$ = 5 μ seconds distance as shown in figure 7.9.

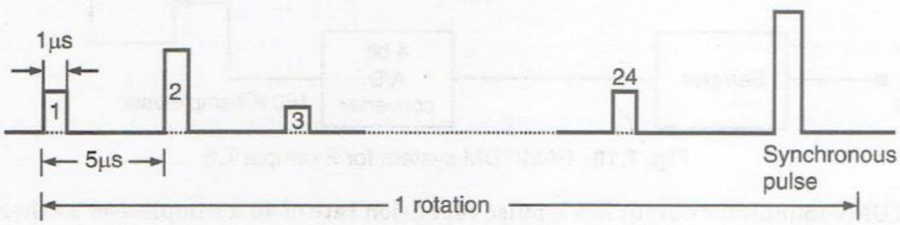


Fig. 7.9

Hence, spacing between successive pulses = 5 – 1 = 4 μs

(ii) Nyquist rate of sampling = 2 × 3.4 kHz = 6.8 kHz.

This means that 6800 samples are produced per second. One rotation of commutator takes 1/6800 = 147 μs time

Therefore, 147 μ sec corresponds to 25 pulses

Therefore, 1 pulse corresponds to 5.88 μ sec.

As the pulse width of each pulse is 1 μ sec, the spacing between adjacent pulses will be 4.88 μ sec and if we assume τ = 0 then the spacing between the adjacent pulses will be 5.88 μ sec.

EXAMPLE 7.4. Six message signals each of bandwidth 5 kHz are time division multiplexed and transmitted. Determine the signalling rate and the minimum channel bandwidth of the PAM/TDM channel.

Solution: The number of channels N = 6

Bandwidth of each channel, f_m = 5 kHz

Therefore, minimum sampling rate = 2 × 5 kHz = 10 kHz

Signaling rate = Number of bits per second

= 6 × 10 kHz = 60 K bits/sec. **Ans.**

Minimum channel bandwidth to avoid cross talk in PAM/TDM is,

$BW = Nf_m = 6 \times 5$ kHz = 30 kHz **Ans.**

EXAMPLE 7.5. Sketch a channel interleaving scheme for the time division multiplexing of the following PAM signals: Five 4 kHz telephone channels and one 20 kHz music channel. Find the pulse repetition rate of the multiplexed signal and estimate the minimum system bandwidth required.*

Solution: Let us note the following points:

(i) Each telephone channel of bandwidth 4 kHz must be sampled at Nyquist rate i.e., 2×4 kHz = 8 kHz using a TDM commutator.

(ii) The 20 kHz music channel must be sampled at 40 kHz (Nyquist rate), hence, a separate sampler is required.

(iii) The sampled signals are applied to two 4-bit A-D converters to obtain the equivalent digital signals.

(iv) These signals are finally multiplexed using a multiplexer as shown in figure 7.10.

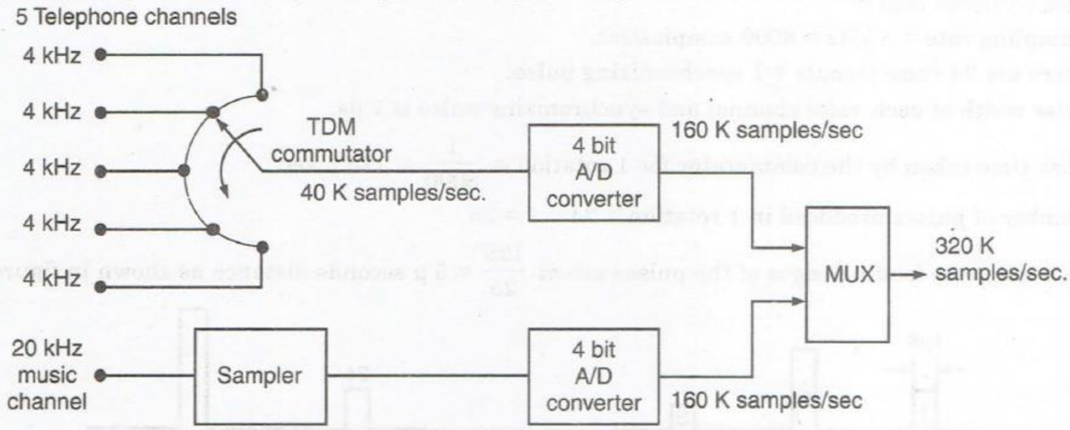


Fig. 7.10. PAM/TDM system for Example 7.5.

