

## Chapter (5)

# Speed Control of D.C. Motors

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### Introduction

Although a far greater percentage of electric motors in service are a.c. motors, the d.c. motor is of considerable industrial importance. The principal advantage of a d.c. motor is that its speed can be changed over a wide range by a variety of simple methods. Such a fine speed control is generally not possible with a.c. motors. In fact, fine speed control is one of the reasons for the strong competitive position of d.c. motors in the modern industrial applications. In this chapter, we shall discuss the various methods of speed control of d.c. motors.

### 5.1 Speed Control of D.C. Motors

The speed of a d.c. motor is given by:

$$N \propto \frac{E_b}{\phi}$$

or 
$$N = K \frac{(V - I_a R)}{\phi} \text{ r.p.m.} \quad (i)$$

where  $R = R_a$  for shunt motor  
 $= R_a + R_{se}$  for series motor

From exp. (i), it is clear that there are three main methods of controlling the speed of a d.c. motor, namely:

- (i) By varying the flux per pole ( $\phi$ ). This is known as flux control method.
- (ii) By varying the resistance in the armature circuit. This is known as armature control method.
- (iii) By varying the applied voltage  $V$ . This is known as voltage control method.

### 5.2 Speed Control of D.C. Shunt Motors

The speed of a shunt motor can be changed by (i) flux control method (ii) armature control method (iii) voltage control method. The first method (i.e. flux control method) is frequently used because it is simple and inexpensive.

## 1. Flux control method

It is based on the fact that by varying the flux  $\phi$ , the motor speed ( $N \propto 1/\phi$ ) can be changed and hence the name flux control method. In this method, a variable resistance (known as shunt field rheostat) is placed in series with shunt field winding as shown in Fig. (5.1).

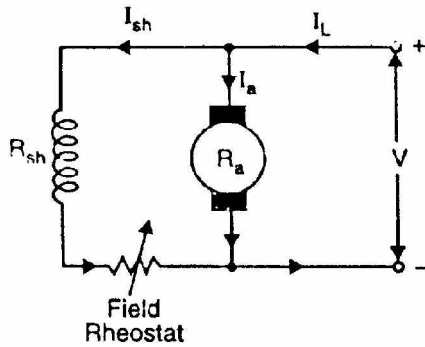


Fig. (5.1)

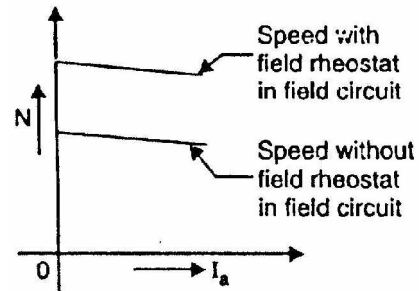


Fig. (5.2)

The shunt field rheostat reduces the shunt field current  $I_{sh}$  and hence the flux  $\phi$ . Therefore, we can only raise the speed of the motor above the normal speed (See Fig. 5.2). Generally, this method permits to increase the speed in the ratio 3:1. Wider speed ranges tend to produce instability and poor commutation.

### Advantages

- (i) This is an easy and convenient method.
- (ii) It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small value of  $I_{sh}$ .
- (iii) The speed control exercised by this method is independent of load on the machine.

### Disadvantages

- (i) Only speeds higher than the normal speed can be obtained since the total field circuit resistance cannot be reduced below  $R_{sh}$ —the shunt field winding resistance.
- (ii) There is a limit to the maximum speed obtainable by this method. It is because if the flux is too much weakened, commutation becomes poorer.

**Note.** The field of a shunt motor in operation should never be opened because its speed will increase to an extremely high value.

## 2. Armature control method

This method is based on the fact that by varying the voltage available across the armature, the back e.m.f and hence the speed of the motor can be changed. This

is done by inserting a variable resistance  $R_C$  (known as controller resistance) in series with the armature as shown in Fig. (5.3).

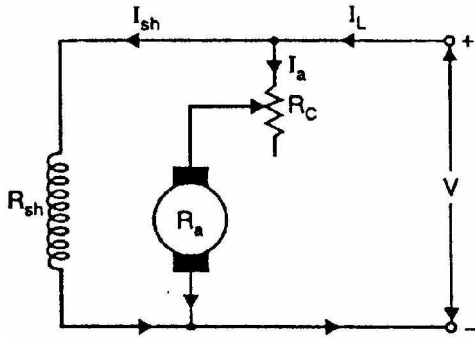


Fig. (5.3)

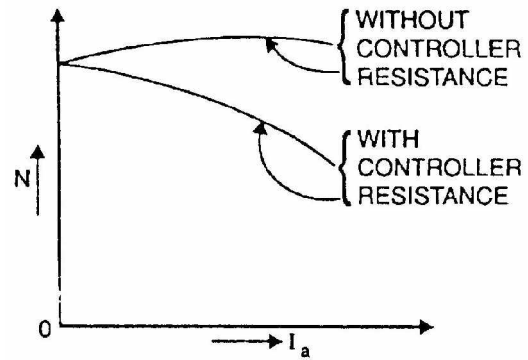


Fig. (5.4)

$$N \propto V - I_a (R_a + R_C)$$

where  $R_C$  = controller resistance

Due to voltage drop in the controller resistance, the back e.m.f. ( $E_b$ ) is decreased. Since  $N \propto E_b$ , the speed of the motor is reduced. The highest speed obtainable is that corresponding to  $R_C = 0$  i.e., normal speed. Hence, this method can only provide speeds below the normal speed (See Fig. 5.4).

### Disadvantages

- (i) A large amount of power is wasted in the controller resistance since it carries full armature current  $I_a$ .
- (ii) The speed varies widely with load since the speed depends upon the voltage drop in the controller resistance and hence on the armature current demanded by the load.
- (iii) The output and efficiency of the motor are reduced.
- (iv) This method results in poor speed regulation.

Due to above disadvantages, this method is seldom used to control the speed of shunt motors.

