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Study and Simulation of least square channel estimation of OFDM systems

A project

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بِسْمِ اللهِ الرَّحْمنِ الرَّحِيمِ

إِنَّ فِي خَلْقِ السَّموَاتِ وَالْأَرْضِ وَاخْتِلافِ اللَّيْلِ وَالنَّهَارِ وَالْفُلْكِ الَّتِي تَجْرِي

فِي الْبَحْرِ بِمَا يَنْفَعُ النَّاسَ وَمَا أَنْزَلَ اللَّهُ مِنَ السماءِ مِنْ ماءٍ فَأَحْيَا بِهِ الأَرْضَ

بَعْدَ مَوْتِهَا وَبَثَّ فِيهَا مِنْ كُلِّ دَآبَّةٍ وَتَصْرِيفِ الرِّيَاحِ وَالسَّحَابِ الْمُسَخَّرِ

بَيْنَ السماءِ وَالأَرْضِ لَآيَاتٍ لِقَوْمٍ يَعْقِلُونَ (164)

صدق الله العلي العظيم

سورة البقرة

Dedication

TO

MY " FAMILY " WITH LOVE

Acknowledgement

We wish to thank our family for their understanding and support including our parents , siblings , our big family and our friends inside and outside university.

We wish to express our deepest gratitude to our Advisor Dr. Montadar Abbas Taher and Mr. Ahmed Mohamed Ahmed for his guidance and friendship during our study. And at last we want to thank the department of communication for giving us the chance to work on as a fine project as this one.

ABSTRACT

The concept of OFDM is not new but receiver designs are constantly improved. With new advances in DSP technologies, OFDM has become popular for the reasons of efficient bandwidth usage and ease of synthesis with new DSP technology. However, it is sensitive to synchronization error and has a relatively large peak to average power ratio. This thesis will provide an overall look into OFDM systems and its developments. It will also look into the challenges OFDM faces and concentrate on one main aspect of an OFDM receiver design, it is the channel estimation.

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LIST OF ABBREVIATIONS

AWGN	–	Additive White Gaussian Noise
ICI	–	Interchannel Interference
ISI	–	Inter symbol Interference
DFT	_	Discrete Fourier Transform
DMT	_	Discrete Modulation
DSL	_	Digital subscariber line
DAB	_	Digital Audio Broadcasting
SNR	_	The Signal-To-Noise Ratio
QAM	_	Quadrature Amplitude Modulation
IDFT	_	Inverse Discrete Fourier Transform
FFT	_	Fast Fourier Transform
IFFT	_	Inverse Fast Fourier Transform
LOS	_	Line of Sight
PDP	_	Power Delay Profile
LS	_	The Least Square
LMMSE	_	The Linear Minimum Mean Squares Error
MSE	_	The mean square error

LIST OF SYMBOLS

P/S	_	Parallel to series
S/P	_	Series to parallel
f_{o}	_	Fundamental frequency
y	_	The received vector of signaling points
x	_	The transmitted signaling points
H_c	_	The diagonalised channel attenuation vector
n	_	A vector of complex
T_s	_	The sampling period of the system
$H[k]$	_	The attenuation
α_k	_	Independent zero mean
τ_k	_	The delay of the *k*th impulse
N_0	_	The noise power density
F	_	Operation frequency

1.1 Introduction

Where did it begin?

In 1966 Robert W. Chang published a paper on the synthesis of band limited orthogonal signals for multichannel data transmission [1]. It describes a method in which signal can be simultaneously transmitted through a band limited channel without ICI (Interchannel Interference) and ISI (Inter symbol Interference). The idea of dividing the spectrum into several channels allowed transmission at a low enough data rate to counter the effect of time dispersion in the channel. As the sub channels are orthogonal they can overlap providing a much more efficient use of the available spectrum.

In 1971, S.B. Weinstein and P.M. Ebert introduced the DFT (Discrete Fourier Transform) to perform the baseband modulation and demodulation [2] This replaced the traditional bank of oscillators and multipliers needed to create and modulate onto each subcarrier.

