Digital Communications II

Third Year, 2ed Semester

Lecture No. 6

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Rayleigh Fading Channel

Let Some Examples:

Let us look at the following example to understand this better. What is the probability? That attenuation of the channel is worse than 20 dB.

Example ①: what is the probability of wireless channel that attenuation is worse than (or loss) -20 dB

Sol.

Power in dB $\Rightarrow 10 \log_{10} a^2 \leq -20$

$\Rightarrow \log_{10} a^2 \leq -2 \Rightarrow a^2 < 10^{-2} \Rightarrow a^2 \leq 0.01$

$\Rightarrow \boxed{a \leq 0.1}$

So the: $P(a \leq 0.1) = \int_0^{0.1} 2a e^{-a^2} da = -e^{-a^2} \Big|_0^{0.1}$

$\therefore P(a \leq 0.1) = 1 - e^{-0.01} = 0.01 = 1\%$

The gain of the channel is attenuation is worse than minus 20 dB if the amplitude of the channel is less than 0.1.

Example 2:

what is the probability of wireless channel that Phase ϕ is $[\frac{-\pi}{3}, \frac{\pi}{3}]$.

Sol.

$P(\frac{-\pi}{3}, \frac{\pi}{3}) = \int_{-\pi/3}^{\pi/3} \frac{1}{2\pi} d\phi = \frac{1}{2\pi}\left(\frac{\pi}{3} - \left(\frac{-\pi}{3}\right)\right)$

$= \frac{1}{2\pi} \cdot \frac{2\pi}{3} = \boxed{\frac{1}{3}}$

Performance of Wireless and Wireline Communication Systems

Bit Error Rate BER

Bit Error Rate? as you know in the communication system, we transmit information bits from the transmitter to the receiver and these information bits can be resulted as a digital stream of binary information symbols?

Performance of wireless and wireline communication systems

Bit-Error Rate (BER)

Performance of comm. system.

1 0 0 1 1 1 0 1 0 0 ----

1 0 0 0 1 1 0 0 0 0 ----

Bit error

So: BER: Probability of Bit error in information stream

Ex.

10000 bits = 10 kbits

and 100 Received in Error

$$BER = \frac{100}{10000} = 10^{-2} = 0.01$$

* To understand the BER of comm. systems Let us compare between wireless and wireline comm. systems.

So, in a communication system 1 transmits information bits such as, for instance, 1 transmits bits of information from the transmitter to receiver these are binary information symbols for instance, 1 0 0 1 1 1 0 1 0 0 0 this is the possible stream of information bits. This is also known as a bit stream which is transmitted from the transmitter to the receiver. However, all the transmitter bits are not received correctly by the receiver frequently there are errors during the reception of these bits. So, there are bit errors for instance, corresponding to this transmit stream one might receive at the receiver the transmit stream might be decoded as 1 0 0 0 1 1 0 0 0 0 0. So, we can clearly see that, this bit 1 transmitted bit has changed to a 0 at the receiver. The Bit Error Rate is that the average rate at which these bit errors are occurred during the communication process.

BER of wireline Communication system

$y = 1x + n$

$\boxed{y = x \oplus n}$ ↑ ADDitive

n is white Gaussian Noise

$n \sim \mathcal{N}(0, \sigma_n^2)$

AWGN = Additive White Gaussian Noise

let assume

$1 : +1 \times \sqrt{P}$

$0 : -1 \times \sqrt{P}$

BPSK ⟹ Binary phase shift keyed system

So $1 \quad \sqrt{P}$, $0 \quad -\sqrt{P}$ } Digital comm system

- Consider the case where the bit zero is transmitted

$y = -\sqrt{P} + n$

Bit-error occurs if $y > 0$

$\Rightarrow n - \sqrt{P} > 0$

$n > \sqrt{P}$

and: $n \sim \mathcal{N}(0, \sigma_n^2)$

$$P(n > \sqrt{P}) = \int_{\sqrt{P}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{x^2}{2\sigma_n^2}} dx$$

let assume:

$\frac{x}{\sigma_n} = t, \; dx = \sigma_n dt$

$$\text{So } P(n > \sqrt{P}) = \int_{\sqrt{P}/\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{t^2}{2}} \cdot \sigma_n dt = \int_{\sqrt{P}/\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

⇓ Gaussian with $\mu = 0$, $\sigma^2 = 1$

The Standard Q-function

$$Q(v) = \int_{v}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

So the Bit-error rate of wireline Comm. system

$$\boxed{= Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right)}$$

remember $y = x + n$

P = signal power

σ_n^2 = noise power

The bit error rate of the wire line communication system that is without multi path proposition with only additive white Gaussian noise without any fading is given as the

probability of bit error is equal to $= Q(\sqrt{\frac{P}{\sigma^2}})$

The probability of that error for BPSK modulated transmission for BPSK modulated transmission with average power P, further you can also see as $\frac{P}{\sigma^2}$ equals the SNR and therefore, this bit error rate is also

$Q(\sqrt{\frac{P}{\sigma^2}}) = Q(\sqrt{SNR})$ that is the Q function of the $\sqrt{SNR}$ Signal to Noise Power Ratio of the communication system.

$$BER = Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right) = Q(\sqrt{SNR})$$

BER of Wireline Comm. System

Example:- At $SNR_{dB} = 10$ dB, what is the BER of wireline communication system?

Sol.

