Filters

9.1 Introduction

Filters are networks that process signals in a frequency-dependent manner. The basic concept of a filter can be explained by examining the frequency dependent nature of the impedance of capacitors and inductors.

Filters have many practical applications:

1. A simple, single-pole, low-pass filter (the integrator) is often used to stabilize amplifiers by rolling off the gain at higher frequencies where excessive phase shift may cause oscillations.
2. A simple, single-pole, high-pass filter can be used to block dc offset in high gain amplifiers or single supply circuits.
3. Filters can be used to separate signals, passing those of interest, and attenuating the unwanted frequencies. An example of this is a radio receiver, where the signal you wish to process is passed through, typically with gain, while attenuating the rest of the signals.
4. In data conversion, filters are also used to eliminate the effects of aliases in A/D systems. They are used in reconstruction of the signal at the output of a D/A as well, eliminating the higher frequency components, such as the sampling frequency and its harmonics, thus smoothing the waveform.

An ideal filter will have an amplitude response that is unity (or at a fixed gain) for the frequencies of interest (called the *pass band*) and zero everywhere else (called the *stop band*). The frequency at which the response changes from passband to stopband is referred to as the *cutoff frequency*.

9.2 Types Of Filters

9.2.1 There are many types of electronic filters and many ways that they can be classified. A filter's frequency selectivity is probably the most common method of classification. A filter can have a low pass, high pass, band pass, or band stop response, where each name indicates how a band of frequencies is affected as explained below:

1. **Low pass filter:** In this filter, the low frequencies are in the pass band and the higher frequencies are in the stop band. An idealized low pass filter is shown in Figure 9.1(a).

2. **High pass filter:** in this filter, the low frequencies are in the stop-band, and the high frequencies are in the pass band. An idealized high pass filter is shown in Figure 9.1(b).
3. **Band Pass Filter**: If a high-pass filter and a low-pass filter are cascaded, a *band pass* filter is created. this filter passes a band of frequencies between a lower cutoff frequency, f l, and an upper cutoff frequency, f h. Frequencies below f l and above f h are in the stop band. An idealized band pass filter is shown in Figure 9.1(C).

4. ***Band-Reject*, Or *Notch* Filter (Band Stop):** Here, the passbands include frequencies below f l and above f h. The band from f l to f h is in the stop band. Figure 9.1(D) shows a notch response.

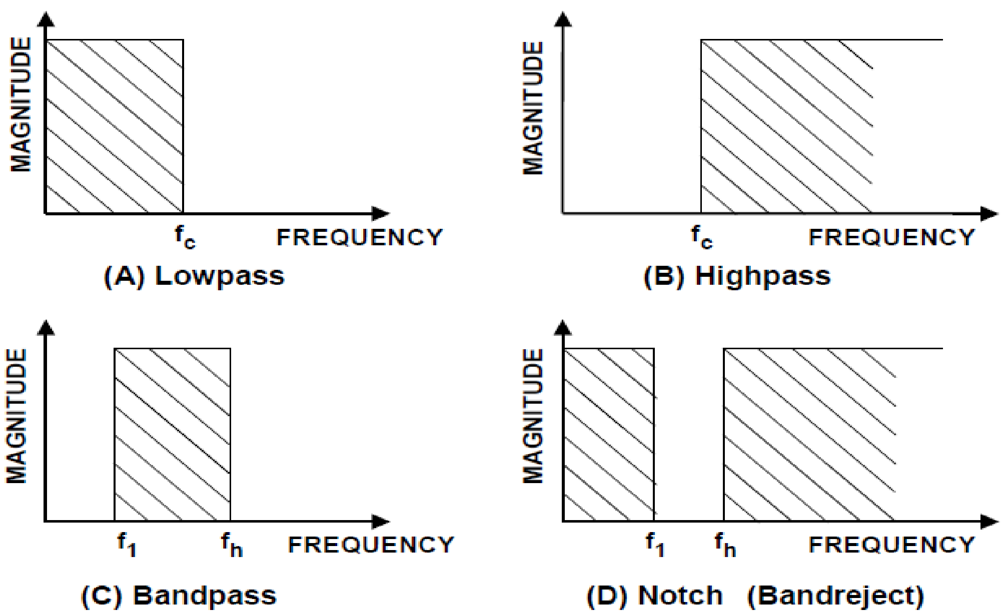


Figure 9.1: *Idealized Filter Responses*

The idealized filters defined above, unfortunately, cannot be easily built. The transition from pass band to stop band will not be instantaneous, but instead there will be a transition region. Stop band attenuation will not be infinite. Figure (9.2) shows the practical(realistic) filters.

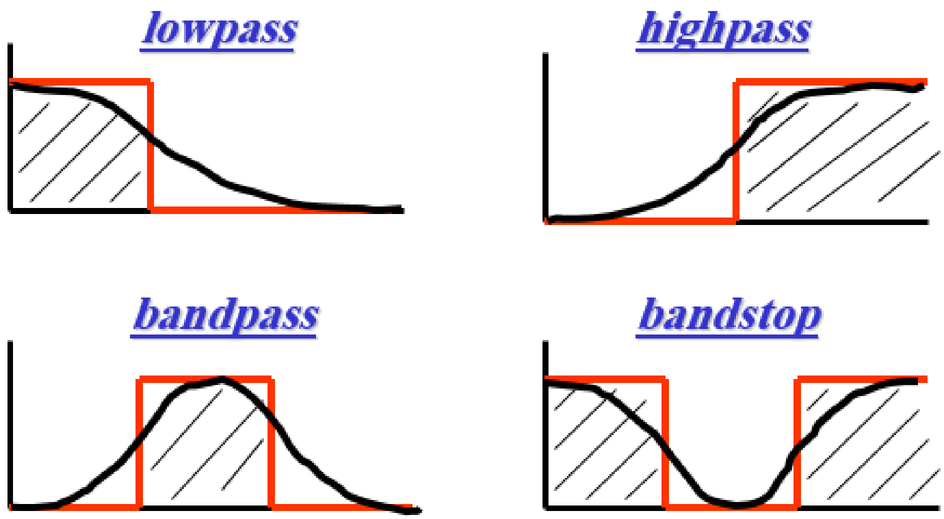


Figure 9.2: *realistic Filter Responses*

9.2.2 Analog and Digital filter

Another means of classifying filters is by the implementation method used. Some filters will be built to filter analog signals using individual components mounted on circuit boards, while other filters might simply be part of a larger digital system which has other functions as well.