In 1980, A.Peled and A. Ruiz introduced the cyclic prefix [3].This takes the last part of the symbol and attaches it to the front. When this extension is longer than the channel impulse response, the channel matrix is seen as circulate and orthogonally of the subcarriers is maintained over the time dispersive channel.

OFDM is currently used in European Digital Audio Broadcasting (DAB.(OFDM is used in DSL (Digital Subscriber Line) where it is know as DMT (Discrete Multitone). It is used in the European standard Hyperlan/2 and in IEEE 802.11a. This thesis will explore concepts and designs which have already been established and look into some newer technologies. However, we will concentrate on what really makes an OFDM system work, for which a certain degree of knowledge and understanding of signal processing and digital communications is necessary. From here, we will launch into important background information which did take a somewhat long time to understand.

1.2 Problem Statement

Multicarrier modulation, which is represented here by the OFDM system, is sensitive to multipath fading channels. In an OFDM system, the transmitter modulates the message bit sequence into PSK/QAM symbols, performs IFFT on the symbols to convert them into time-domain signals, and sends them out through a (wireless) channel. The received signal is usually distorted by the channel characteristics. In order to recover the transmitted bits, the channel effect must be estimated and compensated in the receiver. Each subcarrier can be regarded as an independent channel, as long as no ICI (Inter-Carrier Interference) occurs، and thus preserving the orthogonality among subcarriers. The orthogonality allows each subcarrier component of the received signal to be expressed as the product of the transmitted signal and channel frequency response at the subcarrier. Thus, the transmitted signal can be recovered by estimating the channel response just at each subcarrier. In general, the channel can be estimated by using a preamble or pilot symbols known to both transmitter and receiver، which employ various interpolation techniques to estimate the channel response of the subcarriers between pilot tones. In general, data signal as well as training signal, or both، can be used for channel estimation. In order to choose the channel estimation technique for the OFDM system under consideration, many different aspects of implementations, including the required performance, computational complexity and time-variation of the channel must be taken into account.

1.3 Objective

The study of OFDM performance over AWGN channel and Rayleigh channel using MATLAB simulation

1.4 Organization Of Research

Chapter 1 introduces a basic history of OFDM systems. Chapter 2 shows the general theory of the OFDM system. Chapter 3 speaks about the channel estimation of the OFDM system and the important methods for channel estimation**.** Chapter 4 includes the results and the discussion of the results, which had obtained by using

Matlab software. While Chapter 5 will talk about the conclusion of the research and a recommendation for future work.

2.1 Introduction

OFDM is a modulation technique in that it modulates data onto equally spaced sub-carriers. The information is modulated onto the sub-carrier by varying the phase, amplitude, or both. Each sub-carrier then combined together by using the inverse fast Fourier transform to yield the time domain waveform that is to be transmitted. To obtain a high spectral efficiency the frequency response of each of the sub-carriers are overlapping and orthogonal. This orthogonally prevents interference between the sub-carriers and is preserved even when the signal passes through a multipath channel by introducing a Cyclic Prefix, which prevents Inter-symbol Interference (ISI) on the carriers. This makes OFDM especially suited to wireless communications application . A simple block diagram for the OFDM system can be seen in Figure 2.1.

