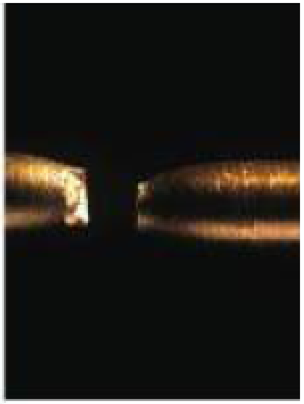
STRAIN

Strength of Materials: Second Class

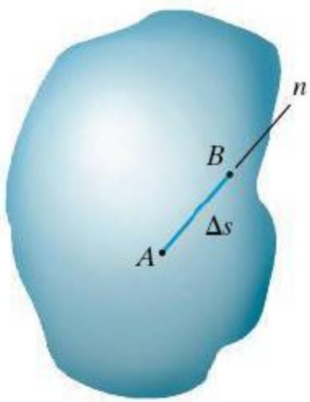
2
Strain 67



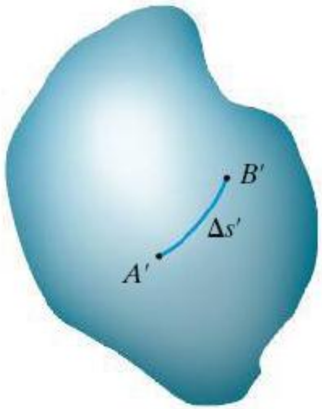
2.2 Strain

In order to describe the deformation of a body by changes in length of line segments and the changes in the angles between them, we will develop the concept of strain.

- Normal Strain: The elongation or contraction of a line segment per unit length

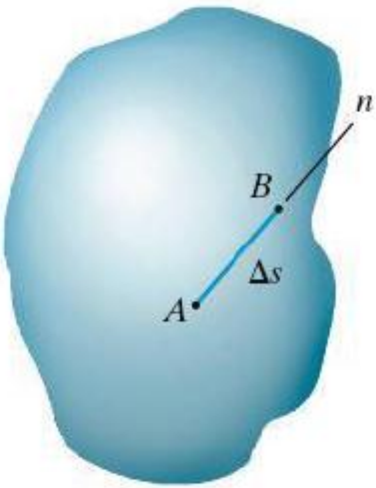


Undeformed body
(a)

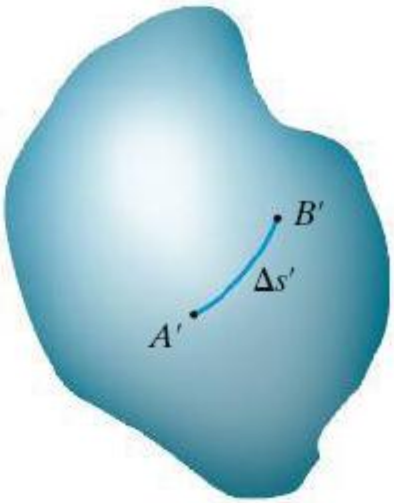


Deformed body
(b)

STRAIN



Undeformed body
(a)



Deformed body
(b)

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$

STRAIN

Hence, when ϵ (or ϵ_{avg}) is positive the initial line will elongate, whereas if ϵ is negative the line contracts.

Note that normal strain is a *dimensionless quantity*, since it is a ratio of two lengths.

SI system is used, then the basic unit for length is the meter (m). most engineering applications ϵ will be very small, so measurements of strain are in micrometers per meter ($\mu m/m$) , where 1 μm = 10^{-6}m .

2.3 Shear Strain

If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between them is referred to as ***shear strain*** .