1. ***Digital filters*** **are implemented using a digital computer or special purpose digital hardware.**
2. ***Analog filters*** **may be classified as either passive or active and are usually implemented with R, L, and C components and operational amplifiers.**

9.3 Specifications of filters

There are two primary sets of specifications necessary to completely define a filter's response, and each of these can be provided in different ways.

- The frequency specifications used to describe the passband(s) and stopband(s) could be provided in hertz (Hz) or in radians/second (rad/sec).

- The other major filter specifications are the gain characteristics of the passband(s) and stopband(s) of the filter response. A filter's gain is simply the ratio of the output signal level to the input signal level. If the filter's gain is greater than 1, then the output signal is larger than the input signal, while if the gain is less than 1, the output is smaller than the input. In most filter applications, the gain response in the stopband is very small. For this reason, the gain is typically converted to decibels (dB) as indicated in (9.1). For example, a filter's passband gain response could be specified as 0.707 or as −3.0103 dB, while the stopband gain might be specified as 0.0001 or −80.0 dB.

Gain(dB) = 20 log(gain) (9.1)

As we can see, the values in decibels are more manageable for very small gains. Some filter designers prefer to use attenuation (or loss) values instead of gain values. Attenuation is simply the inverse of gain. For example, a filter with a gain of 1/2 at a particular frequency would have an attenuation of 2 at that frequency. If we express attenuation in decibels we will find that it is simply the negative of the gain in decibels as indicated in (9.2). Gain values expressed in decibels will be the standard quantities used as filter specifications, although the term attenuation (or loss) will be used occasionally when appropriate.

Attn.(dB) = 20* log (1/gain) = −20 *log(gain) = $-\text{gain}_{dB}$ (9.2)

The five parameters of a practical filter are defined in Figure 9.3, opposite.

1. The *cutoff frequency* (Fc) is the frequency at which the filter response leaves the error band (or the −3 dB point for a Butterworth response filter).
2. The *stop band frequency* (Fs) is the frequency at which the minimum attenuation in the stopband is reached.
3. The *pass band ripple* (Amax) is the variation (error band) in the pass band response.
4. The *minimum pass band attenuation* (Amin) defines the minimum signal attenuation within the stop band.
5. The steepness of the filter is defined as the *order* (n) of the filter. N is also the number of poles in the transfer function. A pole is a root of the denominator of the transfer function. Conversely, a zero is a root of the numerator of the transfer function.

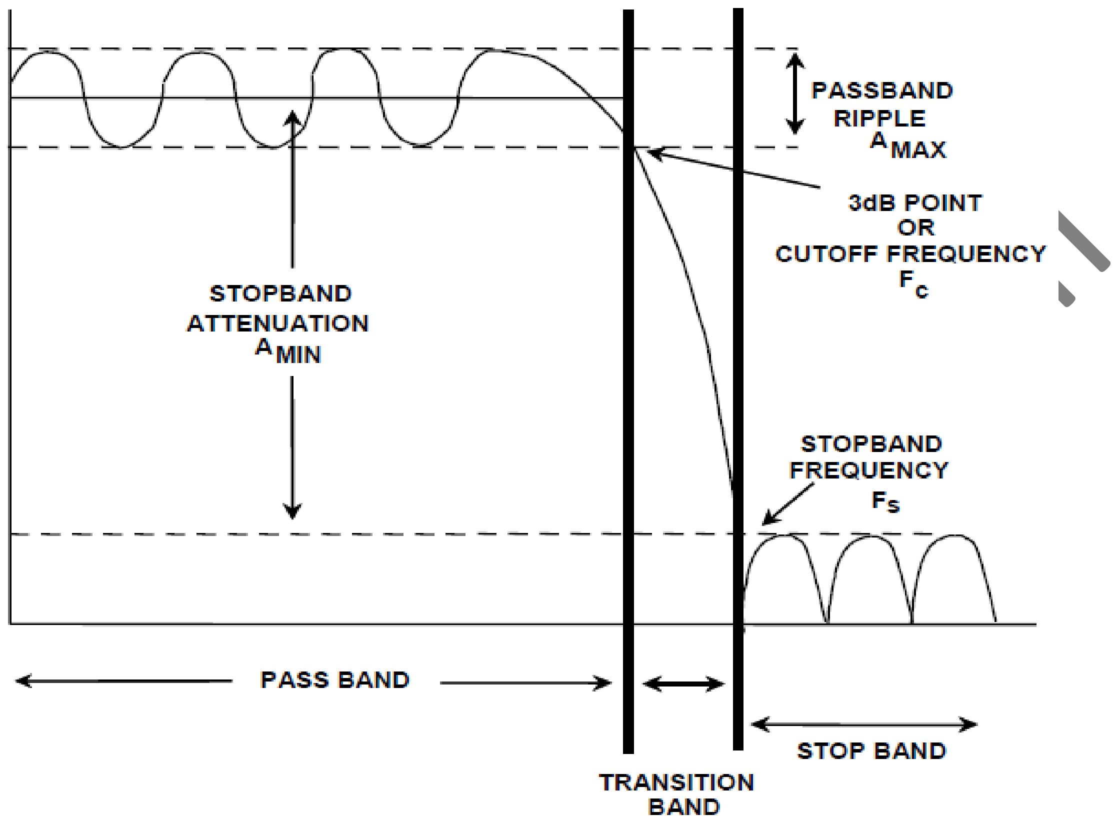


Figure 9.3 filter parameters

- Note that not all filters will have all these features

9.4 FILTER TRANSFER FUNCTIONS

An analog filter is a linear system that has an input and output signal. This system's primary purpose is to change the frequency response characteristics of the input signal as it moves through the filter. The characteristics of this filter system could be studied in the time domain or the frequency domain. From a systems point of view, the impulse response $h(t)$ could be used to describe the system in the time domain. The impulse response of a system is the output of a system that has had an impulse applied to the input. Of course, many systems would not be able to sustain an infinite spike (the impulse) being applied to the input of the system, but there are ways to determine $h(t)$ without actually applying the impulse. A filter system can also be described in the frequency domain by using the transfer function $H(s)$. The transfer function of the system can be determined by finding the Laplace transform of $h(t)$. Figure 9.4 indicates that the filter system can be considered either in the time domain or in the frequency domain. However, the transfer function description is the predominant method used in filter design, and we will perform most of our filter design using it.