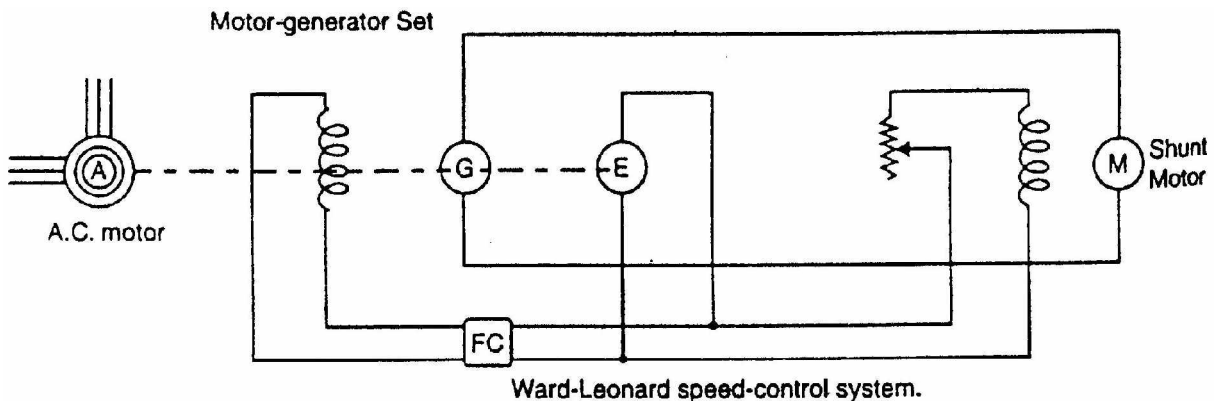
**Note.** The armature control method is a very common method for the speed control of d.c. series motors. The disadvantage of poor speed regulation is not important in a series motor which is used only where varying speed service is required.

### 3. Voltage control method

In this method, the voltage source supplying the field current is different from that which supplies the armature. This method avoids the disadvantages of poor speed regulation and low efficiency as in armature control method. However, it

is quite expensive. Therefore, this method of speed control is employed for large size motors where efficiency is of great importance.

- (i) **Multiple voltage control.** In this method, the shunt field of the motor is connected permanently across a fixed voltage source. The armature can be connected across several different voltages through a suitable switchgear. In this way, voltage applied across the armature can be changed. The speed will be approximately proportional to the voltage applied across the armature. Intermediate speeds can be obtained by means of a shunt field regulator.
- (ii) **Ward-Leonard system.** In this method, the adjustable voltage for the armature is obtained from an adjustable-voltage generator while the field circuit is supplied from a separate source. This is illustrated in Fig. (5.5). The armature of the shunt motor M (whose speed is to be controlled) is connected directly to a d.c. generator G driven by a constant-speed a.c. motor A. The field of the shunt motor is supplied from a constant-voltage exciter E. The field of the generator G is also supplied from the exciter E. The voltage of the generator G can be varied by means of its field regulator. By reversing the field current of generator G by controller FC, the voltage applied to the motor may be reversed. Sometimes, a field regulator is included in the field circuit of shunt motor M for additional speed adjustment. With this method, the motor may be operated at any speed upto its maximum speed.



**Fig. (5.5)**

### Advantages

- (a) The speed of the motor can be adjusted through a wide range without resistance losses which results in high efficiency.
- (b) The motor can be brought to a standstill quickly, simply by rapidly reducing the voltage of generator G. When the generator voltage is reduced below the back e.m.f. of the motor, this back e.m.f. sends current through the generator armature, establishing dynamic braking. While this takes

place, the generator G operates as a motor driving motor A which returns power to the line.

- (c) This method is used for the speed control of large motors when a d.c. supply is not available.

The disadvantage of the method is that a special motor-generator set is required for each motor and the losses in this set are high if the motor is operating under light loads for long periods.

### 5.3 Speed Control of D.C. Series Motors

The speed control of d.c. series motors can be obtained by (i) flux control method (ii) armature-resistance control method. The latter method is mostly used.

#### 1. Flux control method

In this method, the flux produced by the series motor is varied and hence the speed. The variation of flux can be achieved in the following ways:

- (i) **Field diverters.** In this method, a variable resistance (called field diverter) is connected in parallel with series field winding as shown in Fig. (5.6). Its effect is to shunt some portion of the line current from the series field winding, thus weakening the field and increasing the speed ( $N \propto 1/\phi$ ). The lowest speed obtainable is that corresponding to zero current in the diverter (i.e., diverter is open). Obviously, the lowest speed obtainable is the normal speed of the motor. Consequently, this method can only provide speeds above the normal speed. The series field diverter method is often employed in traction work.

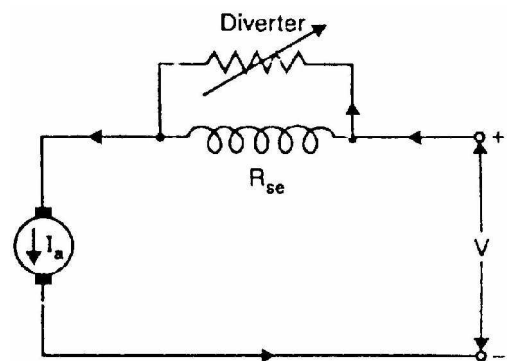


Fig. (5.6)

- (ii) **Armature diverter.** In order to obtain speeds below the normal speed, a variable resistance (called armature diverter) is connected in parallel with the armature as shown in Fig. (5.7). The diverter shunts some of the line current, thus reducing the armature current. Now for a given load, if  $I_a$  is decreased, the flux  $\phi$  must increase ( $T \propto \phi I_a$ ). Since  $N \propto 1/\phi$ , the motor speed is decreased. By adjusting the armature diverter, any speed lower than the normal speed can be obtained.
- (iii) **Tapped field control.** In this method, the flux is reduced (and hence speed is increased) by decreasing the number of turns of the series field winding as shown in Fig. (5.8). The switch S can short circuit any part of the field

winding, thus decreasing the flux and raising the speed. With full turns of the field winding, the motor runs at normal speed and as the field turns are cut out, speeds higher than normal speed are achieved.