(v) The TDM commutator output has a pulse repetition rate of 40 k samples/sec as there are 5 channel and sampling rate is 8 kHz.

(vi) Similarly, the output of the separate sampler has a pulse repetition rate of 40 K samples sec.

(vii) The outputs of A-D converters has pulse repetition rates of $40 \times 4 = 160$ K samples/sec.

(viii) Therefore, pulse repetition rate at the output of a multiplexer is

$$160 + 160 = 320 \text{ K samples/sec.}$$

Hence, pulse repetition rate of the system = 320 kHz

Therefore, bandwidth required = bit rate = 320 kHz. **Ans.**

DO YOU KNOW?

TDM is used extensively in telephony. There are many different standards for TDM. One commonly used arragement is the DS-1 signal.

Ex. Design a time-division multiplexer that will accommodate 11 sources. Assume that the sources have the following specifications:

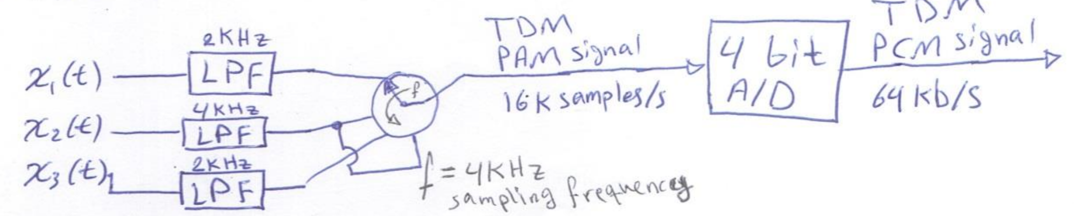
$x_1(t)$ Source 1: analog 2KHz bandwidth.
$x_2(t)$ Source 2: analog 4KHz bandwidth.
$x_3(t)$ Source 3: analog 2KHz bandwidth.
$x_4(n) \to x_{11}(n)$ Source 4-to-11: Digital, synchronous at 7200 bits/s.

Assuming 4-Bit analog to digital (ADC) converter to convert the PAM of the TDM multiplexed analoge signal to digital.

Solution:- we have two parts: analog part and digital part

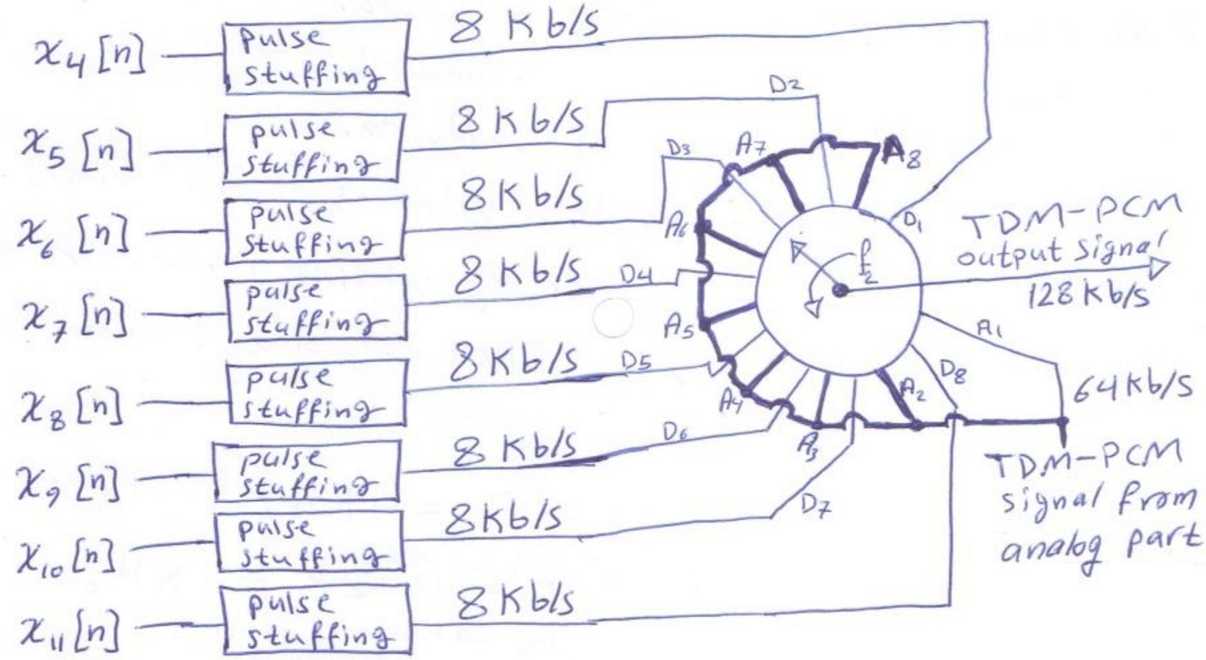
Part 1: we have $x_1(t)$ bandwidth = 2 KHz : $f_s = 4$ KHz
$x_2(t)$ bandwidth = 4 KHz : $f_s = 8$ KHz
$x_3(t)$ bandwidth = 2 KHz : $f_s = 4$ KHz

