Digital Communications ||

Third Year, 2ed Semester

Lecture No. 5

Ass. Lecturer: Yousif Allbadi
M.Sc. of Communications Engineering
Yousif_allbadi_eng@uodiyala.edu.iq
yousifallbadi@uodiyala.edu.iq

University of Diyala
College of Engineering
Department of Communications Engineering
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Fading Channels

Examples of Constructive and Distractive interference:

Let us try to understand this channel coefficient better. Let us consider a simple scenario, for **L = 2**, we have h, which is given as,

$$h = \sum_{i=0}^{1} a_i e^{-j2\pi F_c \tau_i}$$

let us consider a scenario in which $a_0 = a_1 = 1$

let us **considered** a simple scenario with $a_0 = a_1 = 1$ and $\tau_0 = 1$ and $\tau_1 = 1/2\ F_C$.

In this scenario:

$h = 1.\ e^0 + 1.\ e^{-j\,2\pi f c}\ 1/2fc$
$= 1 + e^{-j\pi}$
$= 1 + (-1)$
$= 0$

where: $e^{-j\pi} = -1 - j0$

Let some Examples :-

$L = 2 \quad , \quad h = \sum_{i=0}^{1} a_i e^{-j2\pi f_c \tau_i}$

$h = a_0 e^{-j2\pi F_c \tau_0} + a_1 e^{-j2\pi F_c \tau_1}$

Now let us consider simple scenario

$a_0 = a_1 = 1$ and $\tau_0 = 0 \quad \tau_1 = 1/2F_c$!!

So $h = 1 . e^0 + 1 . e^{-j\cancel{2}\pi \cancel{f_c} \cdot 1/\cancel{2f_c}}$

$= 1 + 1 e^{-j\pi} = 1 + (-1) = 0$

$\boxed{Exp(-j\pi) = -1 - j0}$

So $y(t) = h . s(t) = 0\ s(t) = 0$

?

this Destructive interferance

another c...

So, what we have is h the channel coefficient reduces to 0 and you can see that because both the components have equal amplitude 1 and they have a phase that is exactly the opposite of each other 1 is the phase of 0 the other has a phase of pie and therefore, these components are canceling each other as a result of these multipath components canceling each other the channel coefficient h is 0 and therefore, the received signal y, Y(t) which is h times S(t) which is equal to 0 times S(t) equals 0 therefore, the received signal is perfectly canceled and this is a case of this is an example of destructive interference. So, in this case, this h is 0 which shows destructive this is destructive interference.

further consider another scenario where again

$a_0 = a_1 = 1$ and $\tau_0 = 0$ and $\tau_1 = 1/fc$.