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

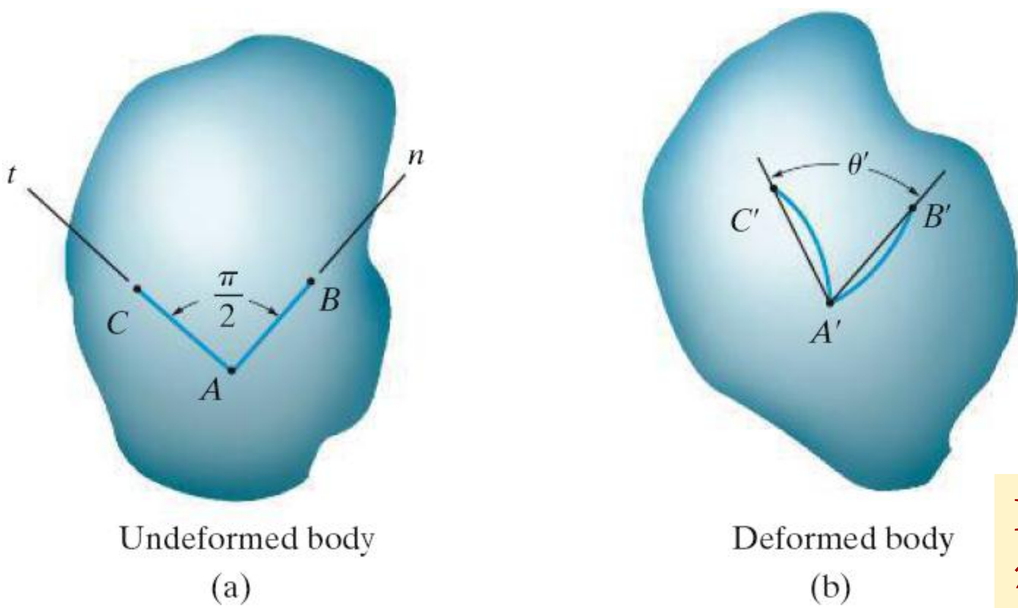
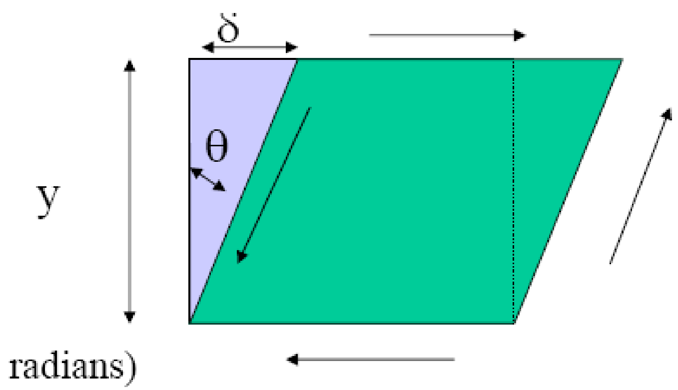


Fig. 2–2

Notice that if θ' is smaller than $\pi/2$ the shear strain is positive, whereas if θ' is larger than $\pi/2$ the shear strain is negative.

STRAIN

Shear Strain: The change in angle between two line segments that were originally perpendicular.



γ= tan θ= δ/ y ≅θ in radians provided that θ is very small

For θ= 3°, tan 3°=0.0524 where 3°= (3 x P_i)/180 =0.0523 radians

γ = π/ 2 -θ'= θ (in radians)

Cartesian Strain Components.

The approximate lengths of the sides of the parallelepiped are

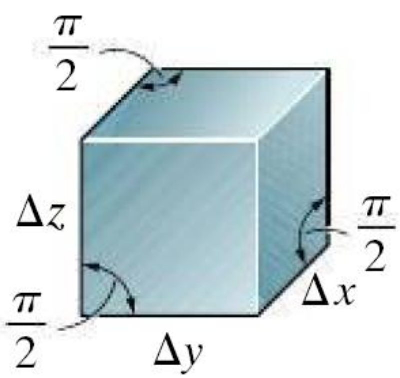
$$(1 + \epsilon_x)\,\Delta x \qquad (1 + \epsilon_y)\,\Delta y \qquad (1 + \epsilon_z)\,\Delta z$$

The approximate angles between sides, again originally defined by the sides Δ x, Δ y and Δ z are