Thus, the second signal will be devided to two Lines as shown in the figure.
Hence, $f_s = 4$ KHz for all the signals.
$r = m f_s = 4$ branches $\times 4$ KHz $= 4 \times 4 = 16$ K samples/second.
- The commutator will take samples at speed of 4K samples/s.
- Each sample will be converted to 4 binary bits, then
- TDM-PCM will be 64 Kbps.



Why $x_2(t)$ sampled two times?

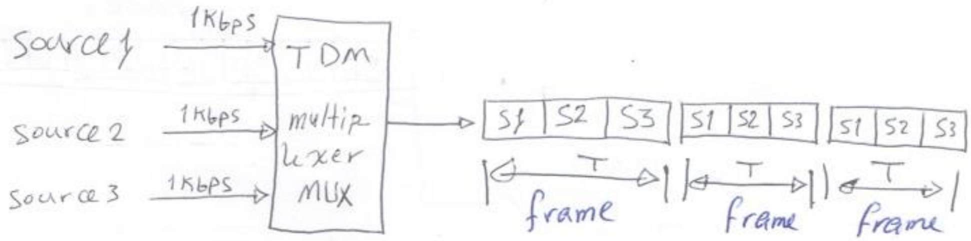
Part 2: we have 8 digital signals, each 7.2 kb/s. we need pulse stuffing to make each digital signal of rate 8 Kb/s.



NOW: $f_2 = 8$ KHz (no need to make it 16 KHz, because it is already digital signal.)

The 64 Kb/s from analog part has been divided into 8 Lines to the Second-commutator.

EX.2 In the following figure, the data rate of each input is 1 Kbps. If one bit at a time is multiplexed, what is the duration of ① Each input slot, ② each output slot ③ each frame.



Solution ① data rate of each input connection is 1 Kbps; Hence the bit duration is $\frac{1}{1000} = 1\ ms$ ∴ $T = 1\ ms$

② the duration of each output time-slot is $\frac{1}{3}T$

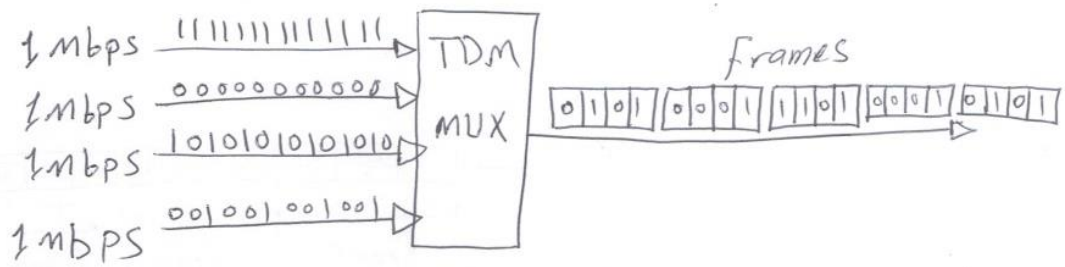
∴ output time-slot = $\frac{1}{3}$ ms.
or duration of $S1 = S2 = S3 = \frac{1}{3} ms$

③ each frame has 3 time-slots
∴ duration of each frame = 3 time-slots × $\frac{1}{3}$ ms

∴ frame duration = 1 ms.

NOTE : The output Frame duration = duration of the input unit.

EX. 3 In the figure shown below, a synchronous TDM system with four 1 Mbps data streams input and one data stream for the output. Find ① The input bit duration ② the output bit duration ③ the output bit rate, and ④ the output frame rate.