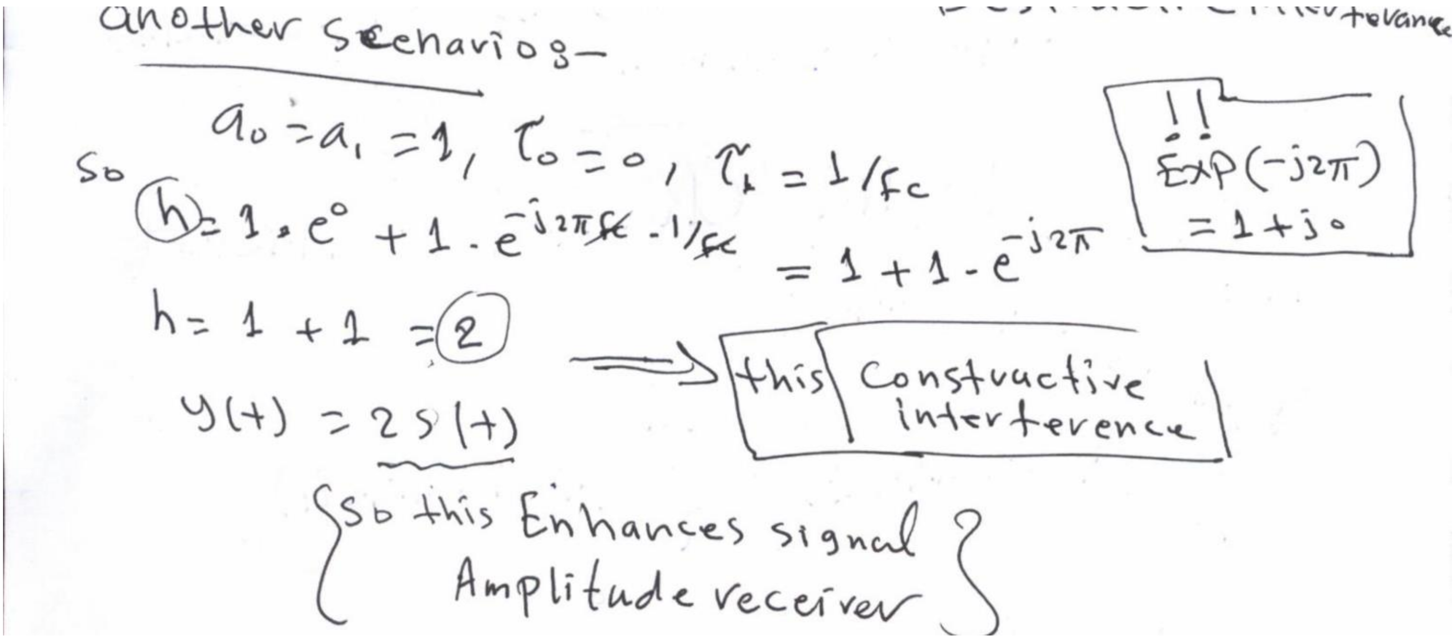
In this case you can see,

$h = 1.\, e^0 + 1.\, e^{-j\,2\,\pi\, f\,c\; 1/fc}$

$= 1 + e^{-j2\pi}$ where: $e^{-j2\pi} = 1 + j0$

$= 1 + 1$

$= 2$



We can see since h is too, we have

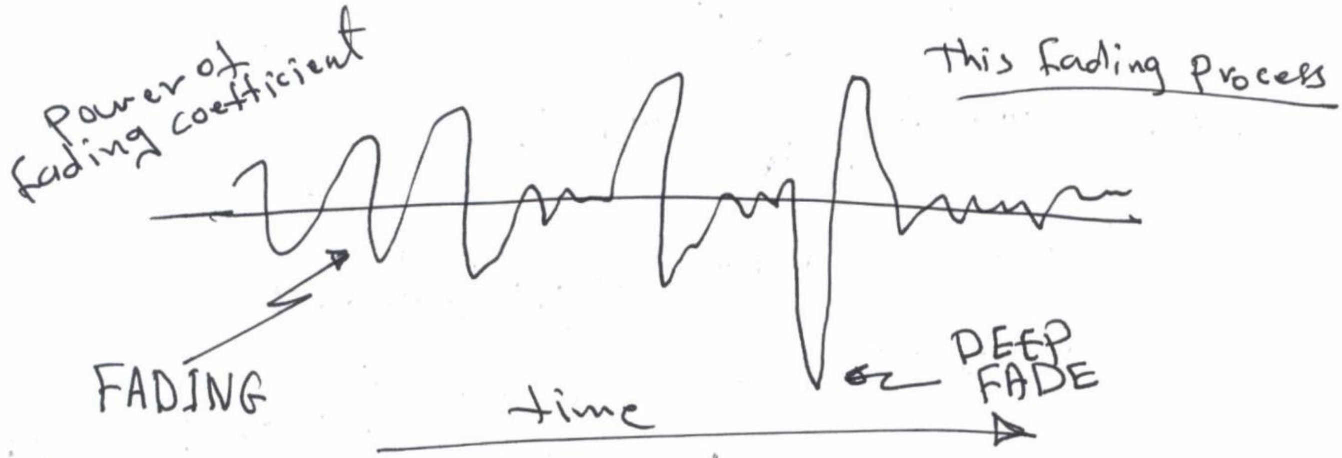
Y(t) = 2. S(t)

That is twice the transmitter signal S(t) and now you can see because these 2 signals are adding up constructively. So, they are adding up to each other constructively. So, this is constructive interference and as a result of this, you see enhanced signal amplitude. So, this is enhanced signal amplitude.

The coefficient h varies depending on the various channel amplitude or various channel attenuation factors a_i and the delays τ_i alright and therefore, as these attenuations and delays are changing the channel h, the expression for h is:

$$h = \sum_{i=0}^{L-1} \mathrm{a_i} e^{-j2\pi F_c \tau_i}$$

The coefficient h varies Depending on the various channel attenuation factor a_i, Delays τ_i



h = fading channel coefficient.

* This fading Process, which causes the reciver power to vary is important and Key aspect of a wireless Communication Systems.