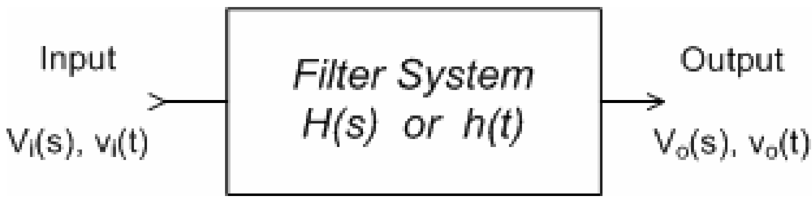


Figure 9.4 The filter as a system

9.4.1 Transfer Function Characterization

The transfer function $H(s)$ for a filter system can be characterized in a number of ways. As shown in (9.4), $H(s)$ is typically represented as the ratio of two polynomials in s where in this case the numerator polynomial is order m and the denominator is a polynomial of order n. G represents an overall gain constant that can take on any value.

$$H(s) = \frac{G \cdot [s^m + a_{m-1} \cdot s^{m-1} + a_{m-2} \cdot s^{m-2} + \cdots + a_1 \cdot s + a_0]}{[s^n + b_{n-1} \cdot s^{n-1} + b_{n-2} \cdot s^{n-2} + \cdots + b_1 \cdot s + b_0]}$$

Transfer function can be represented as:

$$H(s) = \frac{G \cdot [(s+z_0) \cdot (s+z_1) \cdots (s+z_{m-2}) \cdot (s+z_{m-1})]}{[(s+p_0) \cdot (s+p_1) \cdots (s+p_{n-2}) \cdot (s+p_{n-1})]}$$

9.5 Analog Filter Design

The design of a digital filter usually begins with the choice of a CT transfer function H (s) : This approach is an historical artifact, owing to the fact that CT .filter design is a mature subject, thoroughly developed during the first half of the 20th century. As we will see, converting from CT to DT is a simple matter, so it is not necessary to reinvent DT filter theory from scratch. Most filter design is based on rational transfer functions. For analog filters, this is necessary because filters are built with electronic components: operational amplifiers, resistors, and capacitors. The physics of such devices dictate that circuits are governed by differential equations, which in turn lead to rational functions. For digital filters, rational functions correspond to difference equations, which may be solved recursively. let us review analog filter design using lowpass prototype transformation. This method converts the analog lowpass filter with a cutoff frequency of 1 radian per second, called the lowpass prototype, into practical analog lowpass, highpass, bandpass, and bandstop filters with their frequency specifications. Let us consider the following first-order lowpass prototype:

$$H_P(s) = \frac{1}{s+1}.$$

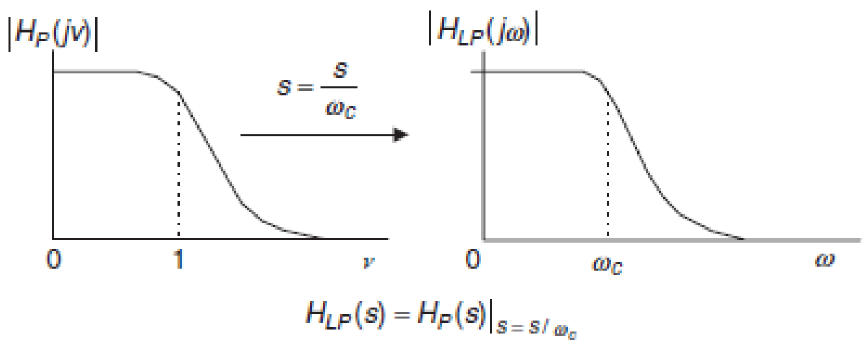


Figure 9.5Analog lowpass prototype transformation into a lowpass filter

The lowpass prototype is a normalized lowpass filter with a normalized cutoff frequency of 1. Applying the prototype transformation **s/w$_c$** in Figure 9.4, we get an analog lowpass filter with a cutoff frequency of w as

$$H(s) = \frac{1}{s/\omega_c + 1} = \frac{\omega_c}{s + \omega_c}.$$

We can obtain the analog frequency response by substituting s=jw into above Equation, that is,

$$H(j\omega) = \frac{1}{j\omega/\omega_c + 1}.$$

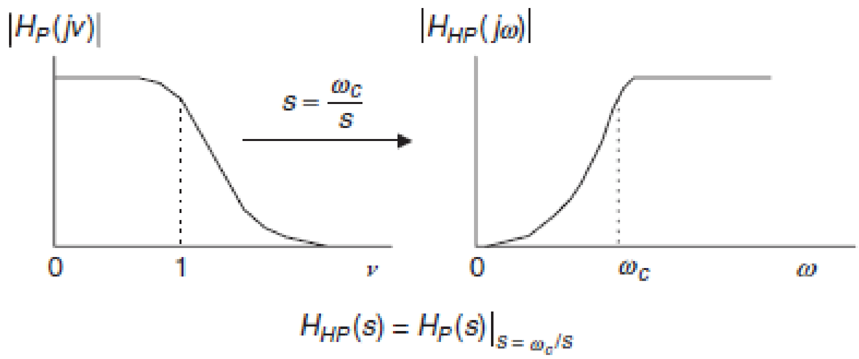
The magnitude response is determined by

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}.$$

This first-order prototype function is used here for an illustrative purpose. We will obtain general functions for Butterworth and Chebyshev lowpassprototypes in a later section.

The highpass, bandpass, and bandstop filters using the specified lowpass prototype transformation can be easily verified. We review them in Figures 9.6, 9.7, and 9.8, respectively.