Solution ① input bit duration $= \frac{1}{\text{input bit rate}} = \frac{1}{1 \times 10^6} = 1\ \mu s$.

② output bit duration $= \frac{1}{4} \times$ input bit duration $= 0.25\ \mu s$.

③ output bit rate $= 4 \times$ input bit rate $= 4$ Mbps

OR

output bit rate $= \frac{1}{\text{output bit duration}} = \frac{1}{0.25 \times 10^{-6}} = 4$ Mbps.

④ The frame rate is always = input rate

∴ frame rate = 1 M frame per second

each frame consists of 4 bits

Ex. 4: A TDM multiplexing system has 4 inputs each of 1kbps. Find ① The duration of each bit before multiplexing ② the transmission rate of the link ③ the duration of a time-slot ④ the duration of a frame.

Solution ① Duration of a bit before multiplexing is $\frac{1}{1\,\text{kbps}} = 1\,ms$.

② The rate of the link is 4 times the input rate = 4 kbps.

③ Duration of each time-slot after multiplexing is $\frac{1}{4}$ of the bit duration before multiplexing

$$T_{bit-after} = T_{bit-before} / (\text{number of input connections})$$

$$= \frac{1\,ms}{4} = 0.25\,ms.$$

$$= 250\,\mu s.$$

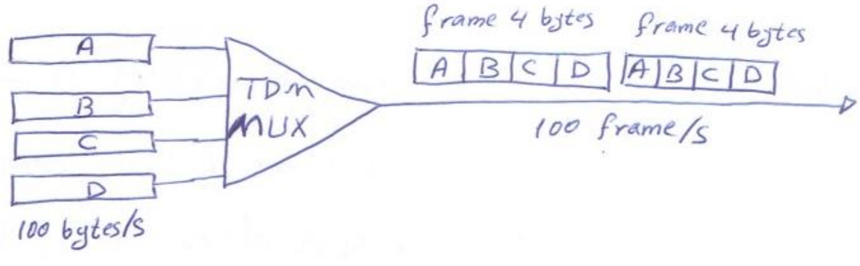
④ $T_{frame} = T_{bit-before} = 1\,ms.$

or

$$T_{frame} = 4\text{ bits} \times 0.25\,ms = 1\,ms$$

Ex.5 Four channels are multiplexed using TDM. If each channel sends 100 bytes/s and we multiplex 1 byte per channel, show the frame traveling on the link, the size of the frame, the frame rate, and the bit rate of the link.

Solution



* Each frame carries 1 Byte from each channel:

∴ Frame size = 4 Bytes = 4 X 8 bits = 32 bits.

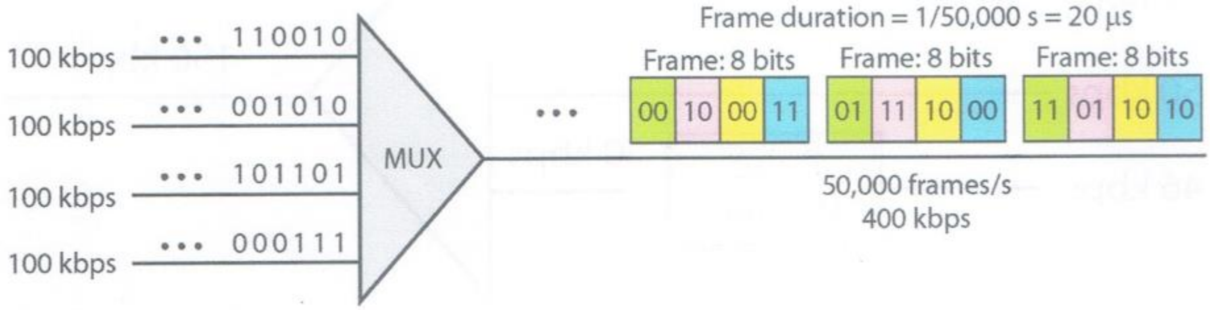
* The frame rate = input channel rate = 100 Frame per second

= 100 X 4 Bytes = 400 Bytes /s

= 400 X 8 bits = 3200 bits/s.