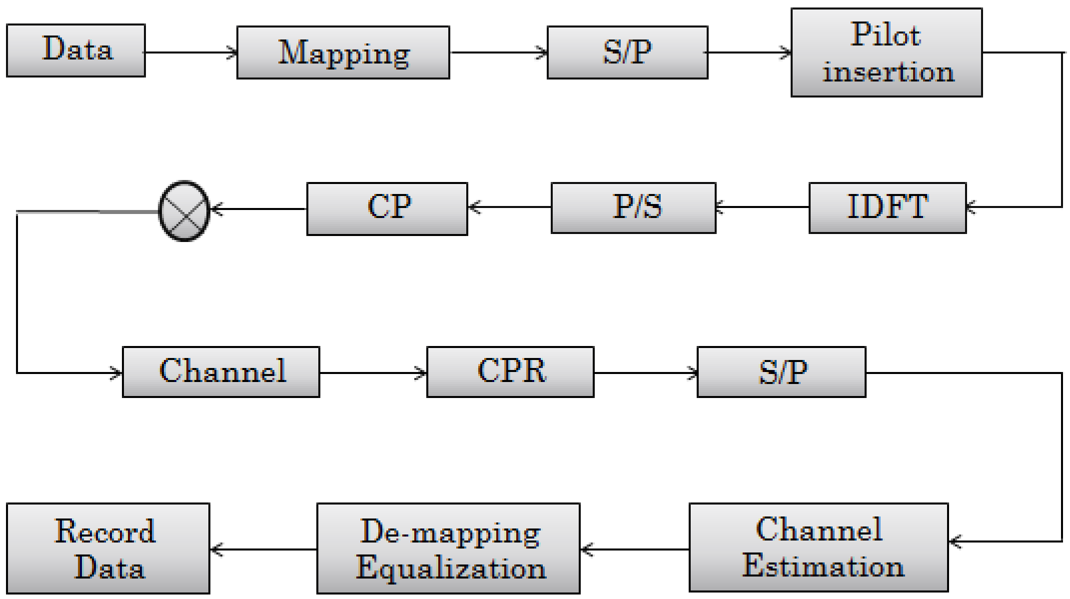

Fig 2.1: Block diagram of OFDM

In Figure (2.1):-

- Data : can be Discrete [0 , 1 , 3 ……. M-1] or binary [01010….], where M is the constellation order, or it is called the baseband modulation order.

- Mapping (baseband modulation) : distributing the data on the construction diagram, in Figure (2.2)

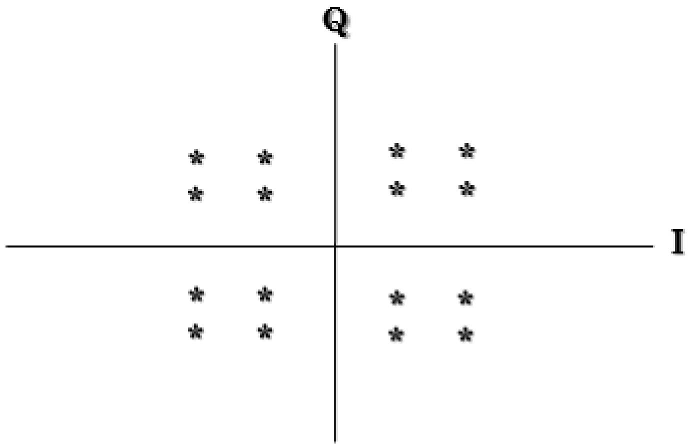

Fig 2.2 : The construction diagram

- S/P (De-Multiplexing) : Any symbol contain set of serial bits entered to serial to parallel convertor , that is register has one input and several output

- IDFT (Inverse Discrete Fourier Transform) :

$$\boldsymbol{X(n)} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi nk}{N}} \qquad (2.1)$$

$$\boldsymbol{e^{j\theta}} = \cos\theta_k + j\sin\theta_k \qquad (2.2)$$

- Cyclic Prefix : this can be (8% to 25%) of the symbol length data copied to the front of the OFDM symbol to prevent the ISI during transmission through the propagation channel as shown in Figure (2.3)

