University of Diyala

Information Theory

Lecture 1

3rd Stage

Communication department / Engineering collage

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Review of Probability

Definition: A probabilistic model is a mathematical description of an uncertain situation.

A probability of an event A: If an experiment has $A1$, $A2$,……. An, outcomes, then:

$$Prob(A_i) = P(A_i) = \lim_{N\to\infty} \frac{n(A_i)}{N}$$

Where $n(Ai)$= no. of times event (outcomes)

(Ai) occurs N= total number of trails.

$$1 \geq P(A_i) \geq 0, \qquad \text{and}$$

$$\sum_{i=1}^{n} P(A_i) = 1$$

If $P(A_i) = 1$ then A_i is certain event

Joint probability

If we have two experiment *A & B*

an experiment *A* has $A1$, $A2$,……. An

and an experiment *B* has *B1, B2,*…….. *Bm*

Then *p(Ai,Bj) = joint prob. that event Ai occur from experiment A*

& event Bj occur from experiment B

$\sum_j^m \sum_i^n p(Ai, Bj)$ =1

$p(Ai, Bj)$ written in the matrix form

$p(Ai, Bj)$ is written in te matrix form

$$p(Ai, Bj) = \begin{bmatrix} A1, B1 & \cdots & A1, Bm \\ \vdots & \ddots & \vdots \\ An, B1 & \cdots & An, Bm \end{bmatrix}$$

$\sum_{i=1}^{n} p(Ai, Bj) = p(Bj)$ sum of the *jth* column

$\sum_{j=1}^{m} p(Ai, Bj) = p(Ai)$ sum of the *ith* row

Conditional Probability

A conditional probability : two experiment *A & B* with their outcomes effect on each other

When A given B, denoted by P(Ai | Bj).

When B given A denoted by P(Bj | Ai).

<u>For example,</u> suppose that all six possible outcomes of a fair die roll are equally likely. If we are told that the outcome is even, we are left with only three possible outcomes, namely, 2, 4, and 6.

P(the outcome is 6 |the outcome is even) = 1/3

The definition of conditional probability when all outcomes are equally likely, is given by:

$P(Ai \mid Bj) = P(Ai)\, P(Bj \mid Ai)$

$$p(Ai, Bj) = \begin{bmatrix} A1, B1 & \cdots & A1, Bm \\ \vdots & \ddots & \vdots \\ An, B1 & \cdots & An, Bm \end{bmatrix}$$