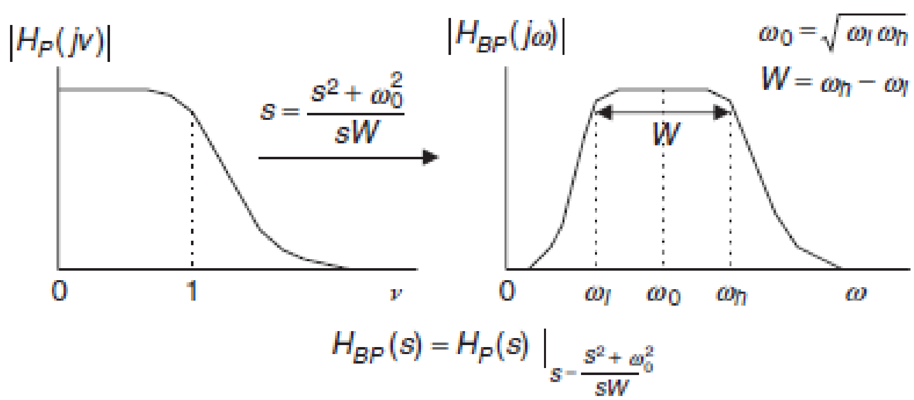
- The transformation from the lowpass prototype to the highpass filter H_{HP} (s) with a cutoff frequency $\mathbf{w_c}$ radians/second is given in Figure 9.6, where $\mathbf{s = w_c /s}$ in the lowpass prototype transformation.



F IGURE 9.6 Analog lowpass prototype transformation to the highpass filter.

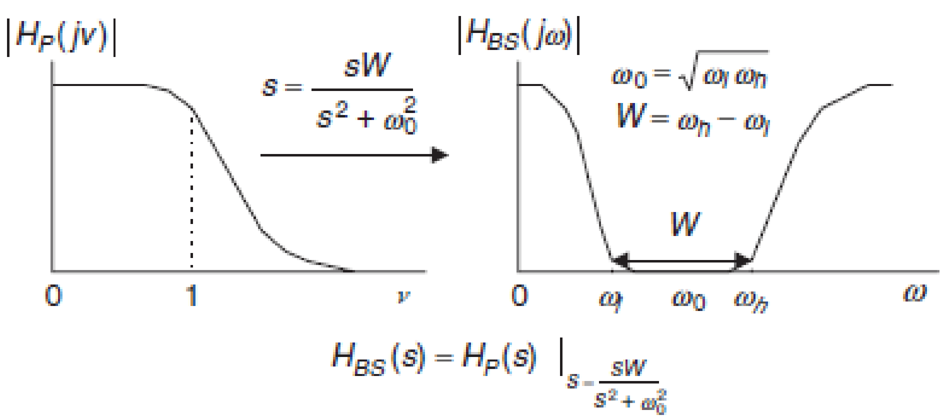
- The transformation of the lowpass prototype function to a bandpass filter with a center frequency $\mathbf{w_o}$, a lower cutoff frequency $\mathbf{w_l}$, and an upper cutoff frequency $\mathbf{w_h}$ in the passband is depicted in Figure 9.7 $\boldsymbol{s = (s^2 + w_0^2)/sW}$ is substituted into the lowpass prototype. As shown in Figure 9.7, w0 is the geometric center frequency, which is defined $\omega_0 = \sqrt{\omega_l \omega_h}$

  while the passband bandwidth is given by $\mathbf{W = w_h - w_l}$



F IGURE 9.7 Analog lowpass prototype transformation to the bandpass filter

- the transformation from the lowpass prototype to a bandstop (band reject) filter is illustrated in Figure 9.8, with $s = s\mathbf{W}/(s^2 + w_0^2)$ substituted into the lowpass prototype. Finally, the lowpass prototype transformations are summarized in Table 9.1.



F IGURE 9.8 Analog lowpass prototype transformation to the bandpass filter.

TABLE 9.1 Analog lowpass prototype transformations.

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Example 1: Given a lowpass prototype $H_P(s) = \frac{1}{s+1}$

Determine each of the following analog filters and plot their magnitude responses from 0 to 200 radians per second.

1. The highpass filter with a cutoff frequency of 40 radians per second.

2. The bandpass filter with a center frequency of 100 radians per second and bandwidth of 20 radians per second.

Solution:

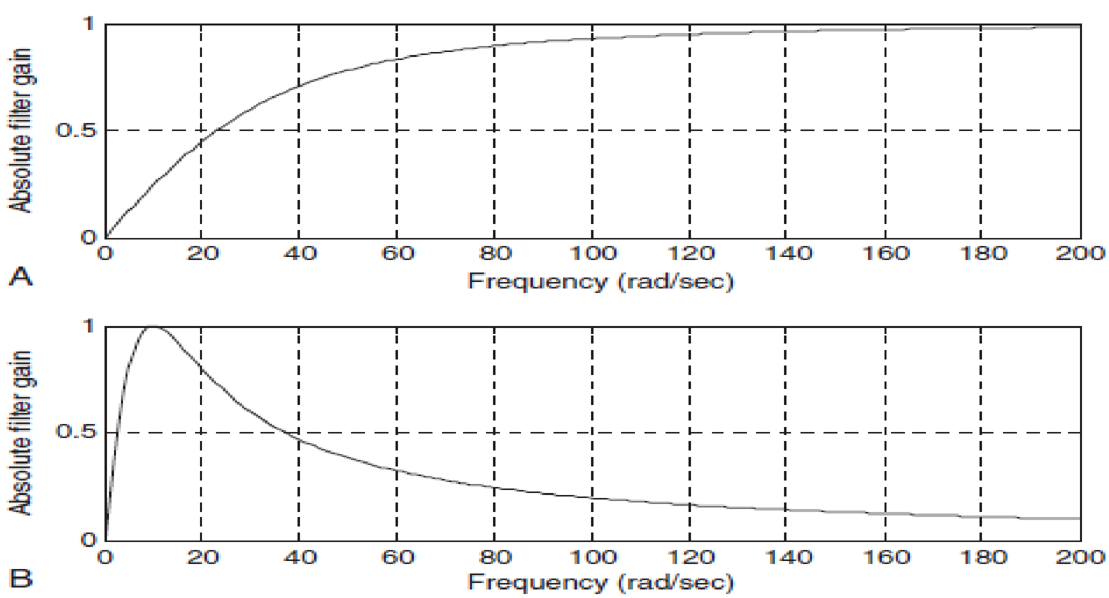
1. Applying the lowpass prototype transformation by substituting **s= 40/s** into the lowpass prototype, we have an analog highpass filter as

$$H_{HP}(s) = \frac{1}{\frac{40}{s}+1} = \frac{s}{s+40}$$

2. Similarly, substituting the lowpass-to-bandpass transformation $s = (s^2+100)/(20s)$ into the lowpass prototype leads to

$$H_{BP}(s) = \frac{1}{\frac{s^2+100}{20s}+1} = \frac{20s}{s^2+20s+100}.$$

- To transfer from a lowpass prototype to a bandpass or bandstop filter, the resultant order of the analog filter is twice that of the lowpass prototype order.