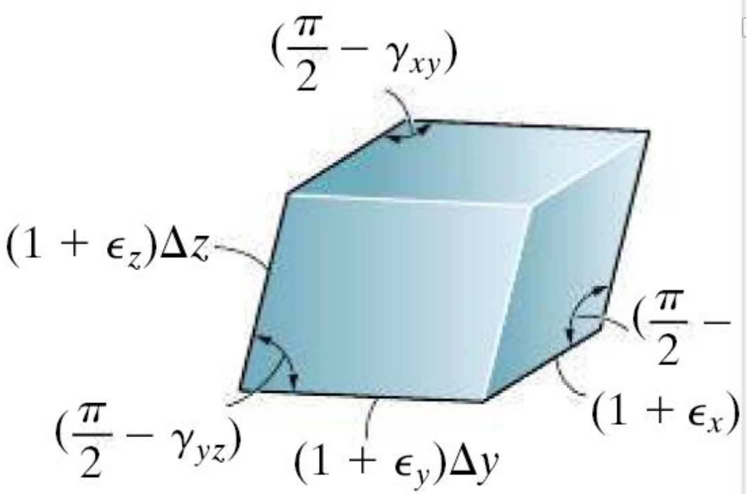
$$\frac{\pi}{2} - \gamma_{xy} \qquad \frac{\pi}{2} - \gamma_{yz} \qquad \frac{\pi}{2} - \gamma_{xz}$$

Strength of Materials: Second Class

Notice that the normal strains cause a change in **volume** of rectangular element, whereas the shear strain cause a change in **shape**



Undeformed element



Deformed element

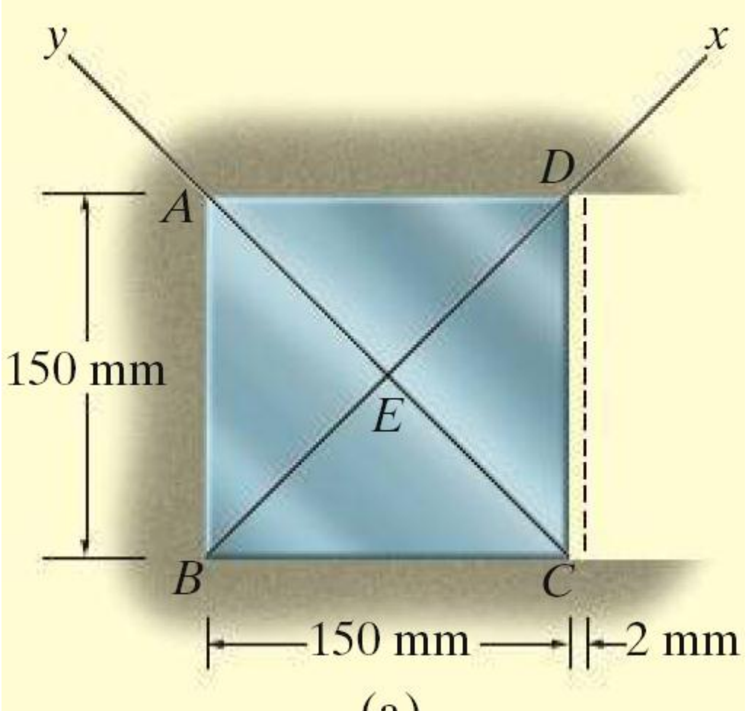
Small Strain Analysis

Most Engineering Materials undergo very small deformations, and so the normal strain $\epsilon \ll 1$. This assumption of "small strain analysis" allows the calculations for normal strain to be simplified, since first-order approximations can be made about their size.

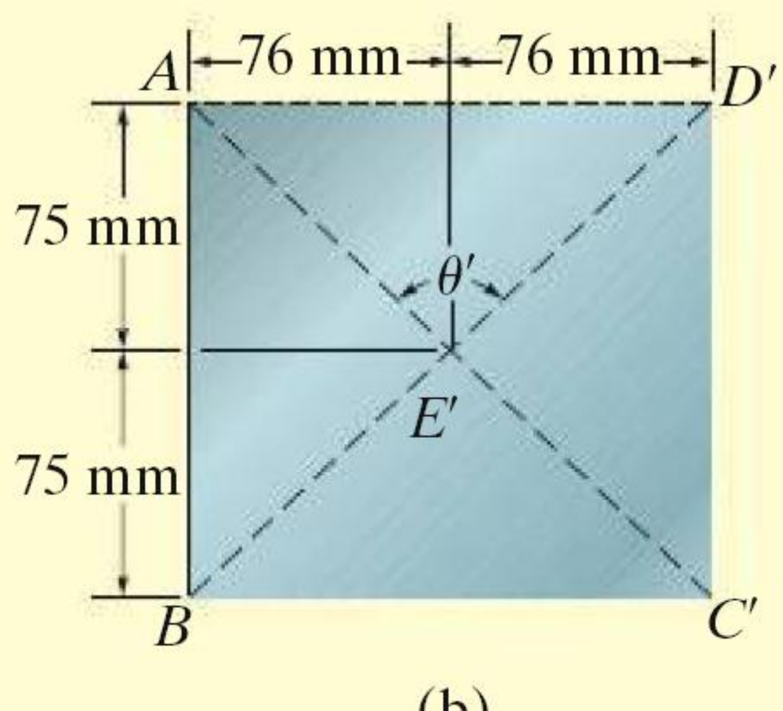
The rubber bearing support under this concrete bridge girder is subjected to both normal and shear strain. The normal strain is caused by the weight and bridge loads on the girder, and the shear strain is caused by the horizontal movement of the girder due to temperature changes.

