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Information Theory

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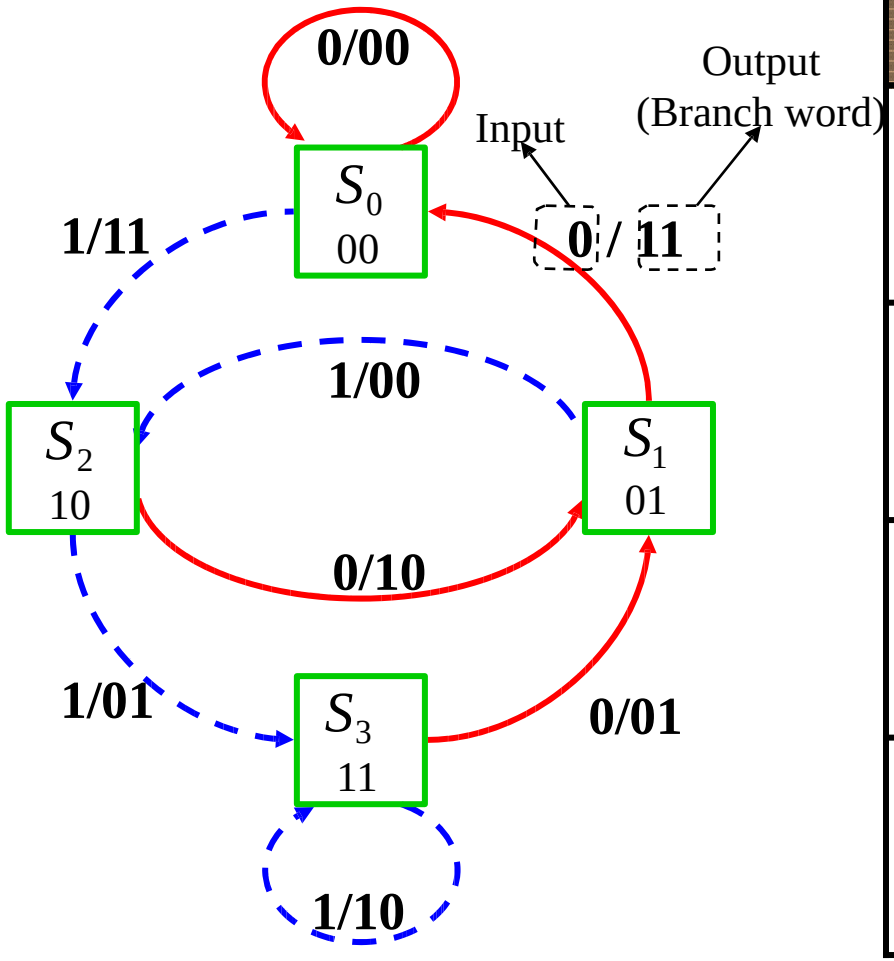
State diagram

- A finite-state machine only encounters a finite number of states.
- State of a machine: the smallest amount of information that, together with a current input to the machine, can predict the output of the machine.
- In a Convolutional encoder, the state is represented by the content of the memory.
- Hence, there are 2^{K-1} states.

State diagram – cont'd

- A state diagram is a way to represent the encoder.
- A state diagram contains all the states and all possible transitions between them.
- Only two transitions initiating from a state
- Only two transitions ending up in a state