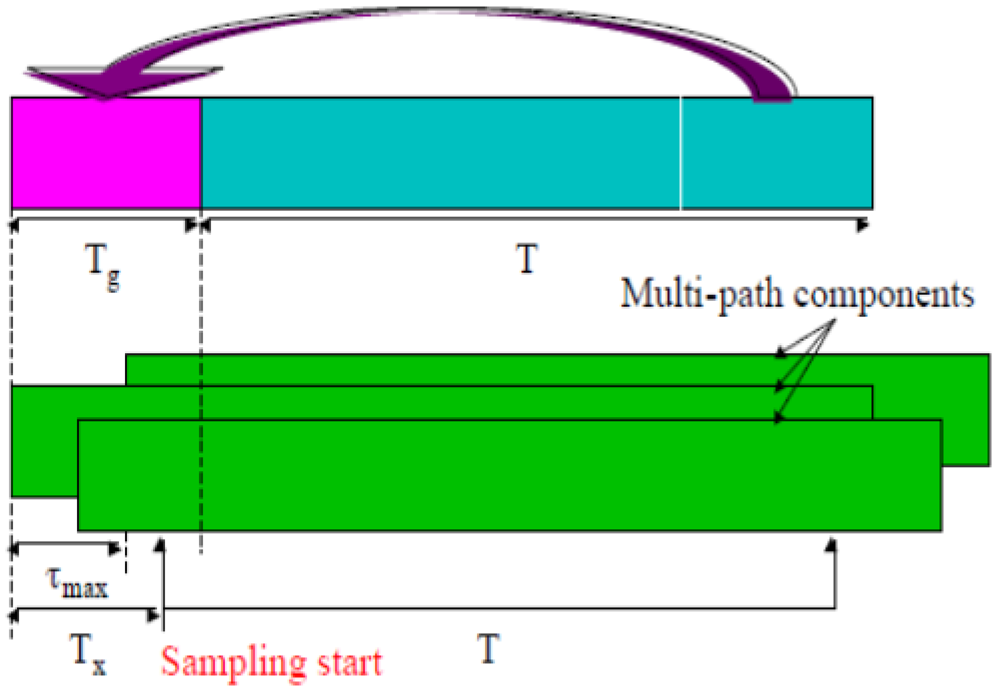

Fig 2.3: Cyclic Prefix of OFDM symbol

- Channel : Is contain multi-path effect and noise, this can be seen in Figure (2.4)

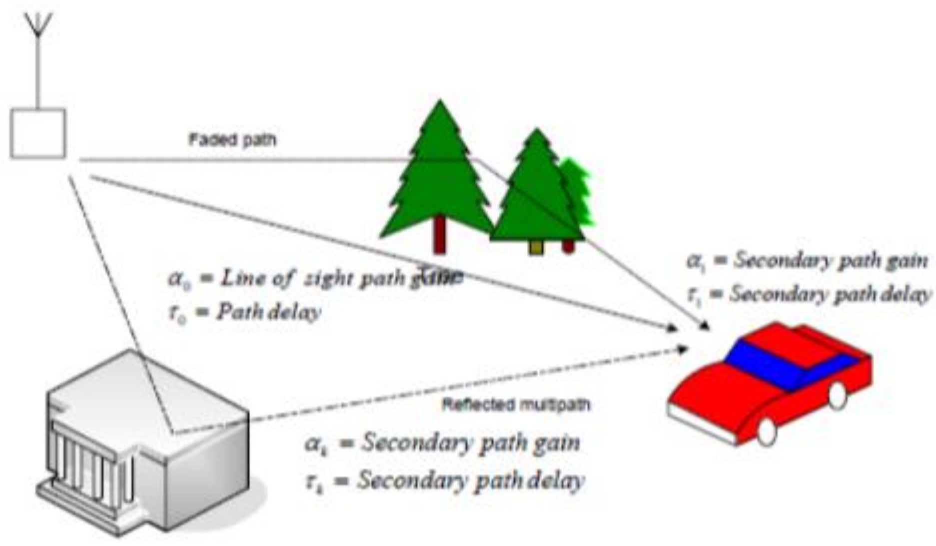

Fig 2.4: Showing the multipath channel

$$\boldsymbol{Y}(\boldsymbol{f}) = H(f)\,X(f) \tag{2.3}$$

- Channel Estimation :

- Pilot should be known value at the receiver and transmitter.

- Pilot value is equal in transmitter and varies in the Receiver because noise. In Figure (2.5), a simple pilot distribution can be seen.