9.6 Analog Filter Approximation Functions

As indicated in the first section, an ideal filter is unattainable; the best we can do is to approximate it. There are a number of approximations we can use based on how we want to define "best." In this section we discuss two methods of approximation, each using a slightly different definition. two sections are devoted to the major approximation methods used in analog filter design: the Butterworth and Chebyshev In each of these sections we determine the order of the filter required given the filter's specifications and the required normalized transfer function to satisfy the specifications.

9.6.1 BUTTERWORTH NORMALIZED APPROXIMATION FUNCTIONS

The Butterworth approximation function is often called the maximally flat response because no other approximation has a smoother transition through the passband to the stopband. The phase response also is very smooth, which is important when considering distortion. The lowpass Butterworth polynomial has an all-pole transfer function with no finite zeros present. It is the approximation method of choice when low phase distortion and moderate selectivity are required. A typical frequency response for a Butterworth low-pass filter of order n is shown in Fig. 9.9.

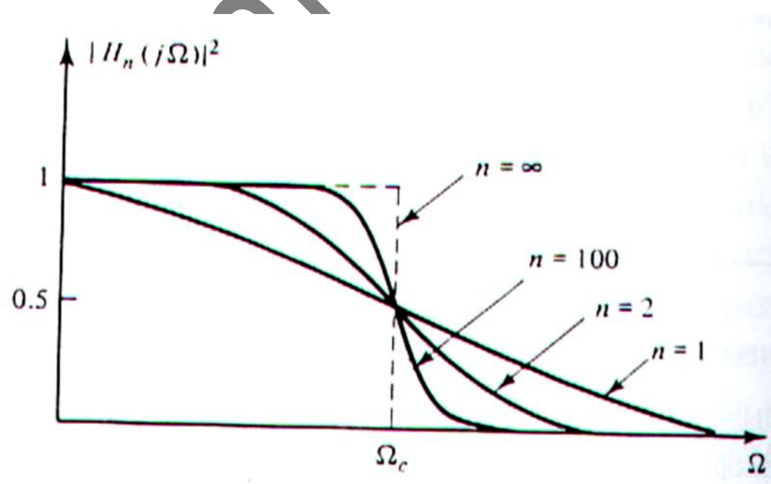


Fig.9.9 Butterworth LPF c/cs

Equation (9. 1) gives the Butterworth approximation's magnitude response

$$\left|H_n(j\Omega)\right|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2n}} \qquad (9.1)$$

The Butterworth approximation has a number of interesting properties

- $$\left|H_n(j\Omega)\right|^2_{\Omega=0} = 1 \ \ for \ \ all \ \ n$$

- $$\left|H_n(j\Omega)\right|^2_{\Omega=\Omega_c} = \frac{1}{2} \ for \ all \ finite \ n$$

- $$\left|H_n(j\Omega)\right|_{\Omega=\Omega_c} = 0.707 \ \ (-3.0103 \ \ dB)$$

- $$|H(j0)|^2 = 1; \qquad |H(j1)|^2 = \frac{1}{2}; \qquad |H(j\infty)|^2 = 0$$

$$-10\log|H(j1)|^2 = -10\log 0.5 = 3.01 \cong 3.0\,dB$$

- $|H_n(j\Omega)|^2$ is monotonically decreasing function of Ω, it is also called maximally flat at the origin since all derivatives exist and are zero. As n → ∞ , we get ideal response. The *normalized* LP Butterworth is obtained when:

Ω_c = 1 rad / sec.

Substituting S = j Ω in eq. (9.1), and rearrange to get the LP Butterworth poles, then:

$$\mathbf{S = (-1)^{[(n+1)/2n]}}$$

For n odd, $S_K = 1\angle k\pi/n \quad , k = 0, 1, 2, \ldots, 2n-1$

For n even, $S_K = 1\angle (k\pi/n) + (\pi/2n) \quad , k = 0, 1, 2, \ldots, 2n-1$

For stable and causal filter:

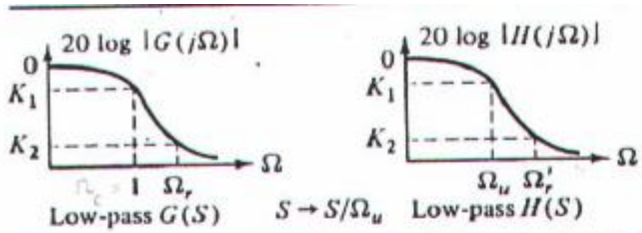
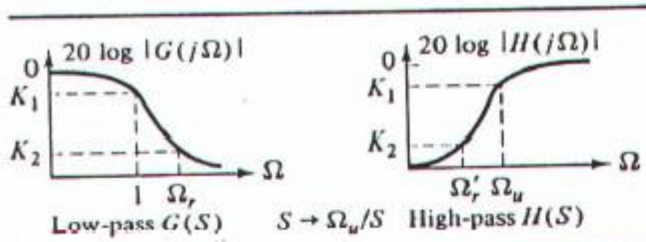
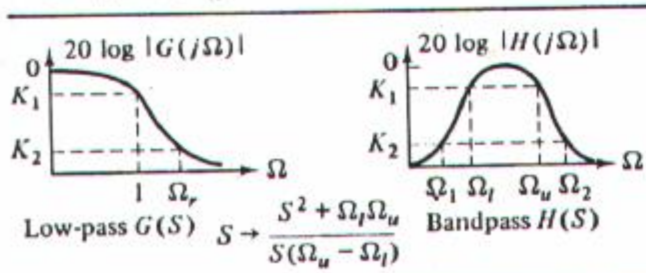
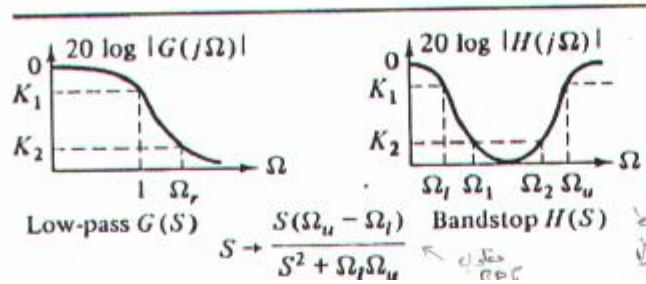
$$H_n(S) = \frac{1}{\prod_{LHP\ poles}(S - S_k)} = \frac{1}{B_n(S)}$$

$B_n(S)$: Butterworth polynomial of order n (see Table (1)).

