FIR and IIR System Implementation

7.1 Definitions

1. If unit sample response h(n) is of finite duration, the system is said to be a finite impulse response (FIR) system. Eq. (9.1) represents FIR system if $a_0 \neq 0$ and $a_k = 0$ for k=1, 2,..N.

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k) \qquad (9.1)$$

2. If unit sample response h(n) is of infinite duration, the system is said to be an infinite impulse response (IIR) system.

3. IIR system is usually implemented by recursive realization (is one in which the present value of the output depends on both the input present and or past values), i.e., ***with feedback***.
4. FIR system is usually implemented by either a non-recursive realization **(*without feedback*)** or an FFT realization.

7.2 A comparison between FIR and IIR systems:

FIR	IIR
1- Finite impulse response h(n) $n_1 \leq n \leq n_2$	1- Infinite impulse response h(n) $n_1 \leq n \leq \infty$
2-Complex requires large number of computations	2- Simple, does not require large number of computations
3- Due to large number of computations, it requires large memory	3- Dose not require large memory
4- Always stable because its poles lie at the origin	4- Stable only if its poles lie inside the unit circle of the Z-plane
5- Linear phase characteristics	5- nonlinear phase characteristics

7.3 Infinite Impulse Response (IIR) format

An IIR is described using the difference equation (9.1) as:

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_M x(n-M) - a_1 y(n-1) - \cdots - a_N y(n-N).$$

The IIR filter transfer function given in eq.(9.2) as:

$$H(Z) = \frac{\sum_{k=0}^{M} b_k \; Z^{-k}}{\sum_{k=0}^{N} a_k \; Z^{-k}} \qquad (9.2)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}},$$

Example (1): Given the following IIR:
y(n) = 0.2 x(n) + 0.4 x(n − 1) + 0.5 y(n − 1),
Determine the transfer function, nonzero coefficients, and impulse response.

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.2 + 0.4z^{-1}}{1 - 0.5z^{-1}}.$$

$$b_0 = 0.2,\ b_1 = 0.4,\ \text{and}\ a_1 = -0.5.$$

7.4 Realization of IIR systems

Transfer function H(z) may be generally realized in the following forms:

1. Direct form I realization.
2. Direct form II realization.
3. Cascade realization.
4. Parallel realization.

7.4.1 Direct-Form I Realization

Transfer function H(z), is given by:

$$H(Z) = \frac{B(Z)}{A(Z)} = \frac{b_0 + b_1 Z^{-1} + \ldots + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + \ldots + a_N Z^{-N}}$$

Let x(n) and y(n) be the digital filter input and output, respectively. Taking z-transform:
Y(Z) = H(Z) X(Z)

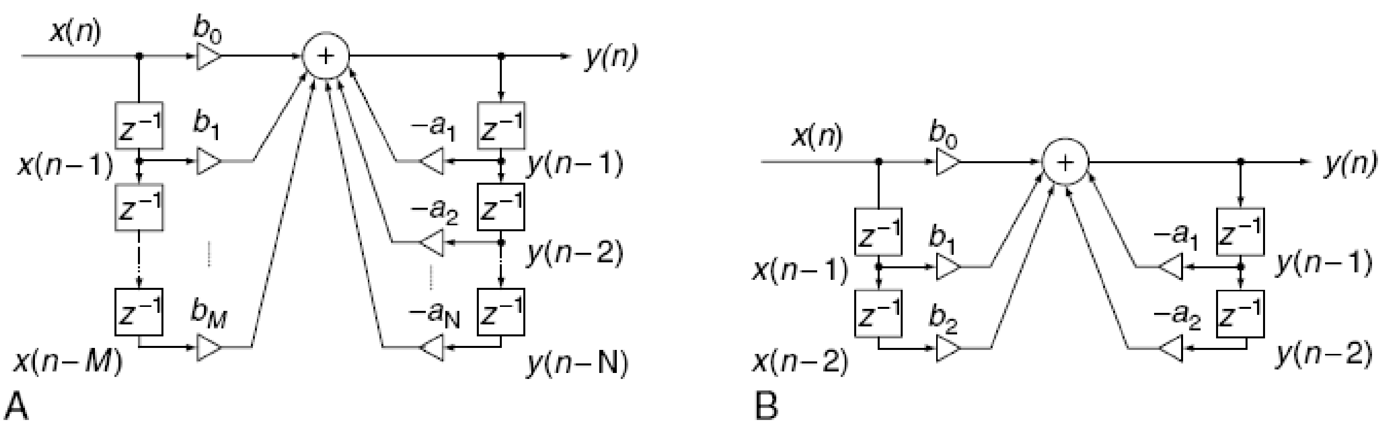
Where X(z) and Y(z) are the z-transforms of x(n) and y(n), respectively. then

$$Y(Z) = \left(\frac{b_0 + b_1 Z^{-1} + \ldots + b_M Z^{-M}}{a_0 + a_1 Z^{-1} + \ldots + a_N Z^{-N}} \right) X(Z)$$

Taking the inverse of the z-transform of Equation above, then:

$$y(n) = b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) - a_1 y(n-1) - a_2 y(n-2) - \ldots - a_N y(n-N)$$

This difference equation thus can be implemented by a direct-form I realization shown in Fig. (9.1A). Figure (9.1B) illustrates the realization of the second-order IIR (M = N = 2).