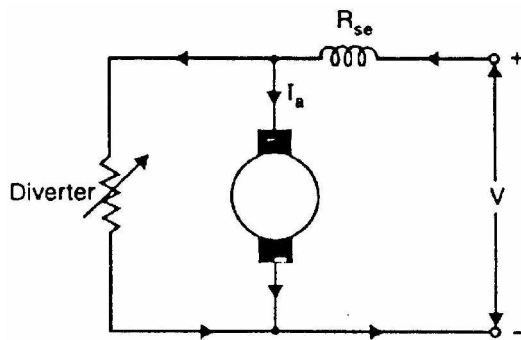


Fig. (5.7)

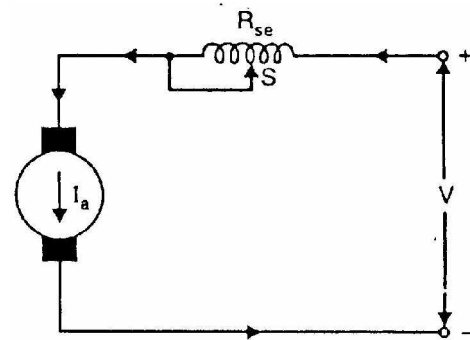


Fig. (5.8)

(iv) **Paralleling field coils.** This method is usually employed in the case of fan motors. By regrouping the field coils as shown in Fig. (5.9), several fixed speeds can be obtained.

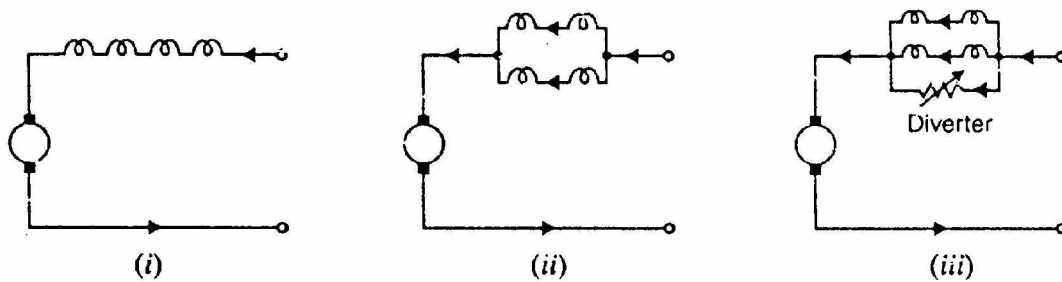


Fig. (5.9)

## 2. Armature-resistance control

In this method, a variable resistance is directly connected in series with the supply to the complete motor as shown in Fig. (5.10). This reduces the voltage available across the armature and hence the speed falls. By changing the value of variable resistance, any speed below the normal speed can be obtained. This is the most common method employed to control the speed of d.c. series motors. Although this method has poor speed regulation, this has no significance for series motors because they are used in varying speed applications. The loss of power in the series resistance for many applications of series motors is not too serious since in these applications,

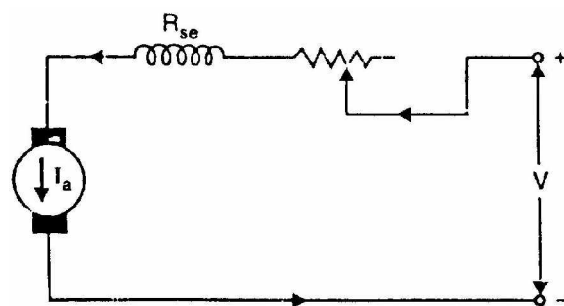


Fig. (5.10)

the control is utilized for a large portion of the time for reducing the speed under light-load conditions and is only used intermittently when the motor is carrying full-load.

## 5.4 Series-Parallel Control

Another method used for the speed control of d.c. series motors is the series-parallel method. In this system which is widely used in traction system, two (or more) similar d.c. series motors are mechanically coupled to the same load.

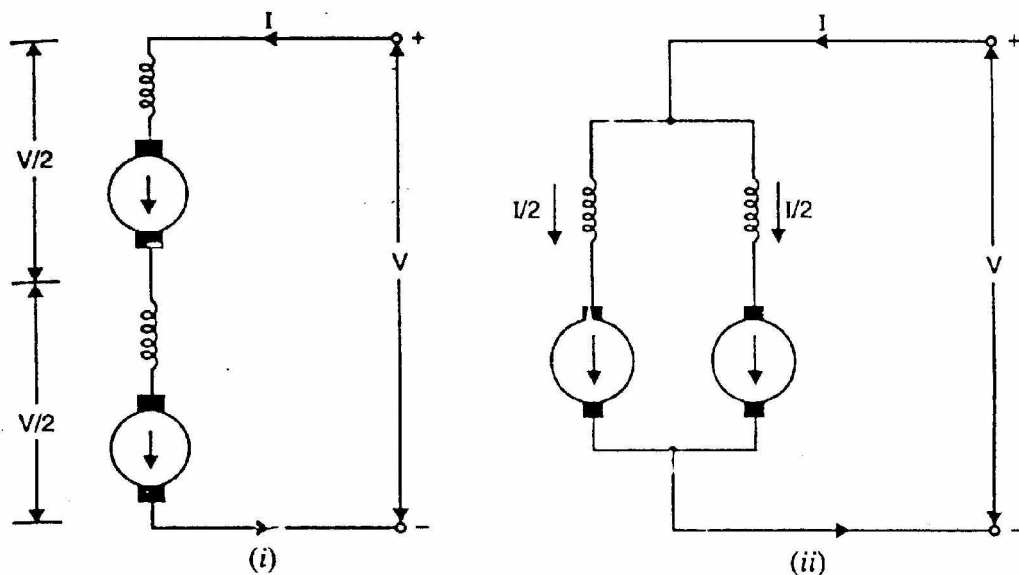


Fig. (5.11)

When the motors are connected in series [See Fig. 5.11 (i)], each motor armature will receive one-half the normal voltage. Therefore, the speed will be low. When the motors are connected in parallel, each motor armature receives the normal voltage and the speed is high [See Fig. 5.11 (ii)]. Thus we can obtain two speeds. Note that for the same load on the pair of motors, the system would run approximately four times the speed when the machines are in parallel as when they are in series.

### Series-parallel and resistance control

In electric traction, series-parallel method is usually combined with resistance method of control. In the simplest case, two d.c. series motors are coupled mechanically and drive the same vehicle.