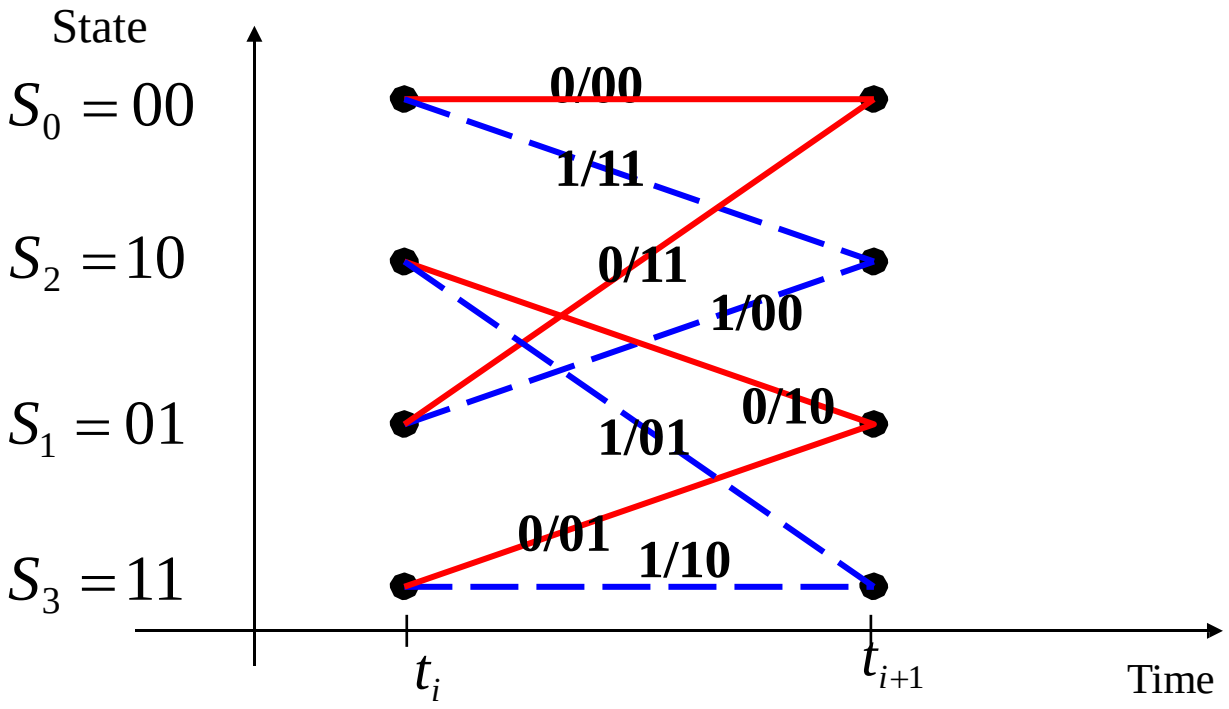
State diagram – cont'd



Current state	input	Next state	output
S_0 00	0	S_0	00
	1	S_2	11
S_1 01	0	S_0	11
	1	S_2	00
S_2 10	0	S_1	10
	1	S_3	01
S_3 11	0	S_1	01
	1	S_3	10