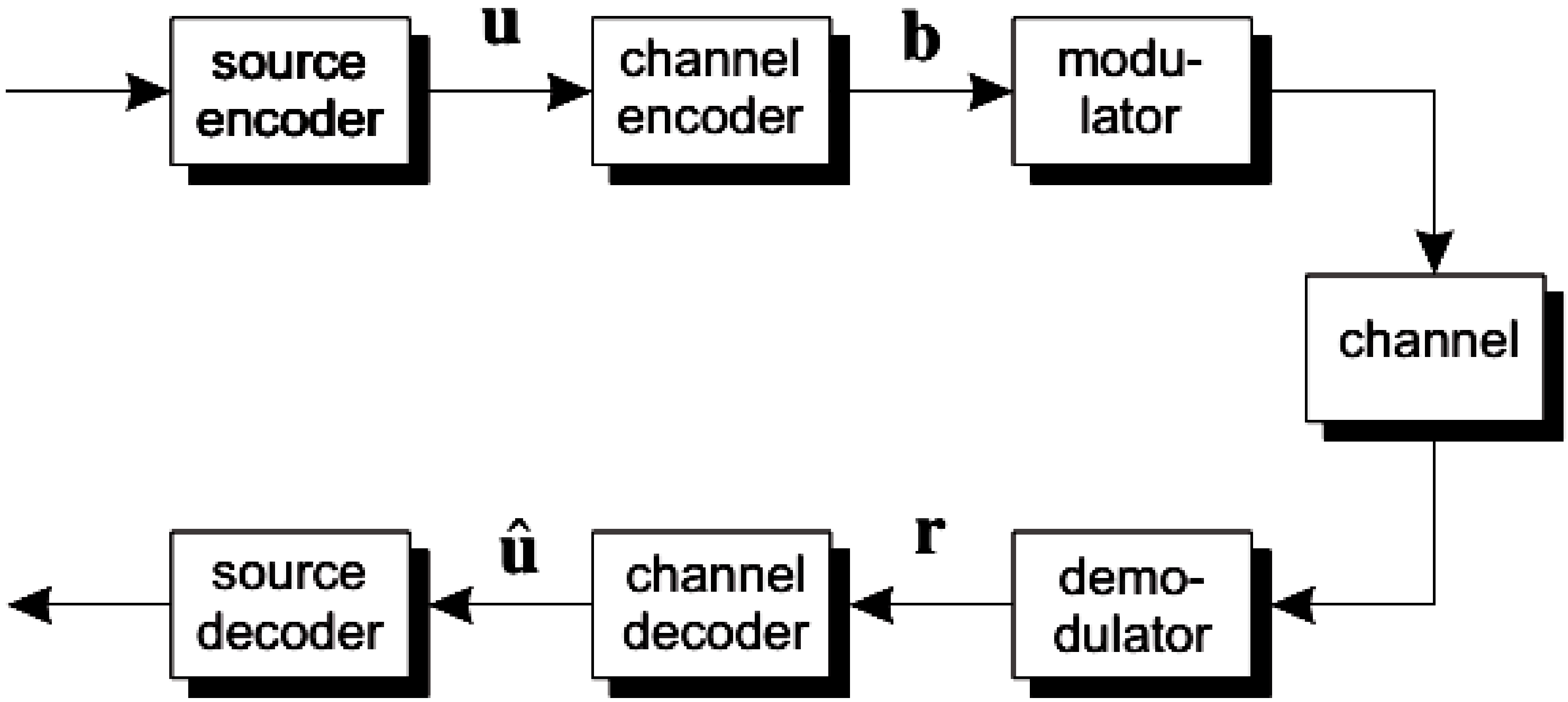
$\sum_{i=1}^{n} P(Ai, Bj) = 1$ and $\sum_{j=1}^{m} P(Ai, Bj) = 1$

If *Ai* has no effect on the probability of *Bj* then they are called **independent** and

$P(Ai \mid Bj) = P(Ai)$ $\quad$ *and* $\quad$ $P(Bj \mid Ai) = P(Bj)$

& $P(Ai\, , Bj) = P(Ai)\, P(Bj)$

Digital Communication system



H.W

- What is the advantage of digital communication system over analog system?

- What is the disadvantage of digital communication system over analog system?

Information theory

Digital **Communication - Information Theory**.

- **Information** is the source of a communication system, whether it is analog or digital.
- **Information theory** is a mathematical approach to the study of coding of information along with the quantification, storage, and communication of information. It is also determine the capacity of communication system to transfer this information from source to destination

Information source

The set of source symbols is called the source alphabet and the element of the set are called symbols or letters

Two type of sources :

1. Source with memory
2. Source without memory – discrete memory less source (DMS)

Information content of DMS :

Suppose that the source of information produces finite set of message $x1, x2, \ldots\ldots. xn$ with prob. $p(x1), p(x2), \ldots\ldots\ldots. P(xn)$

$$\sum_{i=1}^{n} P(x_i) = 1$$

Self- information

self-information is a measure of the information content of symbol xi and it denoted by $I(xi)$.

$$I(x_i) = -\log_a P(x_i)$$

Where $I(xi)$ is self information of (xi)

$P(xi)$ is probability of occurrence of symbol xi

Note that :

- $I(xi) = 0$ for $P(xi)$=1 (certain event)
- $I(xi) \geq o$
- $I(xi) > I(x\text{j})$ if $P(xi) < P(x\text{j})$
- $I(xi, x\text{j}) = I(xi) + I(x\text{j})$ if xi & xj are independent

Also note:

- If "a" =2 , then $I(xi)$ has the unit of bits
- If "a"= e = 2.71828, then $I(xi)$ has the unit of nats
- If "a"= 10, then $I(xi)$ has the unit of hartly

Recall that $\log_a x = \frac{lnx}{lna}$

Example 1:

A fair die is thrown, find the amount of information gained if you are told that 4 will appear.

Solution:

$$P(1) = P(2) = \cdots \ldots \ldots . = P(6) = \frac{1}{6}$$

$$I(4) = -\log_2\left(\frac{1}{6}\right) = \frac{\ln(\frac{1}{6})}{ln2} = 2.5849\ bits$$