- (i) At standstill, the motors are connected in series via a starting rheostat. The motors are started up in series with each other and starting resistance is cut out step by step to increase the speed. When all the resistance is cut out (See Fig. 5.12), the voltage applied to each motor is about one-half of the line voltage. The speed is then about one-half of what it would be if the full line voltage were applied to each motor.

- (ii) To increase the speed further, the two motors are connected in parallel and at the same time the starting resistance is connected in series with the combination (See Fig. 5.12). The starting resistance is again cut out step by step until full speed is attained. Then field control is introduced.

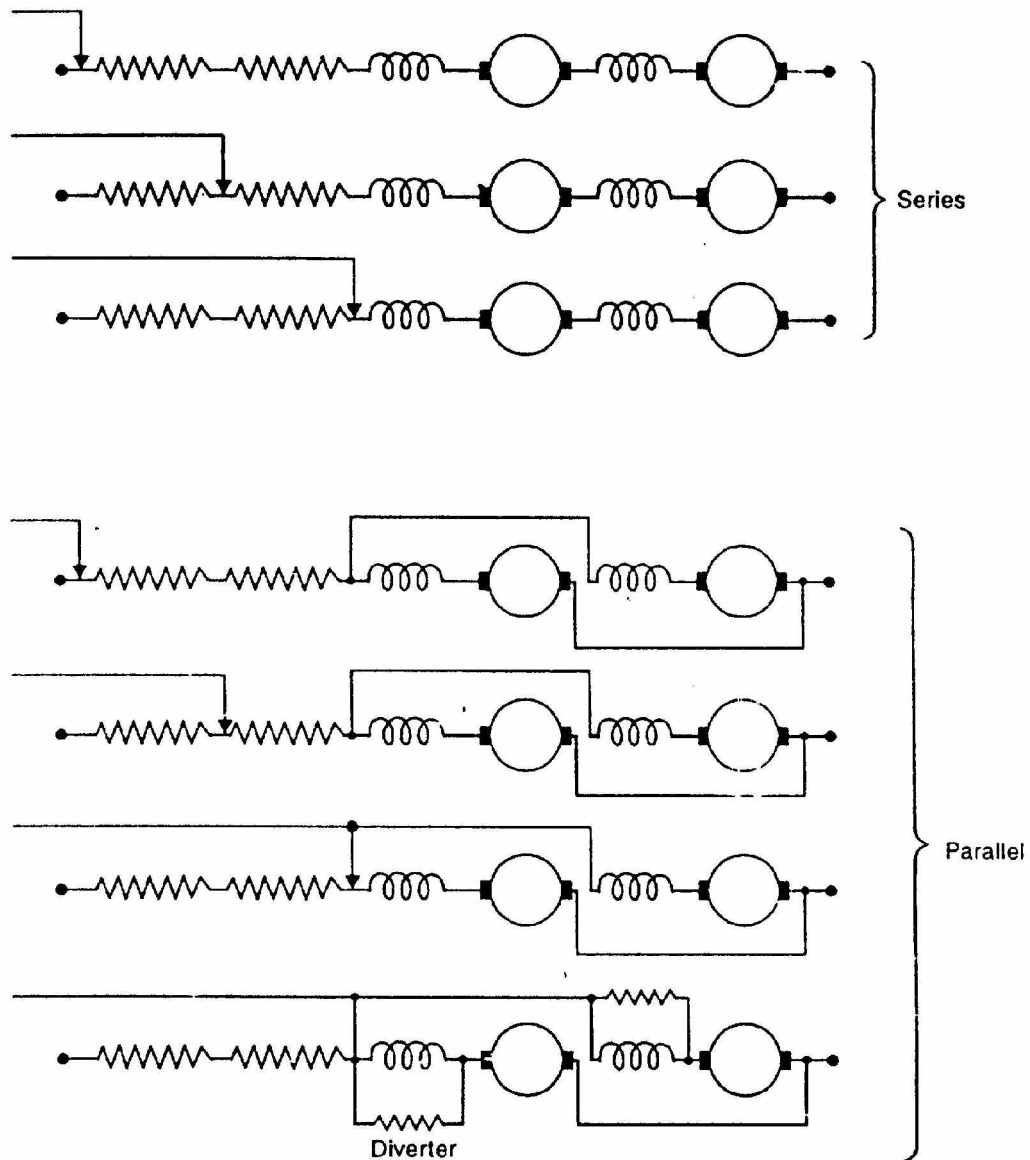


Fig. (5.12)

## 5.5 Electric Braking

Sometimes it is desirable to stop a d.c. motor quickly. This may be necessary in case of emergency or to save time if the motor is being used for frequently repeated operations. The motor and its load may be brought to rest by using either (i) mechanical (friction) braking or (ii) electric braking. In mechanical braking, the motor is stopped due to the friction between the moving parts of the motor and the brake shoe i.e. kinetic energy of the motor is dissipated as heat. Mechanical braking has several disadvantages including non-smooth stop and greater stopping time.

In electric braking, the kinetic energy of the moving parts (i.e., motor) is converted into electrical energy which is dissipated in a resistance as heat or alternatively, it is returned to the supply source (Regenerative braking). For d.c. shunt as well as series motors, the following three methods of electric braking are used:

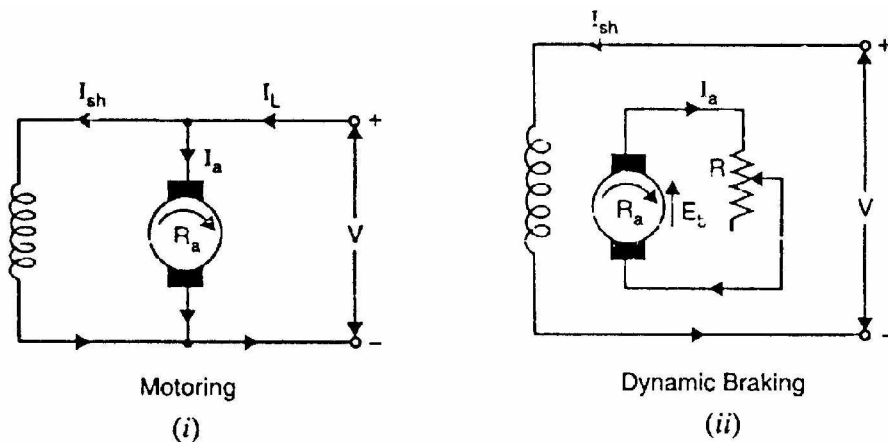
- (i) Rheostatic or Dynamic braking
- (ii) Plugging
- (iii) Regenerative braking

It may be noted that electric braking cannot hold the motor stationary and mechanical braking is necessary. However, the main advantage of using electric braking is that it reduces the wear and tear of mechanical brakes and cuts down the stopping time considerably due to high braking retardation.