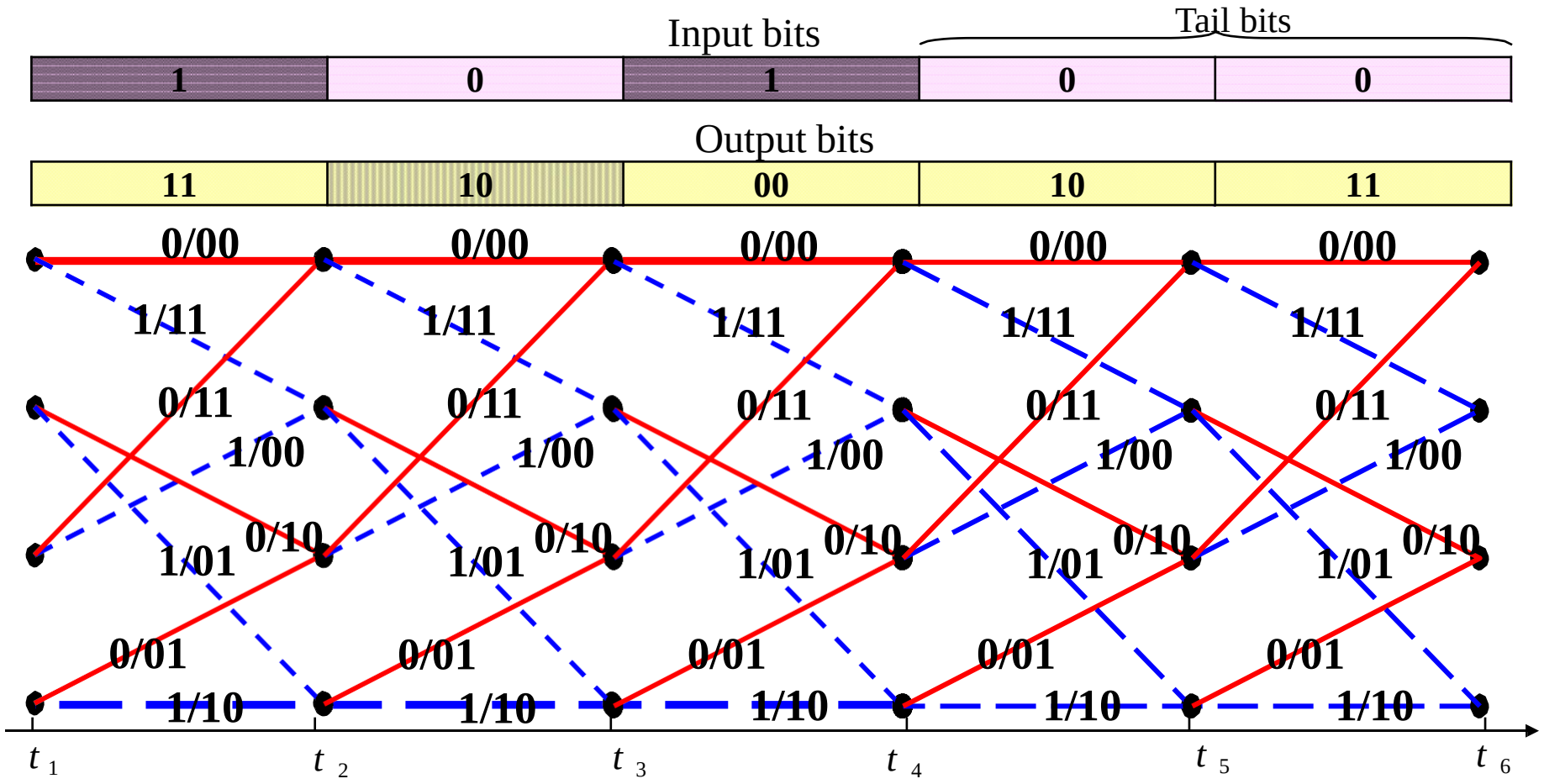
Trellis – cont'd

- Trellis diagram is an extension of the state diagram that shows the passage of time.
  - Example of a section of trellis for the rate ½ code

