Department of Communications Engineering, College of Engineering, University of Diyala

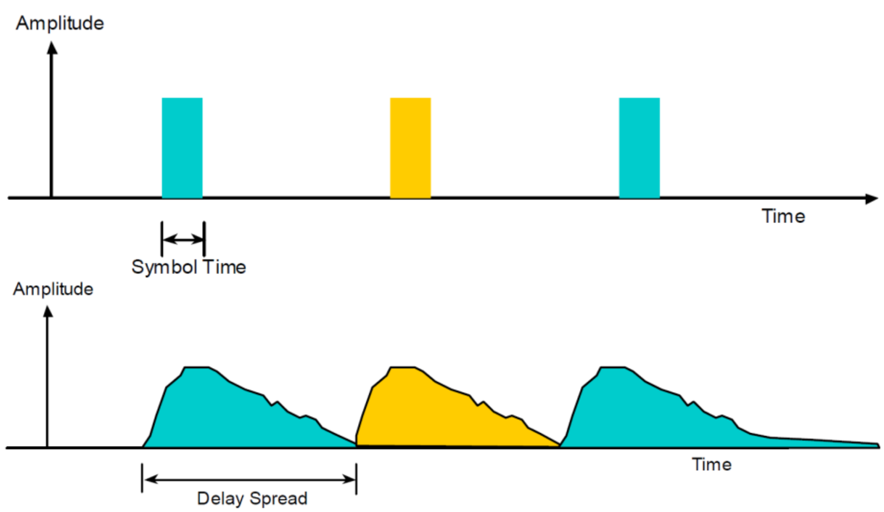
Digital Communication I

Lecture # 8

Optimum Receivers for the AWGN Channel: Receiver for Signals Corrupted by AWGN, Performance of Memoryless Modulation, Trade off of power, bandwidth, data rate, and error probability.

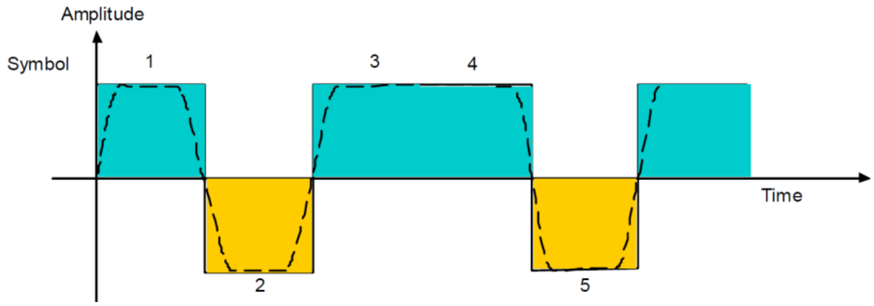
Inter Symbol Interference (ISI):

- **I**nter-**s**ymbol **i**nterference (**ISI**) is an unavoidable consequence of both wired and wireless communication systems.
- **Morse** first noticed it on the transatlantic telegraph cables transmitting messages using dots and dashes.



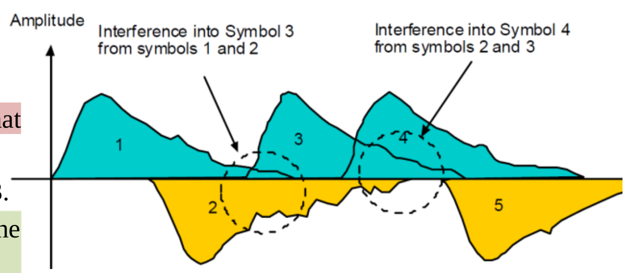
- It was noticed that the received signals tended to get ***elongated and smeared into each other***.
- The problem appeared to be related to the **properties of the medium** used and the **distance** of signal travel.
- To counter this undesired effect, **intermediate repeating stations were established** and ways had to be devised to reduce this smearing.