**(i) Rheostatic or Dynamic braking**

In this method, the armature of the running motor is disconnected from the supply and is connected across a variable resistance  $R$ . However, the field winding is left connected to the supply. The armature, while slowing down, rotates in a strong magnetic field and, therefore, operates as a generator, sending a large current through resistance  $R$ . This causes the energy possessed by the rotating armature to be dissipated quickly as heat in the resistance. As a result, the motor is brought to standstill quickly.

Fig. (5.13) (i) shows dynamic braking of a shunt motor. The braking torque can be controlled by varying the resistance  $R$ . If the value of  $R$  is decreased as the motor speed decreases, the braking torque may be maintained at a high value. At a low value of speed, the braking torque becomes small and the final stopping of the motor is due to friction. This type of braking is used extensively in connection with the control of elevators and hoists and in other applications in which motors must be started, stopped and reversed frequently.



**Fig. (5.13)**

We now investigate how braking torque depends upon the speed of the motor. Referring to Fig. (5.13) (ii),

$$\text{Armature current, } I_a = \frac{E_b}{R + R_a} = \frac{k_1 N \phi}{R + R_a} \quad (\text{b } E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left( \frac{k_1 N \phi}{R + R_a} \right) = k_3 N \phi^2$$

where  $k_2$  and  $k_3$  are constants

For a shunt motor,  $\phi$  is constant.

$$\therefore \text{ Braking torque, } T_B \propto N$$

Therefore, braking torque decreases as the motor speed decreases.

### (ii) Plugging

In this method, connections to the armature are reversed so that motor tends to rotate in the opposite direction, thus providing the necessary braking effect. When the motor comes to rest, the supply must be cut off otherwise the motor will start rotating in the opposite direction.

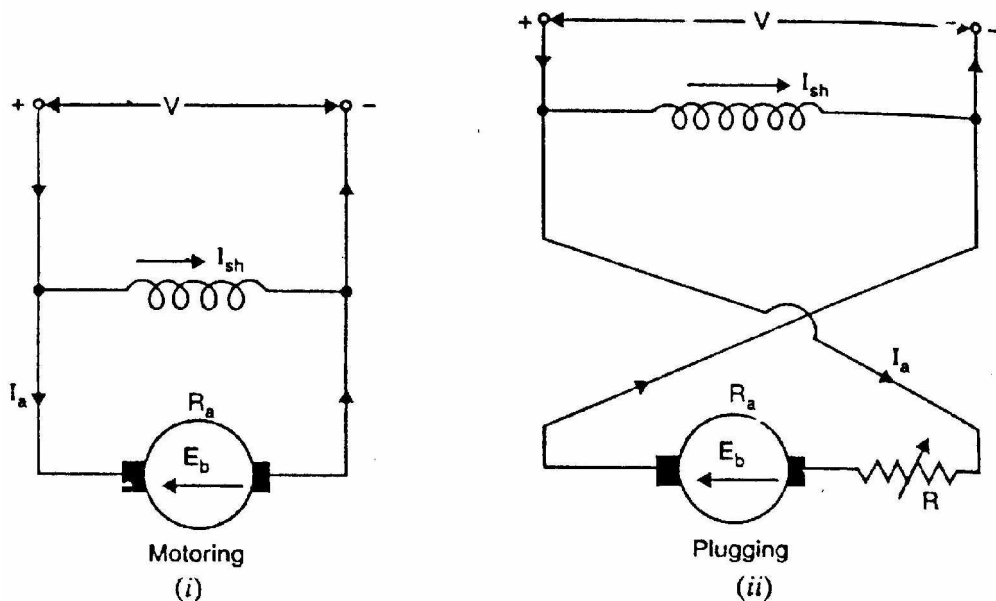


Fig. (5.14)

Fig. (5.14) (ii) shows plugging of a d.c. shunt motor. Note that armature connections are reversed while the connections of the field winding are kept the same. As a result the current in the armature reverses. During the normal running of the motor [See Fig. 5.14 (i)], the back e.m.f.  $E_b$  opposes the applied voltage  $V$ . However, when armature connections are reversed, back e.m.f.  $E_b$  and  $V$  act in the same direction around the circuit. Therefore, a voltage equal to

$V + E_b$  is impressed across the armature circuit. Since  $E_b \simeq V$ , the impressed voltage is approximately  $2V$ . In order to limit the current to a safe value, a variable resistance  $R$  is inserted in the circuit at the time of changing armature connections.

We now investigate how braking torque depends upon the speed of the motor. Referring to Fig. (5.14) (ii),

$$\text{Armature current, } I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{k_1 N \phi}{R + R_a} \quad (\text{b } E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left( \frac{V}{R + R_a} + \frac{k_1 N \phi}{R + R_a} \right) = k_3 \phi + k_4 N \phi^2$$

For a shunt motor,  $\phi$  is constant.

$$\therefore \text{ Braking torque, } T_B = k_5 + k_6 N$$

Thus braking torque decreases as the motor slows down. Note that there is some braking torque ( $T_B = k_5$ ) even when the motor speed is zero.

### (iii) Regenerative braking

In the regenerative braking, the motor is run as a generator. As a result, the kinetic energy of the motor is converted into electrical energy and returned to the supply. Fig. (5.15) shows two methods of regenerative braking for a shunt motor.