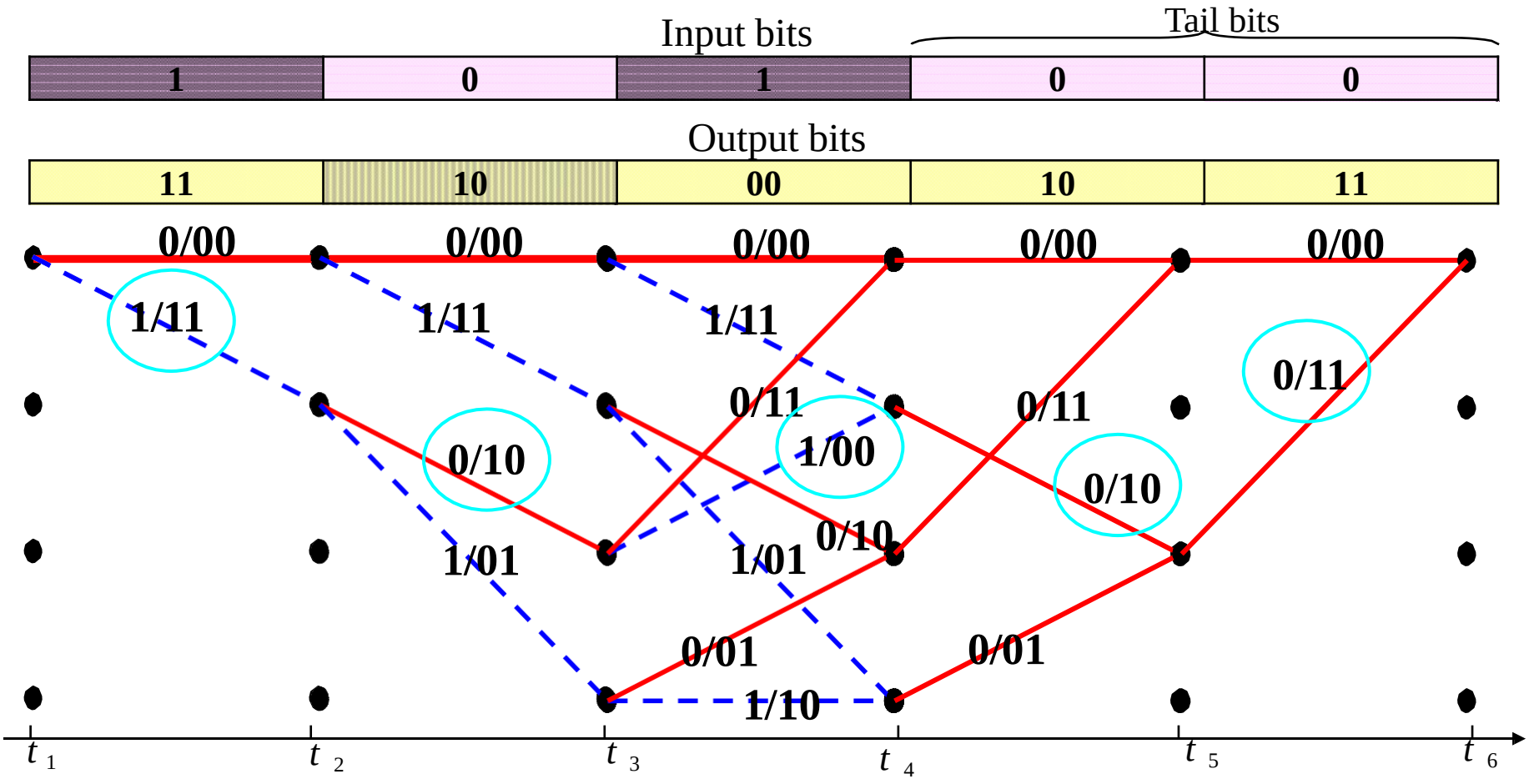


Trellis –cont'd

- A trellis diagram for the example code.



Trellis – cont'd



The Viterbi Decoding Algorithm

- The Viterbi algorithm performs Maximum Likelihood decoding.
- It finds a path through trellis with the largest metric (maximum correlation or minimum distance).
  - It processes the demodulator outputs in an iterative manner.
  - At each step in the trellis, it compares the metric of all paths entering each state, and keeps only the path with the largest metric, called the survivor, together with its metric.
  - It proceeds in the trellis by eliminating the least likely paths.
- It reduces the decoding complexity to $L2^{K-1}$!

The Viterbi algorithm - cont'd

- Viterbi algorithm:

A. Do the following set up:
   - For a data block of L bits, form the trellis. The trellis has $L+K-1$ sections or levels and starts at time t_1 and ends up at time t_{L+K} .
   - Label all the branches in the trellis with their corresponding branch metric.
   - For each state in the trellis at the time t_i which is denoted by $S(t_i) \in \{0,1,...,2^{K-1}\}$, define a parameter $\Gamma(S(t_i), t_i)$

B. Then, do the following:

The Viterbi algorithm - cont'd

1. Set $\Gamma(0, t_1) = 0$ and $i = 2.$
2. At time t_i , compute the partial path metrics for all the paths entering each state.
3. Set $\Gamma\big(S(t_i), t_i\big)$ equal to the best partial path metric entering each state at time t_i.

   Keep the survivor path and delete the dead paths from the trellis.
4. If $i < L + K$, increase i by 1 and return to step 2.