Ex:-1

The plate shown in Fig. 2–6*a* is fixed connected along *AB* and held in the horizontal guides at its top and bottom, *AD* and *BC*. If its right side *CD* is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal *AC*, and (b) the shear strain at *E* relative to the *x*, *y* axes.



(a)



(b)

SOLUTION

Part (a). When the plate is deformed, the diagonal AC becomes AC', Fig. 2–6b. The lengths of diagonals AC and AC' can be found from the Pythagorean theorem. We have

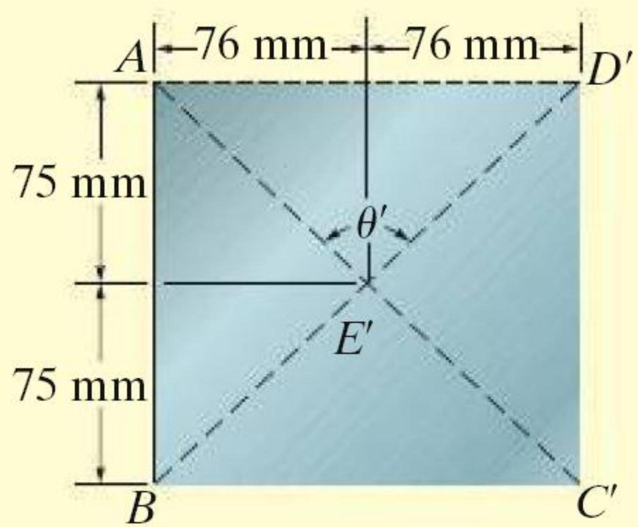
$$AC = \sqrt{(0.150\text{ m})^2 + (0.150\text{ m})^2} = 0.21213\text{ m}$$

$$AC' = \sqrt{(0.150\text{ m})^2 + (0.152\text{ m})^2} = 0.21355\text{ m}$$

Therefore the average normal strain along the diagonal is

$$(\epsilon_{AC})_{\text{avg}} = \frac{AC' - AC}{AC} = \frac{0.21355\text{ m} - 0.21213\text{ m}}{0.21213\text{ m}}$$

$$= 0.00669\text{ mm/mm}$$



(b)

Part (b). To find the shear strain at E relative to the x and y axes, it is first necessary to find the angle θ' after deformation, Fig. 2–6b. We have

$$\tan\left(\frac{\theta'}{2}\right) = \frac{76 \text{ mm}}{75 \text{ mm}}$$

$$\theta' = 90.759° = \left(\frac{\pi}{180°}\right)(90.759°) = 1.58404 \text{ rad}$$

Applying Eq. 2–3, the shear strain at E is therefore

$$\gamma_{xy} = \frac{\pi}{2} - 1.58404 \text{ rad} = -0.0132 \text{ rad} \qquad \textit{Ans.}$$

The *negative sign* indicates that the angle θ' is *greater than* 90°.

NOTE: If the x and y axes were horizontal and vertical at point E, then the 90° angle between these axes would not change due to the deformation, and so $\gamma_{xy} = 0$ at point E.

Homework

1:- The rigid beam is supported by a pin at ***A*** and wires ***BD*** and ***CE***. If the load **P** on the beam causes the end ***C*** to be displaced 10 mm downward, determine the normal strain developed in wires ***CE* and *BD*.**