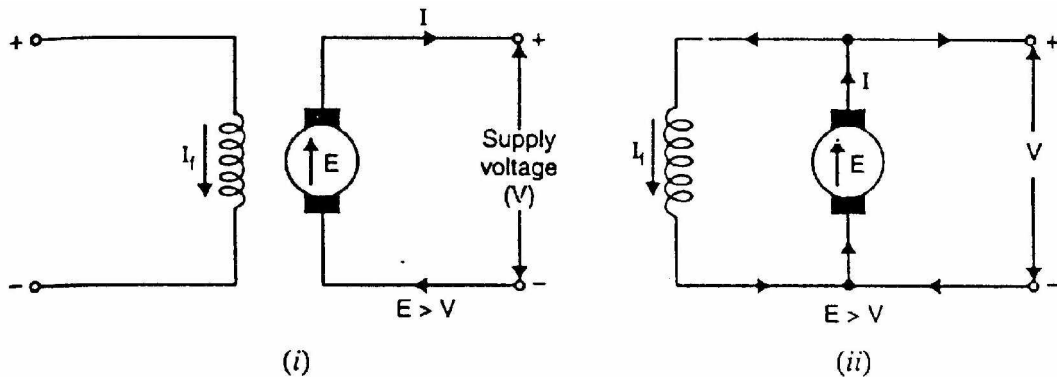


Fig. (5.15)

- (a) In one method, field winding is disconnected from the supply and field current is increased by exciting it from another source [See Fig. 5.15 (i)]. As a result, induced e.m.f.  $E$  exceeds the supply voltage  $V$  and the machine feeds energy into the supply. Thus braking torque is provided up to the speed at which induced e.m.f. and supply voltage are equal. As the machine slows down, it is not possible to maintain induced e.m.f. at a higher value

than the supply voltage. Therefore, this method is possible only for a limited range of speed.

- (b) In a second method, the field excitation does not change but the load causes the motor to run above the normal speed (e.g., descending load on a crane). As a result, the induced e.m.f.  $E$  becomes greater than the supply voltage  $V$  [See Fig. 5.15 (ii)]. The direction of armature current  $I$ , therefore, reverses but the direction of shunt field current  $I_f$  remains unaltered. Hence the torque is reversed and the speed falls until  $E$  becomes less than  $V$ .

## 5.6 Speed Control of Compound Motors

Speed control of compound motors may be obtained by any one of the methods described for shunt motors. Speed control cannot be obtained through adjustment of the series field since such adjustment would radically change the performance characteristics of the motor.

## 5.7 Necessity of D.C. Motor Starter

At starting, when the motor is stationary, there is no back e.m.f. in the armature. Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current ( $I_a = V/R_a$ ) because of small armature resistance. As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5  $\Omega$ . If this motor is directly switched on to supply, it would take an armature current of  $220/0.5 = 440$  A which is 22 times the full-load current. This high starting current may result in:

- (i) burning of armature due to excessive heating effect,
- (ii) damaging the commutator and brushes due to heavy sparking,
- (iii) excessive voltage drop in the line to which the motor is connected. The result is that the operation of other appliances connected to the line may be impaired and in particular cases, they may refuse to work.

In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit. This resistance is gradually reduced as the motor gains speed (and hence  $E_b$  increases) and eventually it is cut out completely when the motor has attained full speed. The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

## 5.8 Types of D.C. Motor Starters

The stalling operation of a d.c. motor consists in the insertion of external resistance into the armature circuit to limit the starting current taken by the motor and the removal of this resistance in steps as the motor accelerates. When

the motor attains the normal speed, this resistance is totally cut out of the armature circuit. It is very important and desirable to provide the starter with protective devices to enable the starter arm to return to OFF position

- (i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.
- (ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil.

There are two principal types of d.c. motor starters viz., three-point starter and four-point starter. As we shall see, the two types of starters differ only in the manner in which the no-volt release coil is connected.

## 5.9 Three-Point Starter

This type of starter is widely used for starting shunt and compound motors.

### Schematic diagram

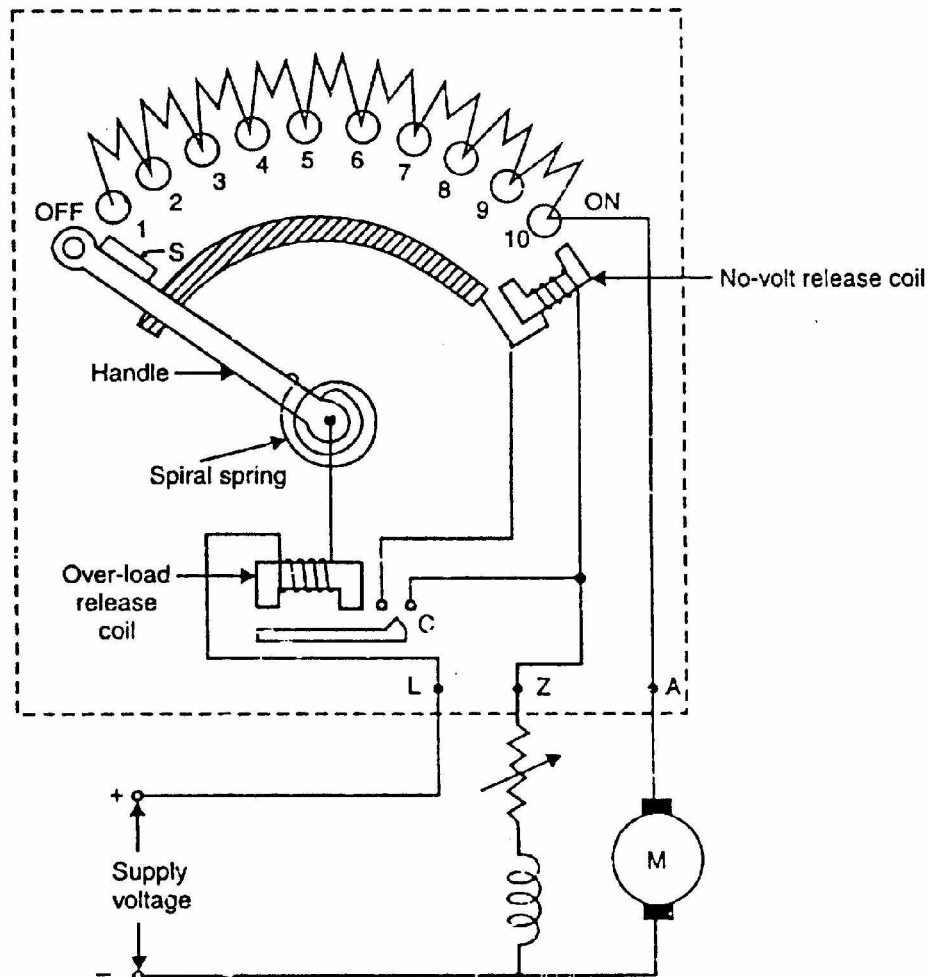
Fig. (5.16) shows the schematic diagram of a three-point starter for a shunt motor with protective devices. It is so called because it has three terminals L, Z and A. The starter consists of starting resistance divided into several sections and connected in series with the armature. The tapping points of the starting resistance are brought out to a number of studs. The three terminals L, Z and A of the starter are connected respectively to the positive line terminal, shunt field terminal and armature terminal. The other terminals of the armature and shunt field windings are connected to the negative terminal of the supply. The no-volt release coil is connected in the shunt field circuit. One end of the handle is connected to the terminal L through the over-load release coil. The other end of the handle moves against a spiral spring and makes contact with each stud during starting operation, cutting out more and more starting resistance as it passes over each stud in clockwise direction.