$SNR_{dB} = 10 \log_{10} SNR$

$10 \log_{10} SNR = 10$ dB

$\log_{10} SNR = 1$

$SNR = 10$

From Table of Q-Function

$BER = Q(\sqrt{10}) = 7.82 \times 10^{-4}$

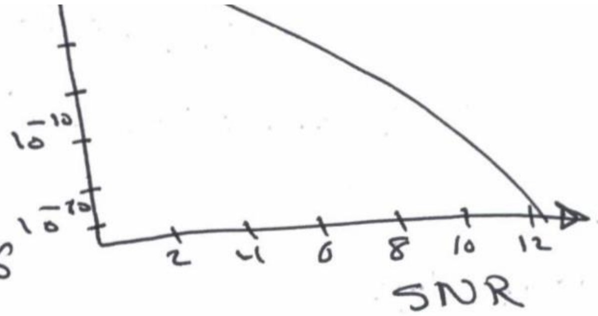
bits in error in 10,000 bits

$= 7.82 \times 10^{-4} \times 10000 = 7.82$ bits

As we have said there is no close formed equation for this Q function correct. Therefore, this has to be evaluated using some software or online tables that are available for this Q function and this gives us the value of 7.82×10^{-4} which is the bit error rate.

One mor Ex.

Compute the SNR_{dB} required for a probability of bit-error rate (BER) $= 10^{-6}$



Sol.

$$10^{-6} = Q(\sqrt{SNR})$$

$$\sqrt{SNR} = Q^{-1}(10^{-6})$$

$$SNR = \left(Q^{-1}(10^{-6})\right)^2$$

from Q-table

$$SNR = (4.7534)^2 = 22.595$$

$$SNR_{dB} = 10 \log_{10}(22.595)$$

$$= 13.6\ dB$$

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BER Analysis of Wireless Communication Systems

we are going to short looking at bit error rate BER of wireless communication system, and as we said previously in a wireless communication system there is fading due to multi path nature of the propagation of the signal in the wireless communication environment.

BER Analysis of wireless comm-system

$$y = hx + n$$

$$h = a e^{j\phi}$$

To You!!

$$BER_{\text{of wireless channel}} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

Example: Compute the BER of a wireless Comm. system at SNR = 20dB

Sol

$$20\,dB = 10 \log_{10} SNR$$

$$\log_{10} SNR = 2$$

$$SNR = 10^2 = 100$$

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{100}{2+100}}\right) = 0.0049 = 4.92 \times 10^{-3} \approx 5 * 10^{-3}$$

Example 2:

Compute SNR_{dB} of a wireless Communication system for $BER = 10^{-6}$?

Wireless	Wireline
SNR = 20dB	SNR = 10dB
$BER = 5 \times 10^{-3}$	$BER = 7.8 \times 10^{-4}$

Sol.

$$10^{-6} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right) \Rightarrow \frac{1}{2}\left(\sqrt{\frac{SNR}{2+SNR}}\right) = \frac{1}{2} - 10^{-6}$$

$$\frac{1}{2}\sqrt{\frac{SNR}{2+SNR}} = \left(\frac{1}{2} - 10^{-6}\right) \Rightarrow \sqrt{\frac{SNR}{2+SNR}} = \left(1 - 2 \times 10^{-6}\right)$$

$$\Rightarrow \frac{SNR}{2+SNR} = (1 - 10^{-6})^2 \Rightarrow SNR = \frac{2 \times (1 - 2 \times 10^{-6})^2}{1 - (1 - 2 \times 10^{-6})^2}$$

$$\therefore SNR = 4.99 \times 10^{5} \quad SNR_{dB} = 20 \log_{10}(4.99 \times 10^{5})$$

$$= 56.98\,dB$$

So, let us do another simple example to understand this better. So, another example is now using our approximate formula, using our approximation that we have recently developed using the formula for approximate bit error rate in the wireless system, again calculate SNR for bit error rate equals 10^{-6} in or wireless system and for this purpose what do we have we have 10^{-6}, remember now we have to use approximate formula, which is approximate formula is if you look at is $\frac{1}{2SNR}$

$$BER = \frac{1}{2}\left(1-\sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1-\sqrt{\frac{1}{1+\frac{2}{SNR}}}\right) \quad \div SNR$$

$$= \frac{1}{2}\left(1-\left(1+\frac{2}{SNR}\right)^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(1-\left(1-\frac{1}{2}\cdot\frac{2}{SNR}\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2}\cdot\frac{2}{SNR}\right)$$

$$\boxed{\approx \frac{1}{2SNR}} \quad \text{BER for Approximate}$$

$$10^{-6} = \frac{1}{2SNR}$$

$$SNR = \frac{1}{2} . 10^{6} = 0.5 \text{ x } 10^{6}$$

$$SNR_{dB} = 10 \log_{10} SNR = 10 \log_{10} (0.5 \text{ x } 10^{6}) = 56.98$$

$$SNR_{dB} = \approx 57 \text{ dB}$$

The Diversity

Simple idea is to consider a system with multiple links that is I have a transmitter. we have multiple links between the transmitter and receiver. Now let us say a few of these links are in the deep fade, let say two of these links are in deep fade. the important thing to notice here is that there are alternative paths right.

The communication is not disrupted in this scenario because there are multiple links even if one or a few links are in a deep fade there are alternative paths for the signal to propagate from the transmitter to receiver hence communication is not disrupted, means that there is diversity in the system.

What is the meaning of diversity?

Diversity means that there are alternatives right means there is not one, but there are many.

Diversity means that there are many alternative paths or there is diversity in the system.

This principle where you have multiple modes between the transmitter and receiver this is the principle and this is an important principle of diversity.