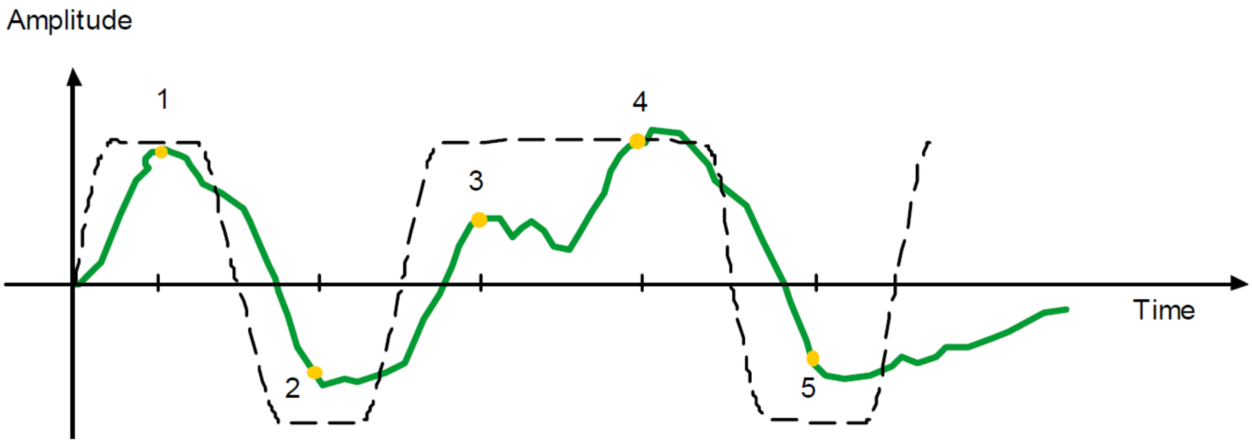
- The following Figure shows a data sequence, **1,0,1,1,0,1**.



- This sequence is in the form of square pulses.
- Square pulses are nice but in practice, they are hard to create and also require far too much bandwidth.
- So we shape them as shown in the dotted line.
- Advantage of shaping is to reduce the bandwidth requirements.

- The transmission medium creates a tail of energy that lasts much longer than intended does.
- The energy from **symbols 1 and 2** goes into **symbol 3**.
- Each symbol interferes with one or more of the subsequent symbols.





- The received signal is the sum of all distorted symbols.
- Compared to the dashed line that was transmitted signal, the received signal looks quite indistinct.
- Notice that for **symbol 3**, this value is approximately half of the transmitted value.

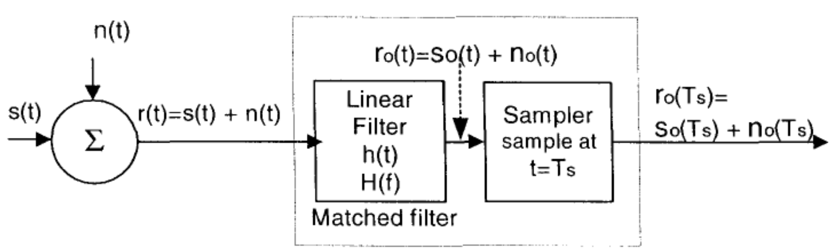
This spreading and smearing of symbols such that the energy from one symbol effects the next ones in such a way that the received signal has a higher probability of being interpreted incorrectly is called Inter Symbol Interference or ISI

- **ISI can be caused by many different reasons:-**
  - It can be caused by filtering effects from hardware
  - Frequency selective fading,
  - From non-linearities
  - From charging effects.
- **Very few systems are immune from it and it is nearly always present in wireless communications.**
- **Communication system designs for both wired and wireless nearly always need to incorporate some way of controlling it.**

Raised **C**osine **P**ulse (or it is also called **R**oot **R**aised **C**osine (***RRC***) filter) is one of the important solutions to the ISI problem.

Matched filter (MF)

- Is defined as a *linear system that maximizes at the sampling instant its output signal to noise power ratio (**SNR**).*
- The objective is to maximize the output signal power at the sampling instant no matter which shape the output signal has.
- Let us consider the system shown in below:



- The transfer function of the linear filter, $\boldsymbol{H}(\mathbf{f})$ that maximizes the output **SNR** should be found.
- The output signal at the sampling instant $\boldsymbol{T_s}$ is given by the convolution of the input signal with the impulse response of the filter $\boldsymbol{h}(\boldsymbol{t})$ at the sampling instant $\boldsymbol{T_s}$ or equivalently:

$$s_0(T_s) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi f T_s}\, df$$

- The output signal power is given by

$$P = |s_0(T_s)|^2$$

- The output noise average power is given by

$$\overline{n_0^2(t)} = R_{n_0}(0) = \int_{-\infty}^{\infty} |H(f)|^2 G_n(f) df$$