Example (1): Given a second-order transfer function

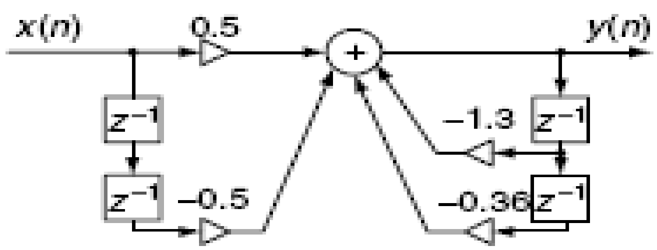
$$H(Z) = \frac{0.5(1 - Z^{-2})}{1 + 1.3Z^{-1} + 0.36Z^{-2}}$$

write the difference equations using: Direct form I

s0l: To perform the realizations using the direct form I

$$H(Z) = \frac{0.5(1 - Z^{-2})}{1 + 1.3Z^{-1} + 0.36Z^{-2}}$$

Where, $a_1 = 1.3$, $a_2 = 0.36$, $b_0 = 0.5$, $b_1 = 0$, and $b_2 = -0.5$. Fig. shows the direct-form I realization .
The difference equation for the direct- form I realization is given by
y(n) = 0.5 x(n) – 0.5 x(n–2) – 1.3 y(n-1) –0.36 y(n–2)



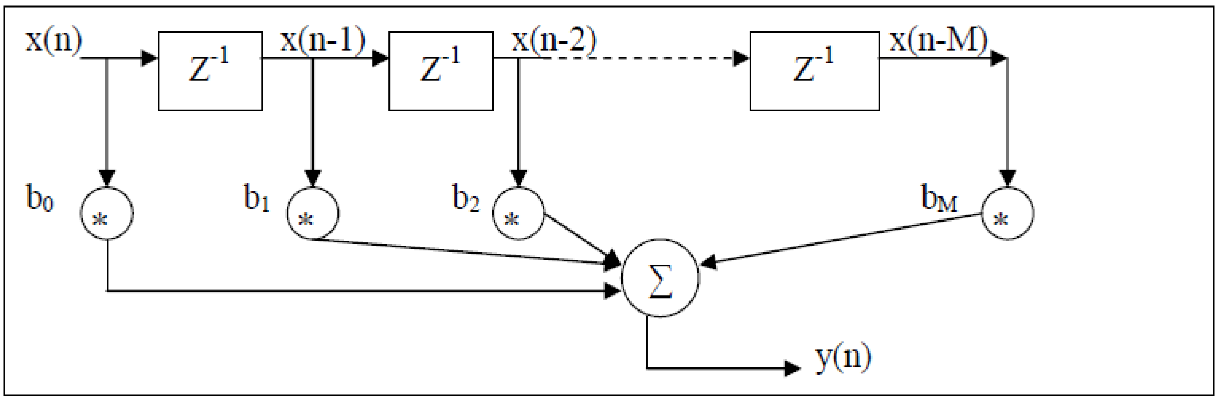
7.5 Realization of FIR

A causal FIR is characterized by:

$$H(Z) = \sum_{k=0}^{M} b_k \, Z^{-k}$$

$$y(n) = \sum_{k=0}^{M} b_k \, x(n-k)$$

The output is simply a weighted sum of present and past input values, as shown in Figure below



H.W 1: Given the following difference equation,

$$y(n) = 0.5x(n) + 0.5x(n-1).$$

find the H(z)

H.W2: Given the following difference equation,

$$y(n) = x(n) - 0.5y(n-1),$$

find the H(z)

H.W3: Convert each of the following transfer functions into its difference equation:

a. $H(z) = \frac{z^2 - 0.25}{z^2 + 1.1z + 0.18}$

b. $H(z) = \frac{z^2 - 0.1z + 0.3}{z^3}$

H.W4: Given the first-order IIR system

$$H(z) = \frac{1 + 2z^{-1}}{1 - 0.5z^{-1}}$$

realize H(z) and develop the difference equations using the direct-form I