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## Centroids and Center of Gravity of Composite Areas and Bodies

Bodies

- The centroid or center of gravity of any area, or bodies can be obtained by means of the principle of moments. If the **area, weight and so on**, can be determined and the moment of these quantities about any axis of plane car also be determined. The methods avoids the necessity for integration

- The composite area can be divided into simple shapes (**ex, rectangles, triangles, circles or other shapes**) who area and centroid coordinates can be readily obtained.

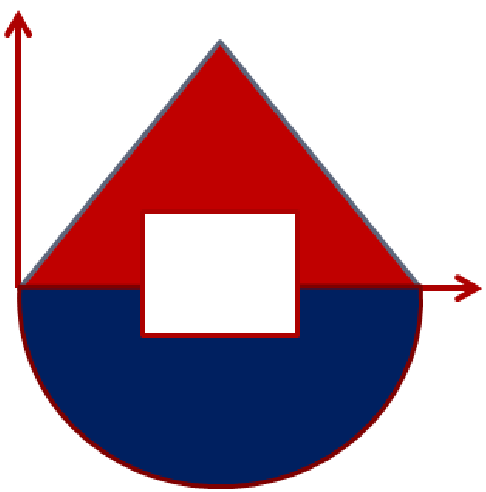
- The total area is the sum of the separate area. The resultant moment about any axis of plane is the algebraic sum of the moments of the component area.

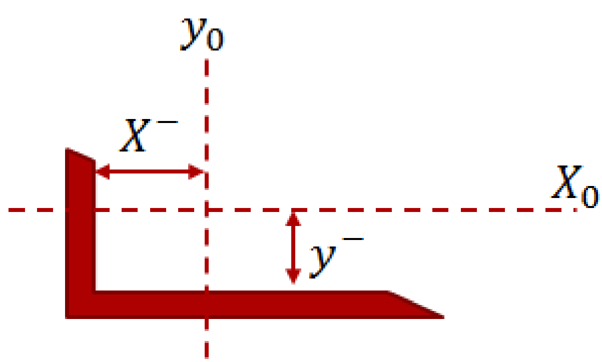
**Notes:-**

1- if some parts are removed the corresponding area must be subtracted

2- See table (7-1) to know the properties of plane shapes.

3- For structural section.





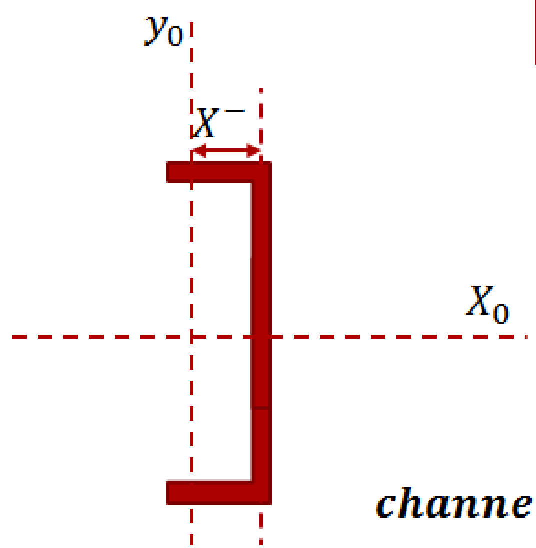



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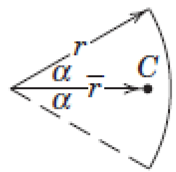
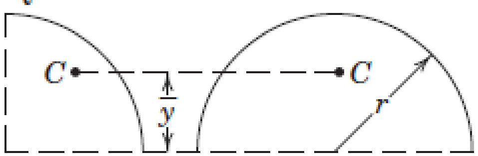
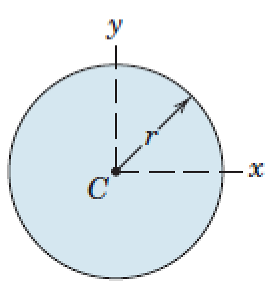
**TABLE D/3 PROPERTIES OF PLANE FIGURES**

| FIGURE | CENTROID | AREA MOMENTS OF INERTIA |
|---|---|---|
| Arc Segment  | $\bar{r} = \frac{r \sin\alpha}{\alpha}$ | — |
| Quarter and Semicircular Arcs  | $\bar{y} = \frac{2r}{\pi}$ | — |
| Circular Area  | — | $I_x = I_y = \frac{\pi r^4}{4}$<br>$I_z = \frac{\pi r^4}{2}$ |