9.6.1.2 Analog- to analog transformation

To obtain Butterworth filters with cutoff frequencies other than 1 rad /sec. It is convenient to use 1 rad /sec. Butterworth filters as prototypes and apply analog-to-analog transformation (see Table (2)). *The transformational method is not limited in its application to Butterworth filters.*

Table 2: analog to analog transformation

	Low-pass $G(S)$ $S \rightarrow S/\Omega_u$ Low-pass $H(S)$	Forward: $\Omega_r' = \Omega_r\Omega_u$ Backward: $\Omega_r = \Omega_r'/\Omega_u$
	Low-pass $G(S)$ $S \rightarrow \Omega_u/S$ High-pass $H(S)$	Forward: $\Omega_r' = \Omega_u/\Omega_r$ Backward: $\Omega_r = \Omega_u/\Omega_r'$
	Low-pass $G(S)$ $S \rightarrow \frac{S^2+\Omega_l\Omega_u}{S(\Omega_u-\Omega_l)}$ Bandpass $H(S)$	Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$ $\Omega_1 = (\Omega_r^2\Omega_{av}^2 + \Omega_l\Omega_u)^{1/2} - \Omega_{av}\Omega_r$ $\Omega_2 = (\Omega_r^2\Omega_{av}^2 + \Omega_l\Omega_u)^{1/2} + \Omega_{av}\Omega_r$ Backward: $\Omega_r = \min\{\lvert A\rvert, \lvert B\rvert\}$ $A = (-\Omega_1^2 + \Omega_l\Omega_u)/[\Omega_1(\Omega_u - \Omega_l)]$ $B = (+\Omega_2^2 - \Omega_l\Omega_u)/[\Omega_2(\Omega_u - \Omega_l)]$
	Low-pass $G(S)$ $S \rightarrow \frac{S(\Omega_u-\Omega_l)}{S^2+\Omega_l\Omega_u}$ Bandstop $H(S)$	Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$ $\Omega_1 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} - \Omega_{av}/\Omega_r$ $\Omega_2 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} + \Omega_{av}/\Omega_r$ Backward: $\Omega_r = \min\{\lvert A\rvert, \lvert B\rvert\}$ $A = \Omega_1(\Omega_u - \Omega_l)/[-\Omega_1^2 + \Omega_l\Omega_u]$ $B = \Omega_2(\Omega_u - \Omega_l)/[\pm\Omega_2^2 \mp \Omega_l\Omega_u]$

9.6.1.3 Butterworth Filter Specifications

A **Butterworth LPF** Filter of order n is given by the following equation:

$$n = \left\lceil \frac{\log_{10}\{(10^{-0.1k_1} - 1)/(10^{-0.1k_2} - 1)\}}{2\log_{10}(1/\Omega_r)} \right\rceil \qquad (9.\ 2)$$

where

k_1 is the pass-band gain

Ω_u is the relative frequency of k_1

K_2 stop-band attenuation

Ω_r is the relative frequency of k_2 As shown in table 2

To satisfy our requirement at Ω_u exactly, then:

$$\Omega_c = \Omega_u / (10^{-0.1k_1} - 1)^{1/2n}$$

To satisfy our requirement at Ω'_r exactly, then:

$$\Omega_c = \Omega'_r / (10^{-0.1k_2} - 1)^{1/2n}$$

Ω_c is the cutoff frequency at – 3dB

Example (1): design an analog Butterworth LPF that has a – 2 dB butter cutoff frequency of 20 rad/sec. and at least 10 dB of attenuation at 30 rad/sec**.**

Solution: k_1= -2 dB, k_2 = -10 dB, Ω_u= 20 rad/sec., and Ω'_r= 30 rad/sec

$$n = \left\lceil \frac{\log_{10}\{(10^{0.2}-1)/(10^{1}-1)\}}{2\log_{10}(20/30)} \right\rceil = \lceil 3.3709 \rceil = 4$$

To satisfy our requirement at Ω_u exactly, then:

$$\Omega_c = 20/(10^{0.2}-1)^{1/8} = 21.3836\ rad/\sec$$

From Table (1) of *normalized* Butterworth LPF (Ω_c =1 rad/ sec) with n = 4 :

$$H_4(S) = \frac{1}{(S^2+0.76536S+1)(S^2+1.84776S+1)}$$

Using Table (2) and applying LP → LP transformation, S→ S / 21.3836, and rearranging:

$$H(S) = \frac{0.20921\times 10^{6}}{(S^2+16.3686S+457.394)(S^2+39.5176S+457.394)}$$

Ex 2: Determine the order of a lowpass Butterworth filter that has a -3-dB bandwidth of 500 Hz and an attenuation of 40 dB at 1000 Hz.

Solution: k_1= -3 dB, k_2 = -40 dB, Ω_u= 500 *2π rad/sec., and Ω'_r= 1000*2 π rad/sec

$$n = \left\lceil \frac{\log_{10}\{(10^{-0.1k_1}-1)/(10^{-0.1k_2}-1)\}}{2\log_{10}(1/\Omega_r)} \right\rceil$$

n=6.64≈ 7

H.W : Find the order of an active low pass Butterworth filter whose specifications are given as: A_{max} = 0.5dB at a pass band frequency (ωp) of 200 radian/sec (31.8Hz), and A_{min} = 20dB at a stop band frequency (ωs) of 800 radian/sec.