This process is known as a fading process and this is known as the h, is an important aspect of wireless communication systems this is known as the fading channel coefficient. As we can see here sometimes when the paths perfectly cancel each other the signal amplitude or the received signal amplitude goes all the way can go all the way up to 0 go to very low amplitude is known as a deep fade. And this has a significant impact on the performance of a wireless communication system. Therefore, it is the fading, this fading process that results from the multipath wireless communication environment.

Fading Channel Coefficient

Fading Channel Coefficient

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

$$= \underbrace{\sum_{i=0}^{L-1} a_i(\cos 2\pi f_c \tau_i)}_{x} - j\underbrace{\sum_{i=0}^{L-1} a_i(\sin 2\pi f_c \tau_i)}_{y}$$

We can write the fading chanel coefficient as:

$$\boxed{h = x + jy}$$

where

$$x = \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i)$$

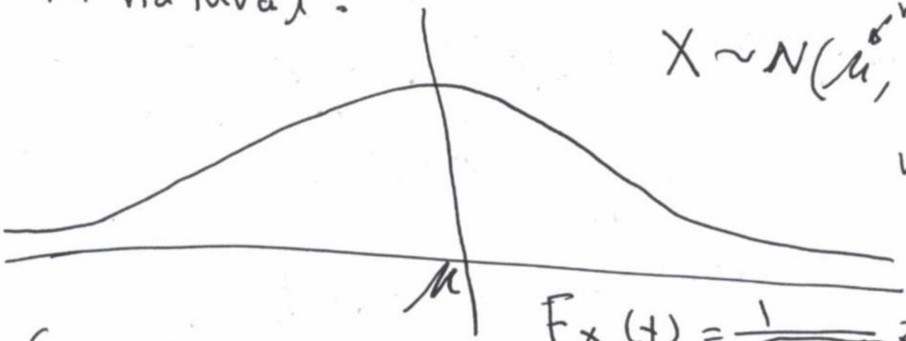
$$y = \sum_{i=0}^{L-1} a_i \sin(2\pi f_c \tau_i)$$

This is depending on the attenuations a_i and the delays τ_i.

x, y : are the sum of a Large number of random component

Hence

x, y : can be assumed to be Gaussian distributed in natural.



$X \sim N(\mu, \sigma^2)$ mean, Variance

$F_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

PDF of Gaussian random

$$\boxed{F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$

This is the PDF of this Gaussian random Variable. Further, We are going to assume that x, y are independent Gaussian random variable with mean 0 and normalized to Variance $\frac{1}{2}$ each. So x is Gaussian random Vairable which is distvibuted with mean $= 0$ and Variance $= \frac{1}{2}$ σ^2

$$X \sim N\left(0, \frac{1}{2}\right) \quad \mu = 0, \quad \sigma^2 = \frac{1}{2}$$

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$F_X(x) = \frac{1}{\sqrt{2\pi \cdot \frac{1}{2}}} \cdot e^{\frac{-x^2}{2 \cdot \frac{1}{2}}} = \frac{1}{\sqrt{\pi}} e^{-x^2}$$

$$\therefore \boxed{F_X(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}} \quad \text{and} \quad \boxed{F_Y(y) = \frac{1}{\sqrt{\pi}} e^{-y^2}}$$

We have the Joint distribution

$$F_{X,Y}(x,y) = \underbrace{F_X(x)\, F_Y(y)}_{\text{is Product}}.$$

$$F_{X,Y}(x,y) = \frac{1}{\sqrt{\pi}} e^{-x^2} \cdot \frac{1}{\sqrt{\pi}} e^{-y^2}$$

$$= \frac{1}{\pi} e^{-(x^2+y^2)}$$

Now we are going to convert h as

$$h = x + jy = a e^{j\phi}$$ where $a = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}\frac{y}{x}$

also: $x = a\cos\phi$
$y = a\sin\phi$

$$F_{X,Y}(x,y) \Longrightarrow F_{A,\phi}(a,\phi)$$

$$F_{A,\phi}(a,\phi) = \frac{1}{\pi} e^{-(x^2+y^2)} \left|J_{XY}\right|$$

$$\boxed{x^2+y^2=a^2}$$

$$= \frac{1}{\pi} e^{-a^2} \left|J_{X,Y}\right| \longrightarrow \text{Jacobian of } X,Y$$

$$J_{X,Y} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{bmatrix}$$

$$\boxed{\begin{aligned} x &= a\cos\phi \\ y &= a\sin\phi \end{aligned}}$$

$$\left|J_{XY}\right| = a\cos^2\phi - (-a\sin^2\phi)$$
$$= a(\cos^2\phi + a\sin^2\phi)$$
$$= a$$