Where $G_n(f)$ is the **P**ower **S**pectral **D**ensity (**PSD**) of the input noise. The **SNR** at the output is given by:

$$SNR_0 = \frac{P}{R_{n_0}(0)}$$

Using *Schwarz inequality*, the $\boldsymbol{SNR_0}$ becomes:

$$SNR_0 \le \int_{-\infty}^{\infty} \frac{|S(f)|^2}{G_n(f)} df$$

- The maximum SNR_0 is obtained when $H(f)$ is chosen such that equality is attained. This occurs when $X(f) = KY^*(f)$, then $H(f)$ reduces to:

$$H(f) = K \frac{S^*(f) e^{-j2\pi f T_s}}{G_n(f)}$$

- The above equation is called ***Matched Filter for colored noise***.
- <u>**M**atched **F**ilter for **W**hite **N**oise</u> is a special case when the noise is a <u>**white noise**</u> with a power spectral density $G_n(f) = \frac{N_0}{2}\ W/Hz$, then $H(f)$ becomes:

$$H(f) = \frac{2K}{N_0} S^*(f) e^{-j2\pi f T_s}$$

- Taking the inverse Fourier transform of $H(f)$:

$$h(t) = \frac{2K}{N_0} [s(T_s - t)]^*$$

- Taking in account that the signal $s(t)$ is real signal and denoting $C = \frac{2K}{N_0}$, the impulse response of the matched filter becomes:

$$h(t) = Cs(T_s - t)$$

The impulse response of the **matched filter** is a delayed version of the mirror image of the signal form.

- However; **The maximum SNR_0 at the output** of *matched filter* in the case of white noise is determined as follows:

$$SNR_0 = \frac{2}{N_0}\int_{-\infty}^{\infty}|S(f)|^2 df$$

$$\boldsymbol{SNR_{max} = \frac{2E_s}{N_0}}$$

Where $\boldsymbol{E_s}$ the energy of finite duration input signal determined by:

$$E_s = \int_{-\infty}^{\infty}|S(f)|^2 df$$

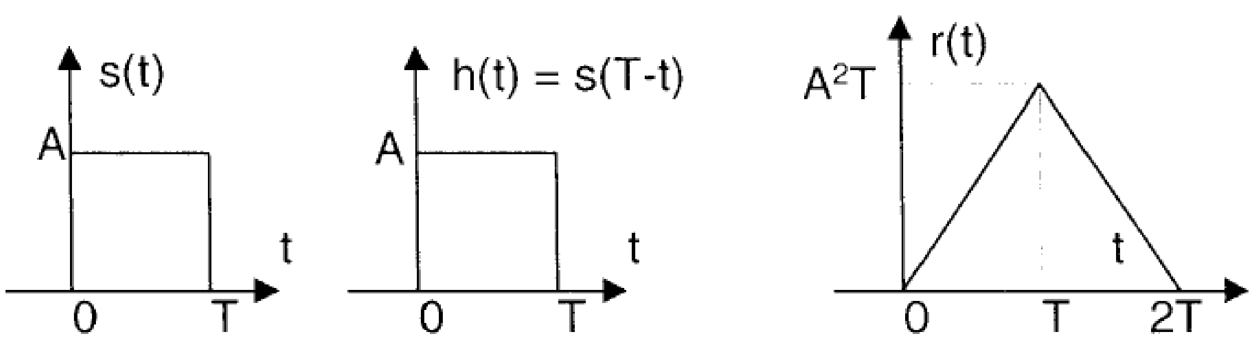
Integrate-And-Dump Matched Filter

- It is the *case when the signal pulse shape is rectangular* pulse such as:

$$s(t) = \begin{cases} A & 0 \le t \le T \\ 0 & otherwise \end{cases}$$

where $\boldsymbol{T}$ is the signal duration. The output of the matched filter is given by convolution so:

$$r(t) = s(t) * h(t) = s(t) * h(T-t) = \int_{-\infty}^{\infty} s(\tau)s(t-\tau)d\tau$$



$$r(t) = \begin{cases} 0 & t < 0 \\ A^2 t & 0 \le t \le T \\ 2A^2T - A^2 t & T \le t \le 2T \\ 0 & 2T \le t \end{cases}$$

It can be noted that the maximum value of $\boldsymbol{r(t)}$ is at $\boldsymbol{t = T}$ and it is:

$$r(t)_{max} = r(T) = A^2T$$

From equation $h(t) = Cs(T_s - t)$ the maximum value of signal to noise ratio at the output is:

$$\boldsymbol{SNR_{max} = \frac{2E_s}{N_0}}$$

Where $E_s = \int_{-\infty}^{\infty} s^2(t)dt = \int_0^T A^2dt = A^2T$, and N_0 is the PSD of the input noise. Hence,

$$SNR_{max} = \frac{2A^2T}{N_0}$$