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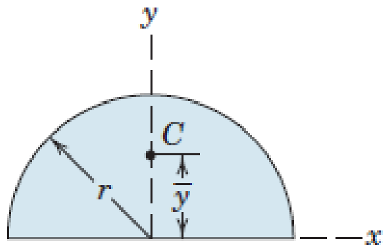
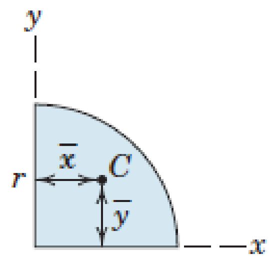
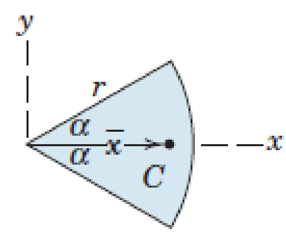
| | | |
|---|---|---|
| Semicircular Area   | $\bar{y} = \frac{4r}{3\pi}$ | $I_x = I_y = \frac{\pi r^4}{8}$ <br> $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ <br> $I_z = \frac{\pi r^4}{4}$ |
| Quarter-Circular Area   | $\bar{x} = \bar{y} = \frac{4r}{3\pi}$ | $I_x = I_y = \frac{\pi r^4}{16}$ <br> $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ <br> $I_z = \frac{\pi r^4}{8}$ |
| Area of Circular Sector   | $\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$ | $I_x = \frac{r^4}{4}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)$ <br> $I_y = \frac{r^4}{4}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$ <br> $I_z = \frac{1}{2} r^4 \alpha$ |

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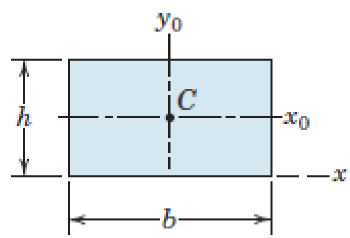
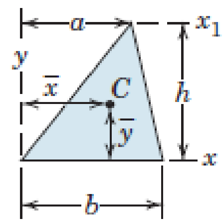
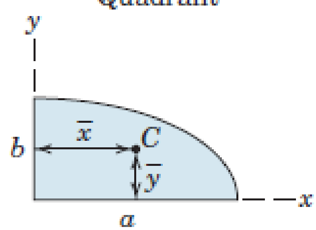
**TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued***

| FIGURE | CENTROID | AREA MOMENTS OF INERTIA |
|---|---|---|
| Rectangular Area   | — | $I_x = \frac{bh^3}{3}$ <br> $\bar{I}_x = \frac{bh^3}{12}$ <br> $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$ |
| Triangular Area   | $\bar{x} = \frac{a+b}{3}$ <br> $\bar{y} = \frac{h}{3}$ | $I_x = \frac{bh^3}{12}$ <br> $\bar{I}_x = \frac{bh^3}{36}$ <br> $I_{x_1} = \frac{bh^3}{4}$ |
| Area of Elliptical Quadrant   | $\bar{x} = \frac{4a}{3\pi}$ <br> $\bar{y} = \frac{4b}{3\pi}$ | $I_x = \frac{\pi ab^3}{16}, \quad \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ <br> $I_y = \frac{\pi a^3 b}{16}, \quad \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ <br> $I_z = \frac{\pi ab}{16}(a^2 + b^2)$ |