### Operation

- (i) To start with, the d.c. supply is switched on with handle in the OFF position.
- (ii) The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature circuit.
- (iii) As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held

magnetically by the no-volt release coil which is energized by shunt field current.

- (iv) If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.
- (v) If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no-volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.



**Fig. (5.16)**

## Drawback

In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current. While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position. This may disconnect the motor from the supply when it is not desired. This drawback is overcome in the four-point starter.

## 5.10 Four-Point Starter

In a four-point starter, the no-volt release coil is connected directly across the supply line through a protective resistance  $R$ . Fig. (5.17) shows the schematic diagram of a 4-point starter for a shunt motor (over-load release coil omitted for clarity of the figure). Now the no-volt release coil circuit is independent of the shunt field circuit. Therefore, proper speed control can be exercised without affecting the operation of no-volt release coil.

Note that the only difference between a three-point starter and a four-point starter is the manner in which no-volt release coil is connected. However, the working of the two starters is the same. It may be noted that the three-point starter also provides protection against an open-field circuit. This protection is not provided by the four-point starter.

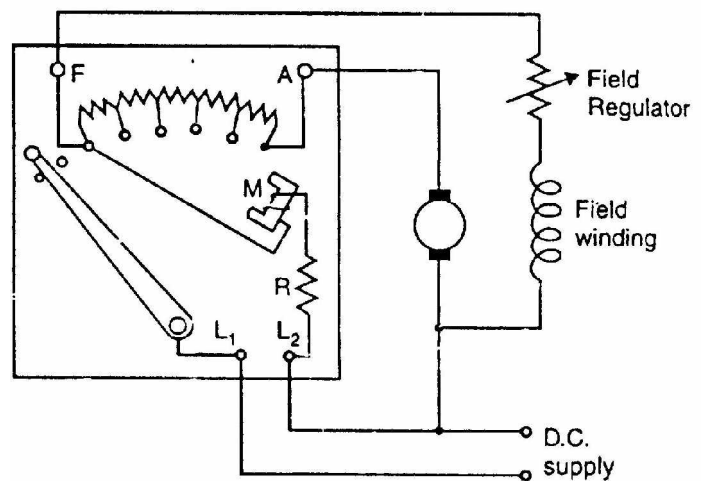
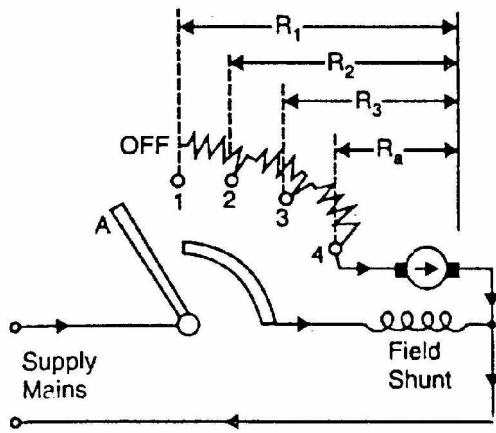


Fig. (5.17)

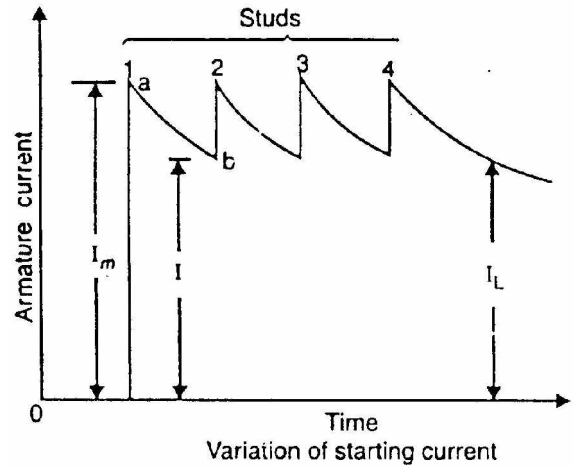
## 5.11 Grading of Starting Resistance—Shunt Motors

For starting the motor satisfactorily, the starting resistance is divided into a number of sections in such a way that current fluctuates between maximum ( $I_m$ ) and minimum ( $I$ ) values. The upper limit is that value established as the maximum permissible for the motor; it is generally 1.5 times the full-load current of the motor. The lower limit is the value set as a minimum for starting operation; it may be equal to full-load current of the motor or some predetermined value. Fig. (5.18) shows shunt-wound motor with starting resistance divided into three sections between four studs. The resistances of

these sections should be so selected that current during starting remains between  $I_m$  and  $I$  as shown in Fig. (5.19).



**Fig. (5.18)**



**Fig. (5.19)**

- (i) When arm A is moved from OFF position to stud 1, field and armature circuits are energized and whole of the starting resistance is in series with the armature. The armature current jumps to maximum value given by;

$$I_m = \frac{V}{R_1}$$

where  $R_1 =$  Resistance of starter and armature

- (ii) As the armature accelerates, the generated e.m.f. increases and the armature current decreases as indicated by curve ab. When the current has fallen to  $I$ , arm A is moved over to stud 2, cutting out sufficient resistance to allow the current to rise to  $I_m$  again. This operation is repeated until the arm A is on stud 4 and the whole of the starting resistance is cut out of the armature circuit.
- (iii) Now the motor continues to accelerate and the current decreases until it settles down at some value  $I_L$  such that torque due to this current is just sufficient to meet the load requirement.