For DESIGNING Butterworth HPF:

1- Put $1/\Omega_r = \Omega'_r / \Omega_u$ in equation (9.2), and find its order n .(see Table(2))

2- Use Table (1) to find the normalized Butterworth LPF equation with order n.

3- Apply LP → HP transformation, S→ Ω_c / S, and rearrange the equation obtained in step 2.

Ex 3:Derive the transfer function of a Butterworth HPF with magnitude /frequency response of -3 dB at frequency of 2kHz and at least -15Db at a freq. of 1 kHz?

Sol:

k_1= -3 dB, k_2 = -15 dB, Ω_u= 2 *2π rad/sec., and Ω'_r= 3*2π rad/sec

$$n = \left[\frac{log_{10}\{(10^{0.3}-1)/(10^{1.5}-1)}{2log_{10}(\frac{1}{2})}\right] = 2.47 \approx 3$$

By using table 1 with n=3

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

9.6.2 CHEBYSHEV NORMALIZED APPROXIMATION FUNCTIONS

Chebyshev filters are analog or digital filters having a steeper roll-off and more passband ripple (type I) or stopband ripple (type II) than Butterworth filters. Chebyshev filters have the property that they minimize the error between the idealized and the actual filter characteristic over the range of the filter but with ripples in the passband. This type of filter is named after Pafnuty Chebyshev because its mathematical characteristics are derived from Chebyshev polynomials. Because of the passband ripple inherent in Chebyshev filters, the ones that have a smoother response in the passband but a more irregular response in the stopband are preferred for some applications.

There are two types of Chebyshev Filters:

1- One containing a ripple in the pass-band (type 1).

2- One containing a ripple in the stop-band (type 2).

9.6.2.1 Type I Chebyshev Filters

Type I Chebyshev Filters are the most common types of Chebyshev Filters. The magnitude squared of the frequency response characteristics of a type I Chebyshev Filters is given as.

$$\left| H_n(j\Omega) \right|^2 = \frac{1}{1+\varepsilon^2 T_n^2(\Omega)}$$

$T_n(\Omega)$ is the nth order Chebyshev polynomial

where $T_0(x) = 1$, and $T_1(x) = x$ as listed in Table (3).

ε^2 is a parameter chosen to provide the proper pass-band ripple. Can be obtained from tables and mathematically

Fig. (9.10) shows *normalized* Chebyshev Filters of both types.

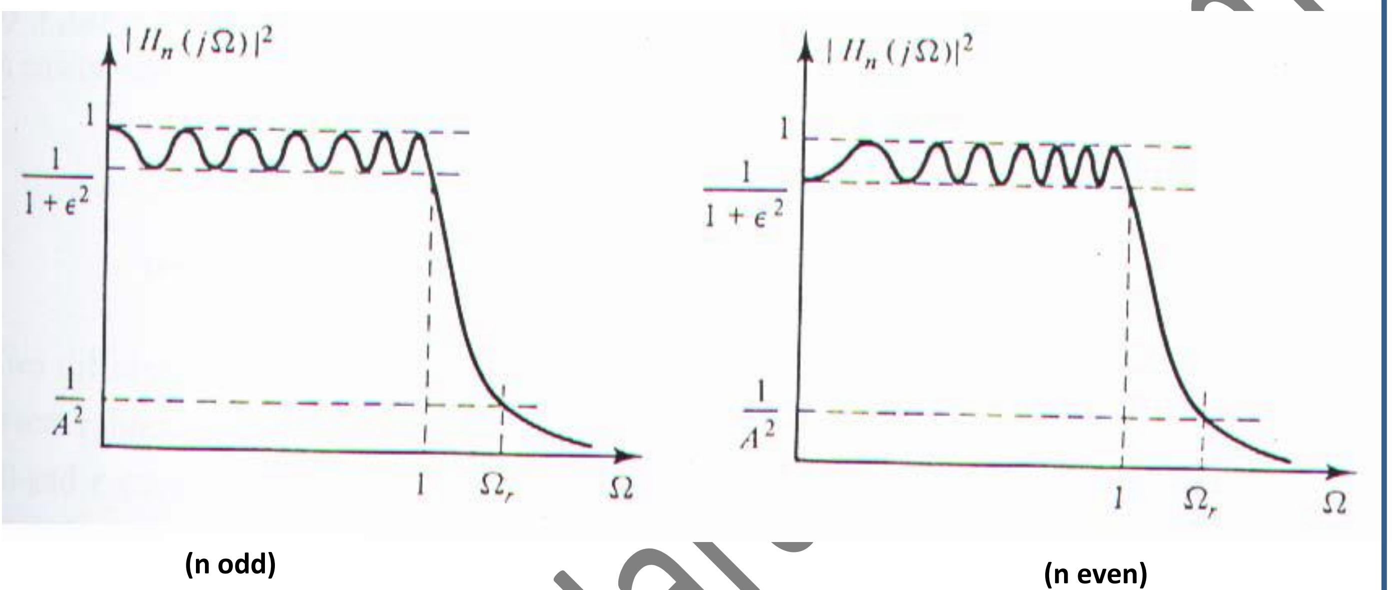


Fig.(9.10) Normalized Chebyshev filters of type 1 for (n odd), and (n even)

9.6.2.1.a Design Equations of Chebyshev Filters:

$$n = \left\lceil \frac{\log_{10}[g + \sqrt{g^2 - 1}\ ]}{\log_{10}[\Omega_r + \sqrt{\Omega_r^{\ 2} - 1}\ ]} \right\rceil \quad \text{....9.3}$$

$$20\log_{10}[1/A^2]^{1/2} = stopband\ attenuation\ (dB) \quad \text{... 9.4.a}$$

$$g = [(A^2 - 1)/\varepsilon^2\ ]^{1/2} \quad \text{...9.4.b}$$

$$|H_n(S)| = \frac{K}{\prod_{\substack{LPF\\poles}} (S - S_K)} = \frac{K}{V_n(S)}$$

$$K = V_n(0) = b_0 \qquad n\ odd$$

$$K = V_n(0)/\sqrt{(1+\varepsilon^2}\qquad n\ even$$

Table (4) gives $V_n(S)$ for n =1 to n =10 and ε corresponding to 0.5, 1, 2, and 3 dB ripples. Table (5) gives the zeros {poles of $H_n(S)$ } for the same n and ε.