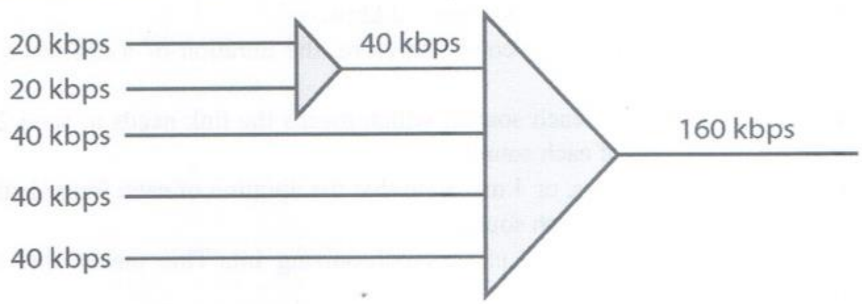
Q1: *A multiplexer combines four 100-kbps channels using a time slot of 2 bits. Show the output with four arbitrary inputs. What is the frame rate? What is the frame duration? What is the bit rate? What is the bit duration?*

Solution:

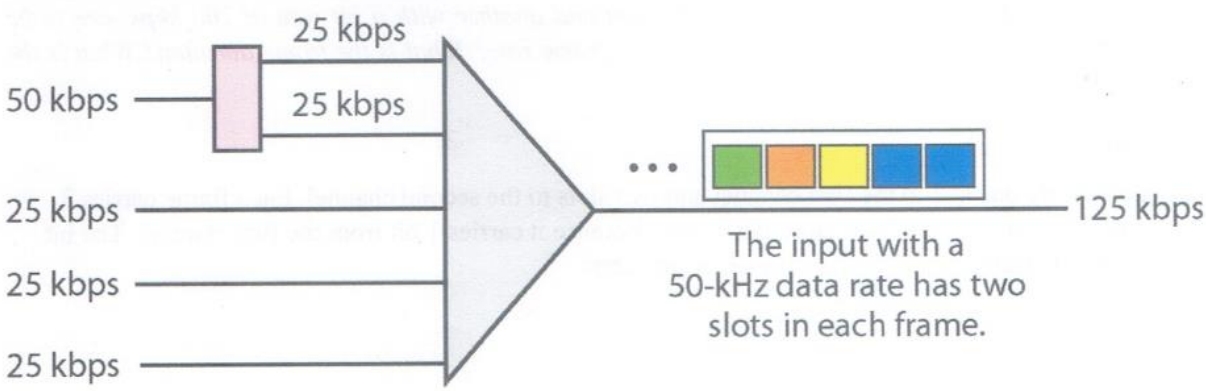


The output (4x100kbps) for 4 arbitrary inputs.
The link carries 400K/(2x4)=50,000
2x4=8bit frames per second.
The frame duration is therefore 1/50,000 s or 20 μs.
The bit duration on the output link is 1/400,000 s, or 2.5 μs.

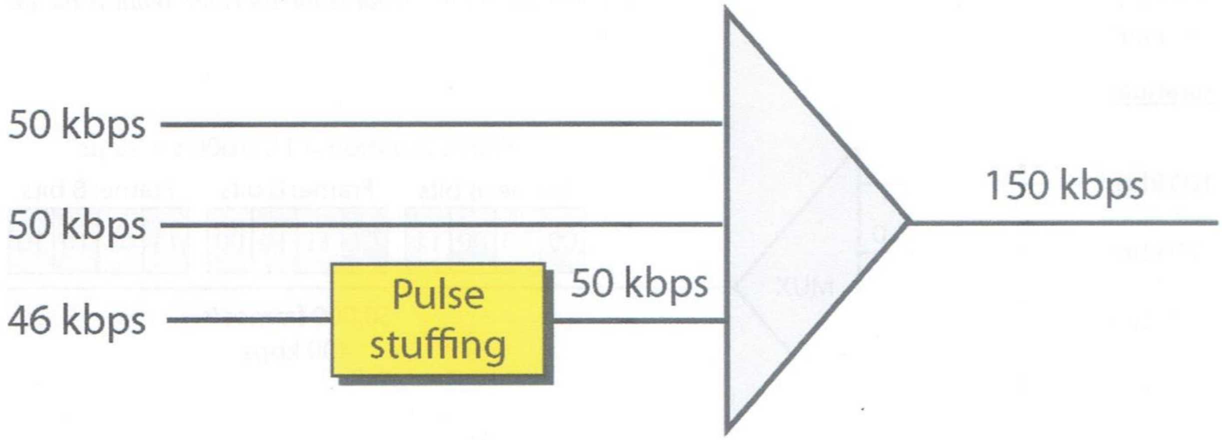
Example: Multilevel TDM Multiplexing



Example: Multiple-slot multiplexing



Example: Pulse stuffing



Q2: *We have four sources, each creating 250 8-bit characters per second. If the interleaved unit is a character and 1 synchronizing bit is added to each frame, find (a) the data rate of each source, (b) the duration of each character in each source, (c) the frame rate, (d) the duration of each frame, (e) the number of bits in each frame, and (f) the data rate of the link.*