Hence, x y can be assumed to be Gaussian, this has to be Gaussian in nature. So, we are

Assuming x and y to be Gaussian random variables. A Gaussian random variable has a PDF that looks like a bell-shaped curve which is a probability density function that is a Gaussian random variable that is centered at the mean μ of the Gaussian random variable. So, x is a Gaussian random variable which is denoted as N that is with mean μ and variance σ^2 that is the spread of this Gaussian random variable; that is the width of this bell curve is related to which variance σ^2, the mean is μ and the PDF of this Gaussian random variable this is given as

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

This is the PDF of this Gaussian random variable.

Further, we are going to assume that this x and y are independent Gaussian random variables with mean 0 and normalized to variance ½

$\mu = 0$ and $\sigma^2 = ½$

Therefore, we have the determinant of this Jacobian matrix is simply equal to **a**; and therefore, the Joint distribution for the magnitude and phase components equals

$$F_{A,ø}(a, ø) = \frac{1}{\pi} e^{-a^2} \left| J_{XY} \right| = \frac{1}{\pi} e^{-a^2} . a$$

$$F_{A,ø}(a, ø) = \frac{a}{\pi} e^{-a^2}$$

This is the Joint distribution of the channel coefficient in terms of the magnitude and phase component. This is the distribution of h or the Joint distribution of A and ø which are the amplitude and phase of the Fading Channel Coefficient h

So $|J_{XY}| = a$

$$F_{A,\phi}(a,\phi) = \frac{1}{\pi} e^{-a^2} |J_{XY}|$$

$$= \frac{1}{\pi} e^{-a^2} \cdot a$$

$$= \frac{a}{\pi} e^{-a^2}$$

rember $h = a e^{j\phi}$ → Phase

→ Amplitude

Now, Let us derive the individual distribution of the Amplitude A and the phase that is ϕ

$$F_A(a) = \int_{-\pi}^{\pi} F_{A,\phi}(a,\phi)\, d\phi$$

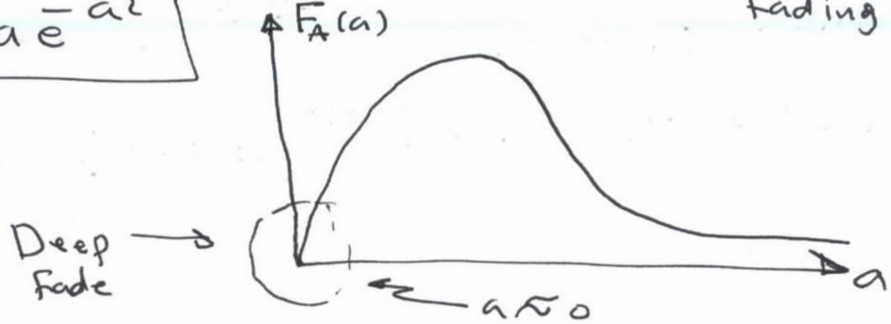
$$= \int_{-\pi}^{\pi} \frac{a}{\pi} e^{-a^2}\, d\phi = \frac{a}{\pi} e^{-a^2} \int_{-\pi}^{\pi} d\phi$$