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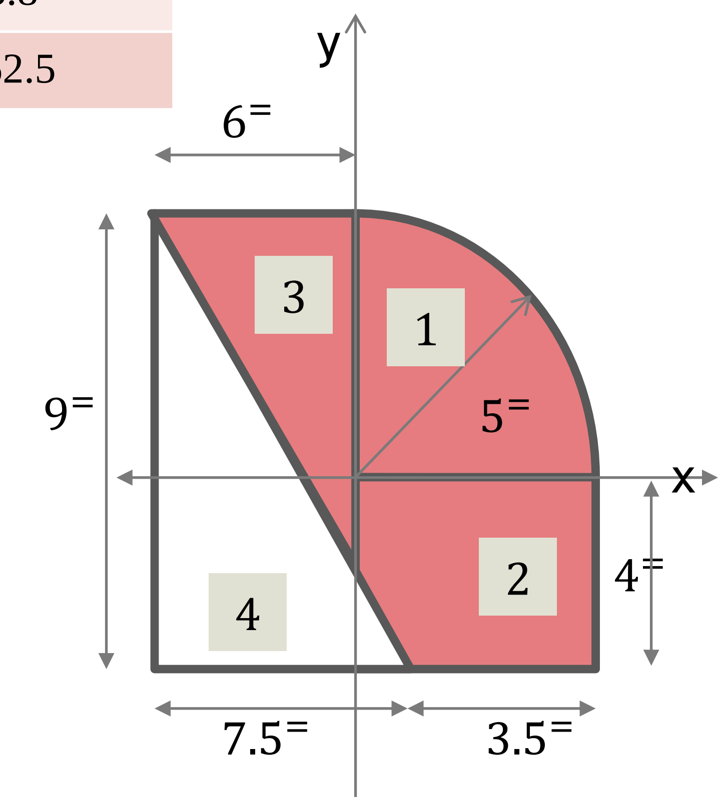
EX:- Determine the position of the centroid of the shaded area shown in fig

| NO | Area | $x^{-}$ | $M_y=A*x^{-}$ | $y^{-}$ | $M_x=A*y^{-}$ |
|---|---|---|---|---|---|
| 1 | 19.65 | 2.12 | 41.7 | 2.12 | 41.7 |
| 2 | 20 | 2.5 | 50 | -2 | -40 |
| 3 | 54.0 | -3.0 | -162 | 0.5 | 27 |
| 4 | -33.75 | -3.5 | 118.2 | -1 | 33.8 |
| | = 59.9 | | = 47.9 | | = 62.5 |




$$x^{-} = \frac{\sum M_y}{\sum A} = \frac{47.9}{59.9} = 0.8\ In$$

$$y^{-} = \frac{\sum M_x}{\sum A} = \frac{62.5}{59.9} = 1.043\ In$$

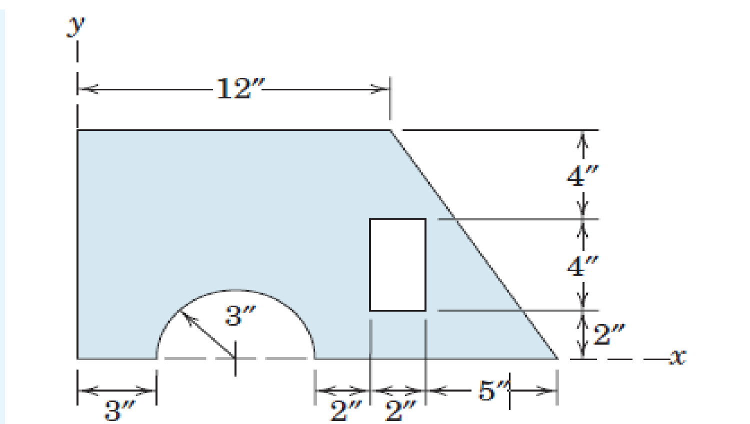
EX:- Determine the position of the centroid of the shaded area shown in fig

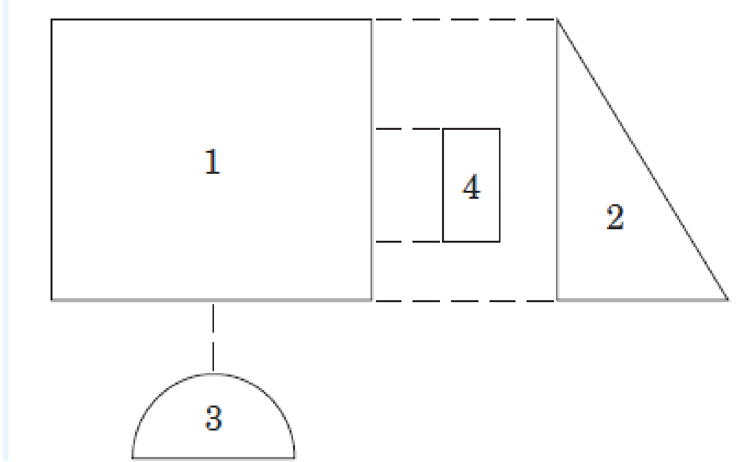
| PART | $A$ in.$^2$ | $\bar{x}$ in. | $\bar{y}$ in. | $\bar{x}A$ in.$^3$ | $\bar{y}A$ in.$^3$ |
|---|---|---|---|---|---|
| 1 | 120 | 6 | 5 | 720 | 600 |
| 2 | 30 | 14 | 10/3 | 420 | 100 |
| 3 | −14.14 | 6 | 1.273 | −84.8 | −18 |
| 4 | −8 | 12 | 4 | −96 | −32 |
| TOTALS | 127.9 | | | 959 | 650 |