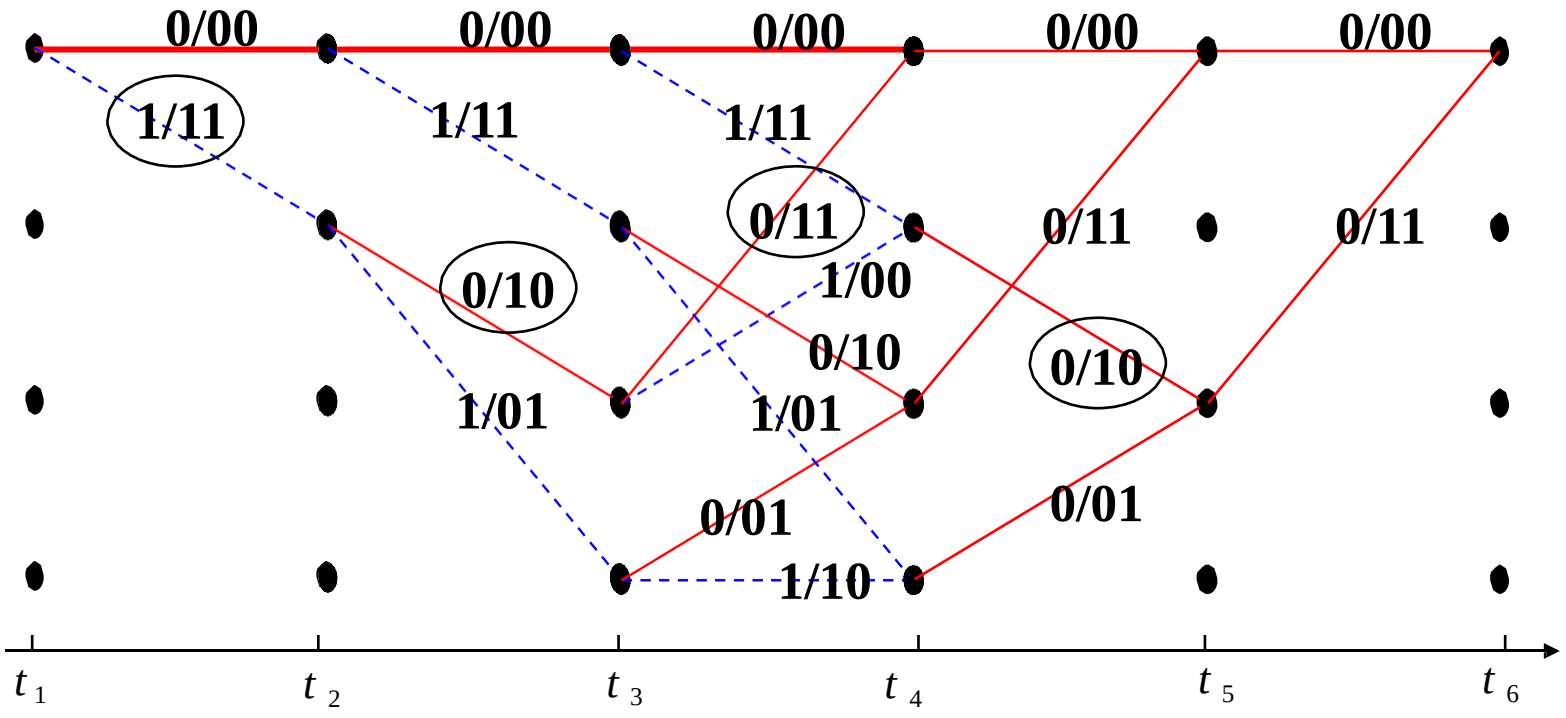
C. Start at state zero at time t_{L+K} . Follow the surviving branches backwards through the trellis. The path thus defined is unique and correspond to the ML codeword.