9.6.2.b Design steps of Chebeshev LPF, HPF, BPF, and BSF :

1- Use the *backward design equations* from Table (2) to obtain normalized LPF requirements (Ω_r).

2-Calculate *A* using eq. (9.4a). $\boldsymbol{20Log\frac{1}{A} = K_2}$ **= Stop band attenuation(dB)**

- $\boldsymbol{20Log\frac{1}{\sqrt{1+\varepsilon^2}} = K_1}$ = pass band ripple.

3-Calculate from eq. (9.4b), then apply eq.(9.3) to find the order n.

4- Use Table (4) and Table (5) to find the Chebeshev Filter equation with order n.

5- Apply LP → LP or HP or BP or BS transformation (Table (2)) and rearrange the equation obtained in step 4.

Example (4): Design a Chebshev filter to satisfy the following specifications:

1-Acceptable pass-band ripple of 2dB

2-Cutoff frequency of 40 rad/sec.

3- stop-band attenuation of 20 dB or more at 52 rad/sec.

Solution: From Table (2)

$$\Omega_r = \Omega'_r / \Omega_u = 52/40 = 1.3 \text{ rad/sec}$$

$$20\log_{10}[1/A^2]^{1/2} = -20$$

Type equation here. $A = 10$

$$-2 = 20Log\frac{1}{\sqrt{1+\varepsilon^2}} \longrightarrow \varepsilon = \sqrt{10^{\{\text{Pass–Band Ripple in dB}\}/10} - 1}$$

ε =0.76 or from Table (4) and Table (5) , ε =0.76478

$$g = [\,(A^2 - 1)\,/\,\varepsilon^2 \,]^{1/2}$$

g=13.01

$$n = \left\lceil \frac{\log_{10}\,[13.01 + \sqrt{(13.01)^2 - 1}\,]}{\log_{10}\,[1.3 + \sqrt{(1.3)^2 - 1}\,]} \right\rceil = \lceil 4.3 \rceil = 5 \quad \text{, n odd}$$

From Table (4) with n = 5 and ε = 2 dB = 0.76478

$$H_5(S) = \frac{0.08172}{S^5 + 0.70646\,S^4 + 1.499\,S^3 + 0.6934\,S^2 + 0.459349\,S + 0.08172}$$

Using poles from Table (5):

$$H_5(S) = \frac{0.08172}{(S + 0.218303)(S^2 + 0.134922\,S + 0.95215)(S^2 + 0.35323\,S + 0.393115)}$$

Using Table (2) and applying LP → LP transformation, S→ S / 40, and rearranging the above equation:

$$H_{LPF}(S) = \frac{8.366 \times 10^6}{(S + 8.73212)\,(S^2 + 5.3969\,S + 1523.44)\,(S^2 + 14.1292\,S + 628.984)}$$

Notes:

1. **Butterworth or maximally flat amplitude; as the order (n) is increased the response becomes flatter in the pass-band and the attenuation is greater in the stop-band.**

2. Chebshev Filter has a sharper cutoff; i.e., a narrower transition band (best amplitude response) than a Butterworth filter of the same order (n)

3. Chebshev Filter provides poorest phase response (most nonlinear). The Butterworth filter compromise between amplitude and phase (this is one of the reasons for its widespread popularity).

Example(5):DeriveTransfer function of denormailized L.P.F. with magnitude/frequency response to

1- 0.5db pass-band ripple
2- stop-band attenuation of 20 dB or more at 10 Khz
3- Cutoff frequency equal to 2 khz

Sol:

From table 4, when pass band ripple =0.5 db, ε =0.3493

Or from $\varepsilon = \sqrt{10^{0.05} - 1} = 0.3493$

From $20\log_{10}[1/A^2]^{1/2} = -20$ A=10

$g = [(A^2 - 1)/\varepsilon^2]^{1/2}$ ⟶ g=28.48

$\boldsymbol{\Omega_r = \frac{\Omega_r}{\Omega_u} = \frac{10}{2} = 5}$

$\boldsymbol{n = \left[\frac{log_{10}^{[28.48+\sqrt{28.48^2-1}]}}{log_{10}^{[5+\sqrt{24}]}}\right] = 1.76 \simeq 2}$**, n is even**

From Table (4) with n = 2 and 0.5 db ripple(ε=0.3493)

$$H_n(s) = \frac{k_n}{V_n(s)}$$

For n even $k_n = \frac{b_0}{(1+\varepsilon^2)^{1/2}}$ from table 4

$$k_n = \frac{1.5162}{(1+0.3493^2)^{1/2}} = 1.4313$$

$$H_2(s) = \frac{1.4313}{1.4256s+1.5162} =$$

Using Table (2) and applying LP → LP transformation, S→ S / 2*2π, and rearranging the above equation:

9.6.2.2-Type II Chebyshev filters

Also known as inverse Chebyshev filters, the Type II Chebyshev filter type is less common because it does not roll off as fast as Type I, and requires more components. It has no ripple in the passband, but does have equiripple in the stopband

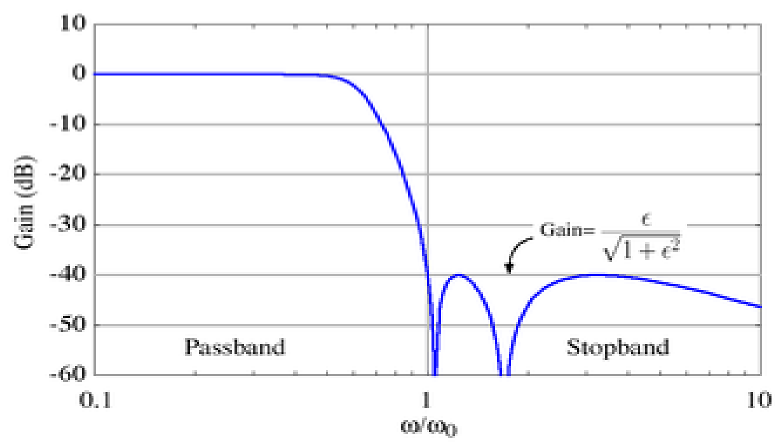


Fig.(9.11) afifth-order type II Chebyshev low-pass filter