Solution:

We can answer the questions as follows:

a. The data rate of each source is 250 × 8 = 2000 bps = 2 kbps.
b. Each source sends 250 characters per second; therefore, the duration of a character is 1/250 s, or 4 ms.
c. Each frame has one character from each source, which means the link needs to send 250 frames per second to keep the transmission rate of each source.
d. The duration of each frame is 1/250 s, or 4 ms. Note that the duration of each frame is the same as the duration of each character coming from each source.
e. Each frame carries 4 characters and 1 extra synchronizing bit. This means that each frame is 4 × 8 + 1 = 33 bits.

Q3: *Two channels, one with a bit rate of 100 kbps and another with a bit rate of 200 kbps, are to be multiplexed. How this can be achieved? What is the frame rate? What is the frame duration? What is the bit rate of the link*?

Solution:

We can allocate one slot to the first channel and two slots to the second channel. Each frame carries 3 bits. The frame rate is 100,000 frames per second because it carries 1 bit from the first channel. The bit rate is 100,000 frames/s × 3 bits per frame, or 300 kbps.

TDM Multiplexing Systems:

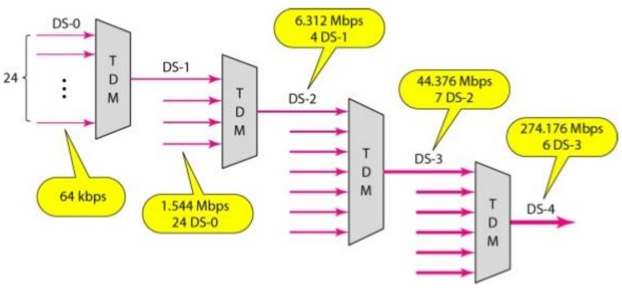
- The famous TDM application is in Telephone systems.
- *Wire and wireless* (mobile cellular phones).
- In 1960, the **T1** carrier, which carries ***24 digital telephone connections*** was used in North America.
- The Digital telephone speech signal is obtained by sampling a speech waveform ***8000 times/sec***.
- Each sample represented by **8 bits**.
- **T1** system's frame **contains 24 slots, each 8 bits**.
- Each slot carries one PCM sample for a single connection.
- Beginning of frame is single bit indicator.
- **T1** transmission speed is:

$$\left[\overbrace{1}^{indicator\ bit} + \overbrace{24}^{slots} \times \overbrace{8}^{PCM\ bits}\right] bits\ pes\ frame \times\ 8000\ frames\ per\ second$$

$$= (1 + 192) \times 8000$$

$$= 193 \times 8000 = 1544000 = 1.544\ Mbps$$

North American Digital Telephone Hierarchy:



Service	Line	Rate (Mbps)	Voice Channels
DS-1	T-1	1.544	24
DS-2	T-2	6.312	96
DS-3	T-3	44.736	672
DS-4	T-4	274.176	4032

- In North America and Japan, ***Digital System* 1** (**DS1**) was introduced:
  1. **DS0:** voice channel 64 Kbps
  2. **DS1:** basic building block (T1) = 1.544 Mbps
  3. **DS2:** 4× DS1 = 6.312 Mbps
  4. **DS3:** 28× DS2 = 44.736 Mbps
- In Europe the CCITT developed **CEPT** system:
  1. **CEPT1 or E1:** is 32 voice channel 64 Kbps = 2.048 Mbps.
  2. **CEPT2** or **E2**: is 4E1 = 8.44 Mbps.
  3. **CEPT3** or **E3**: is 4E2 = 16E1 = 34.368 Mbps.
  4. **CEPT4** or **E4**: is 4E3 = 64E1 = 139.264 Mbps.
- In **E1** only 30 channels of the 32 channels are used for voice channels.
- Thus, the two other channels are used as follows:
  - The first channel for signaling,
  - The second channel for alignment and link maintenance.