Example of Viterbi decoding

m = (101) Source message

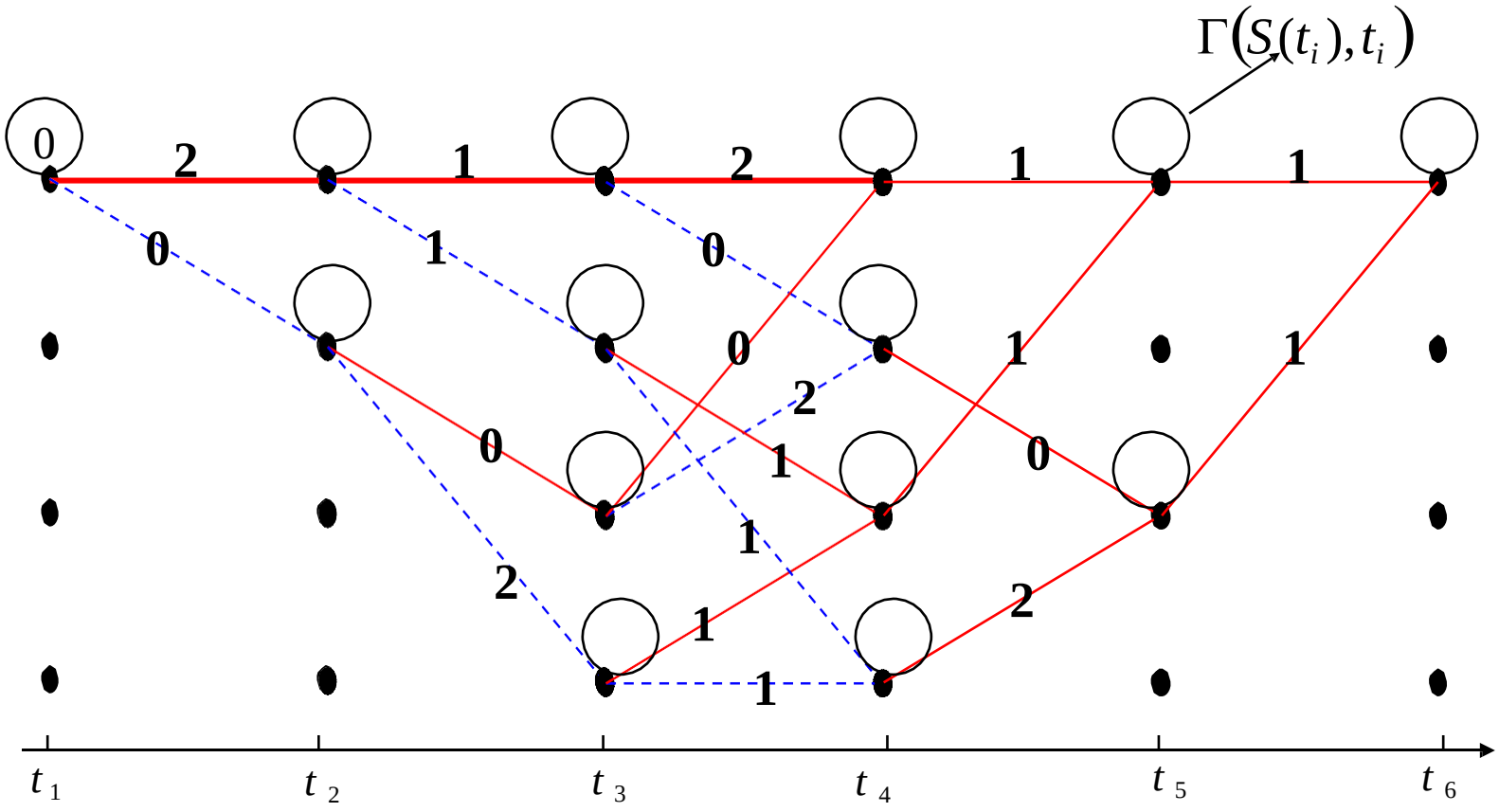
U = (11 10 00 10 11) Codeword to be transmitted