The area counterparts to Eqs. 5/7 are now applied and yield

$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959}{127.9} = 7.50 \text{ in.} \qquad \textit{Ans.}$$

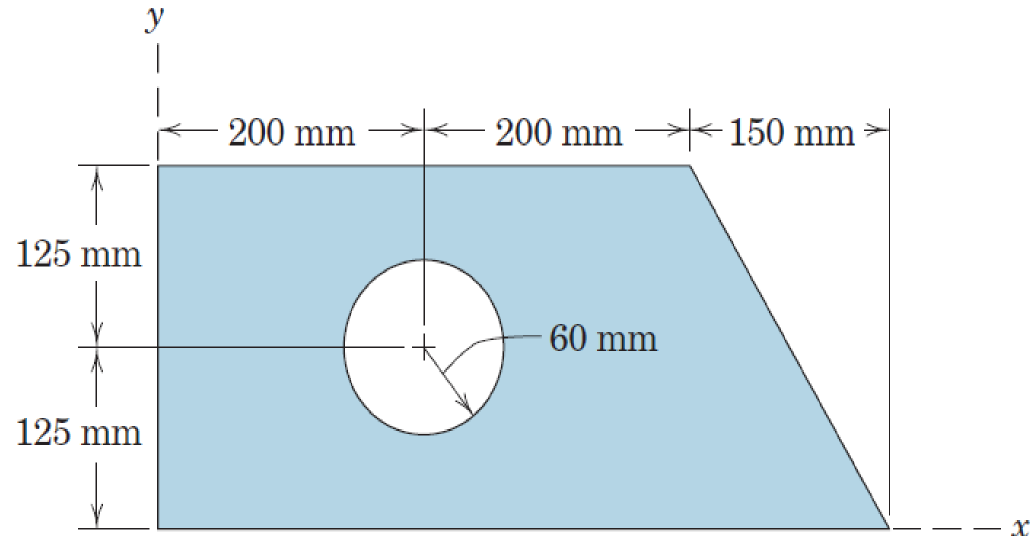
$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.} \qquad \textit{Ans.}$$



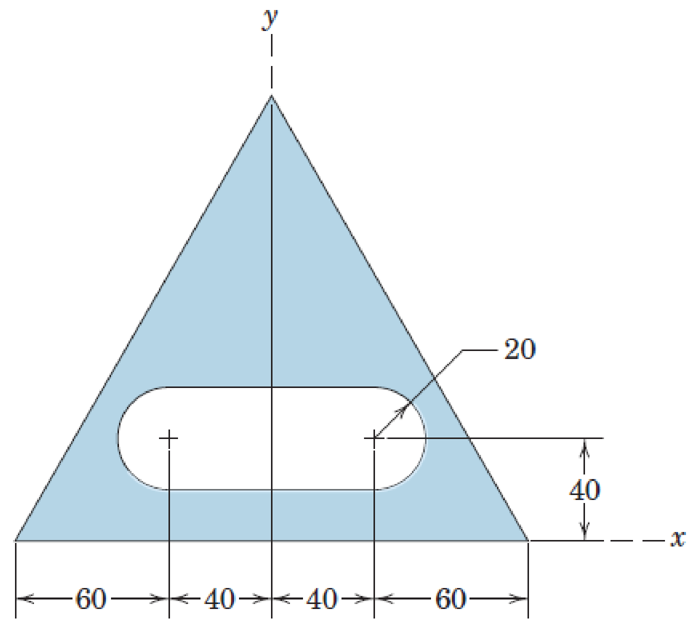



**H.w.**

Q1\ Determine the coordinates of the centroid of the shaded area.




Q2\ Determine the *y*-coordinate of the centroid of the shaded area. The triangle is equilateral.




Dimensions in millimeters