## 5.12 Starter Step Calculations for D.C. Shunt Motor

Fig. (5.20) shows a d.c. shunt motor starter with  $n$  resistance sections and  $(n + 1)$  studs.

Let  $R_1 =$  Total resistance in the armature circuit when the starter arm is on stud no. 1 (See Fig. 5.20)

$R_2 =$  Total resistance in the armature circuit when the starter arm is on stud no. 2 and so on

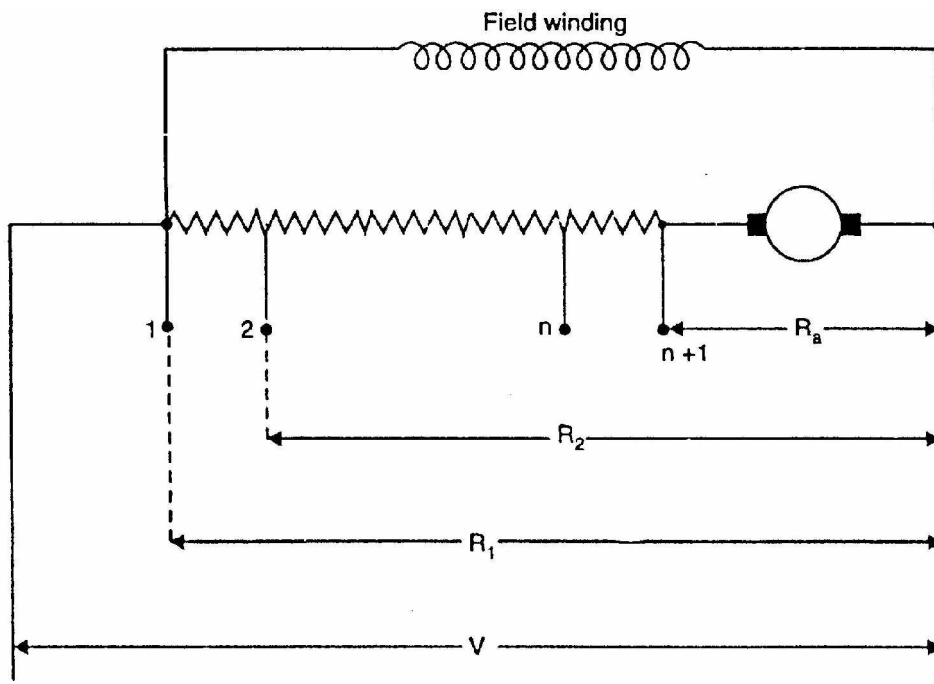
- $I_m$  = Upper current limit
- $I$  = Lower current limit
- $n$  = Number of sections in the starter resistance
- $V$  = Applied voltage
- $R_a$  = Armature resistance

**On stud 1.** When the starter arm-moves to stud 1, the total resistance in the armature circuit is  $R_1$  and the circuit current jumps to maximum values  $I_m$  given by;

$$I_m = \frac{V}{R_1} \quad (i)$$

Since torque  $\propto \phi I_a$ , it follows that the maximum torque acts on the armature to accelerate it. As the armature accelerates, the induced e.m.f. (back e.m.f.) increases and the armature current decreases. When the current has fallen to the predetermined value  $I$ , the starter arm is moved over to stud 2. Let the value of back e.m.f. be  $E_{b1}$  at the instant the starter arm leaves the stud 1. Then  $I$  is given by;

$$I = \frac{V - E_{b1}}{R_1} \quad (ii)$$



**Fig. (5.20)**

**On stud 2.** As the starter arm moves over to stud 2, sufficient resistance is cut out (now total circuit resistance is  $R_2$ ) and current rises to maximum value  $I_m$  once again given by;

$$I_m = \frac{V - E_{b1}}{R_2} \quad (\text{iii})$$

The acceleration continues and the back e.m.f. increases and the armature current decreases. When the current has fallen to the predetermined value  $I$ , the starter arm is moved over to stud 3. Let  $E_{b2}$  be the value of back e.m.f. at the instant the starter arm leaves the stud 2. Then,

$$I = \frac{V - E_{b2}}{R_2} \quad (\text{iv})$$

**On stud 3.**

$$\text{As the starter arm moves to stud 3, } I_m = \frac{V - E_{b2}}{R_3} \quad (\text{v})$$

$$\text{As the starter arm leaves stud 3, } I = \frac{V - E_{b3}}{R_3} \quad (\text{vi})$$

**On nth stud.**

$$\text{As the starter arm leaves } \underline{n}\text{th stud, } I = \frac{V - E_{bn}}{R_n}$$

**On (n + 1)th stud.** When the starter arm moves over to (n + 1)th stud, all the external starting resistance is cut out, leaving only the armature resistance  $R_a$ .

$$\therefore I_m = \frac{V - E_{bn}}{R_a} \quad \text{and} \quad I = \frac{V - E_b}{R_a}$$

Dividing Eq.(ii) by Eq.(iii), we get,

$$\frac{I}{I_m} = \frac{R_2}{R_1}$$

Dividing Eq.(iv) by Eq. (v), we get,

$$\frac{I}{I_m} = \frac{R_3}{R_2}$$

Continuing these divisions, we have finally,

$$\frac{I}{I_m} = \frac{R_a}{R_n}$$

$$\text{Let } \frac{I}{I_m} = k. \quad \text{Then } \frac{R_2}{R_1} = \frac{R_3}{R_2} = \dots = \frac{R_a}{R_n} = k$$

If we multiply these n equal ratios together, then,

$$\frac{R_2}{R_1} \times \frac{R_3}{R_2} \times \frac{R_4}{R_3} \times \dots \times \frac{R_a}{R_n} = k^n$$

$$\therefore \frac{R_a}{R_1} = k^n$$

Thus we can calculate the values of  $R_2$ ,  $R_3$ ,  $R_4$  etc. if the values of  $R_1$ ,  $R_a$  and  $n$  are known.