Z = (11 10 11 10 01) Received codeword



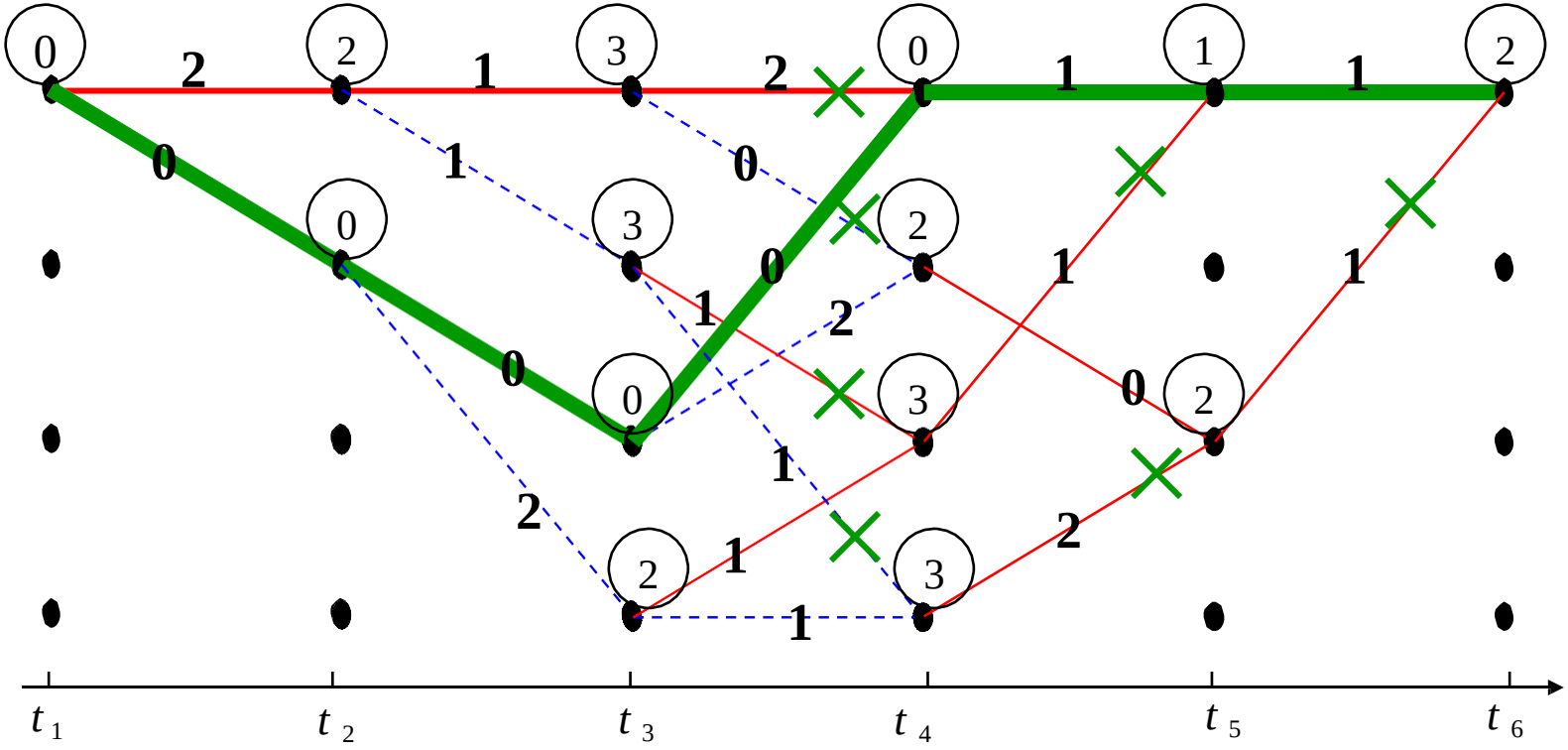
Example of Viterbi decoding-cont'd

- Label all the branches with the branch metric (Hamming distance)



Example of Viterbi decoding-cont'd

- Trace back and then:

$\hat{\mathbf{m}} = (100)$

$\hat{\mathbf{U}} = (11 \;\; 10 \;\; 11 \;\; 00 \;\; 00)$ There are some decoding errors.