$$= \frac{a}{\pi} e^{-a^2} \cdot 2\pi \quad \boxed{= 2a e^{-a^2}}$$

This is known as Rayleigh fading or Rayleigh Distribution

Hence: The Coefficient channel h is ~~also~~ Rayleigh fading channel

$$\boxed{F_A(a) = 2a e^{-a^2}}$$



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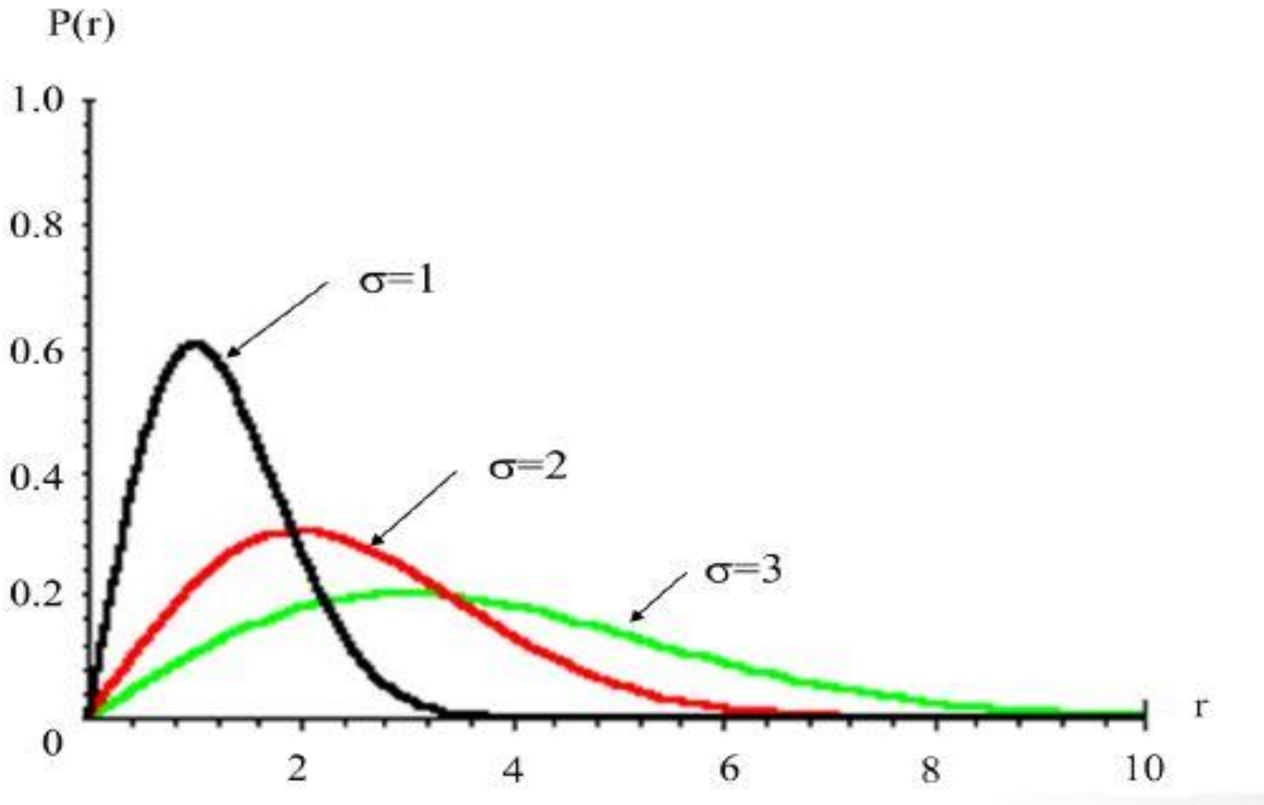
Rayleigh Fading

- Rayleigh fading is a model for urban environments where many objects in the environment scatter the radio signal before it arrives at the receiver
- There is no dominant propagation along line-of-sight LOS between the transmitter and receiver.
- The cover of the channel response will be Rayleigh distributed.



$$F_\phi(\phi) = \int_0^\infty \frac{a}{\pi} e^{-a^2} da = \int_0^\infty \frac{1}{2\pi}\left(2a e^{-a^2}\right) da$$

$$= \frac{1}{2\pi}\left(-e^{-a^2}\Big|_0^\infty\right) = \frac{1}{2\pi}$$

So $\boxed{f(\phi) = \frac{1}{2\pi}}$ $\Leftarrow$ Uniform distirbution in $[-\pi, \pi]$

$$F_{A,\phi}(a,\phi) = \frac{a}{\pi} e^{-a^2} = \underbrace{\left(\frac{1}{2\pi}\right)}_{f_\phi(\phi)} \underbrace{\left(2a e^{-a^2}\right)}_{F_A(a)}$$

So

A, ϕ are Independent

$$\boxed{\begin{array}{lll} \ast \text{ Distirbution of } a: & 2ae^{-a^2} & \text{---- } 0 \le a \le \infty \\ \ast \text{ Density of } \phi: & \frac{1}{2\pi} & \text{---- } -\pi \le \phi \le \pi \end{array}}$$

is very important *

we have these are the distributions, the densities of the amplitude, and the phase; that is the amplitude a and phase ø of the wireless channel. These can now be used to characterize, and derive various properties of the wireless channel.

What we have is that the joint density for the phase and amplitude is equal to the product of the marginal densities for the amplitude and the phase.

The joint density is equal to the product of the marginal densities therefore; this implies that the amplitude and the phase are independent random variables. The amplitude and phase of the Rayleigh fading channels are independent random variables. This means that A, ø the amplitude, and phase are independent. And these densities can be used to derive valuable properties of the fading channel for instance.