- $Y(f) = H(f)\,X(f)$

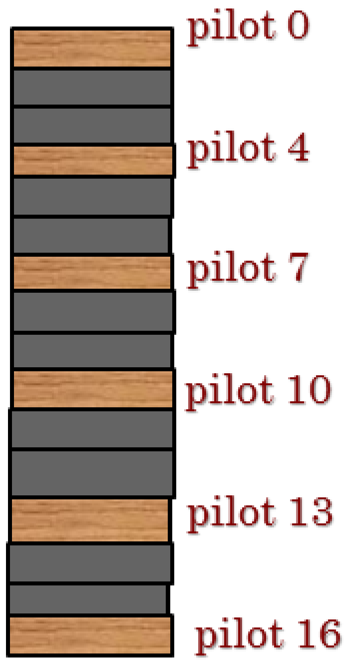

Fig 2.5: Pilot value is equal in transmitter and varies in the Receiver because noise.

- In The Receiver:
  - The cyclic prefix will be removed
  - Transform from time domain to frequency domain will be achieved by Discrete Fourier Transform (DFT).
  - P/S (Multiplexing)
  - Output data

2.2 Advantages / Disadvantages

OFDM has the following advantages[4]:

- OFDM is an efficient way to deal with multipath; implementation complexity is significantly lower than single carrier with equalizer.

- In relatively slow time-varying channels, performance can be enhanced by the adaptability of the data rate according to the SNR ratio of that sub-carrier.

- OFDM is robust against narrowband interference, because such interference affects only a small number of sub-carriers.

- OFDM makes single-frequency networks possible, which is especially attractive for broadcasting applications.

On the other hand, OFDM has the following disadvantages compared to single-carrier modulation:

- OFDM is more sensitive to frequency offsets and phase noise.

- OFDM has a relatively large peak-to-average power ratio, which reduces the power efficiency of the RF amplifier.

2.3 Orthogonality

Two periodic signals are orthogonal when the integral of their product over one period is equal to zero

For the case of continuous time:

$$\int_0^T \cos(2\pi n f_o t)\cos(2\pi n f_o t)dt = 0,\ (m \neq n) \tag{2.4}$$

For the case of discrete time:

$$\sum_{K=0}^{N-1} \cos\left(\frac{2\pi kn}{N}\right)\cos\left(\frac{2\pi km}{N}\right) dt = 0,\ (m \neq n) \tag{2.5}$$

To maintain orthogonality between sub-carriers, it is necessary to ensure that the symbol time contains one or more multiple cycles of each sinusoidal carrier waveform. In the case of OFDM, the sinusoids of our sub-carriers will satisfy this requirement since each is a multiple of a fundamental frequency. Orthogonality is critical since it prevents inter-carrier interference (ICI). ICI occurs when the integral of the carrier products are no longer zero over the integration period, so signal components from one sub-carrier causes interference to neighbouring sub-carriers. As such, OFDM is highly sensitive to frequency dispersion caused by Doppler shifts, which results in loss of orthogonality between sub-carriers.

2.3.1 OFDM Sub-Carriers

Each sub-carrier in an OFDM system is a sinusoid with a frequency that is an integer multiple of a fundamental frequency $f_\circ$. Each sub-carrier is like a Fourier series component of the composite signal, an OFDM symbol. In Figure (2.1), all the sub-carriers have the same amplitude and phase, but in practice these will be modulated separately through the use of Quadrature Amplitude Modulation (QAM).

The sub-carrier waveform can be expressed as the following equation:

$$\begin{aligned}\boldsymbol{s(t)} &= [cos(2\pi f_o t + \theta_k)] \\ &= a_n cos(2\pi n f_o t) + b_n sin(2\pi n f_o t) \\ &= \sqrt{a_n^2 + b_n^2}\cos(2\pi n f_o\, t + \varphi_n),\ \ \text{where } \varphi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)\end{aligned} \tag{2.6}$$

The sum of the sub-carriers is then the baseband OFDM signal:

$$\boldsymbol{s_B(t)} = \sum_{n=0}^{N-1}\{a_n \cos(2\pi n f_o t) - b_n \sin(2\pi f_o t)\} \tag{2.7}$$

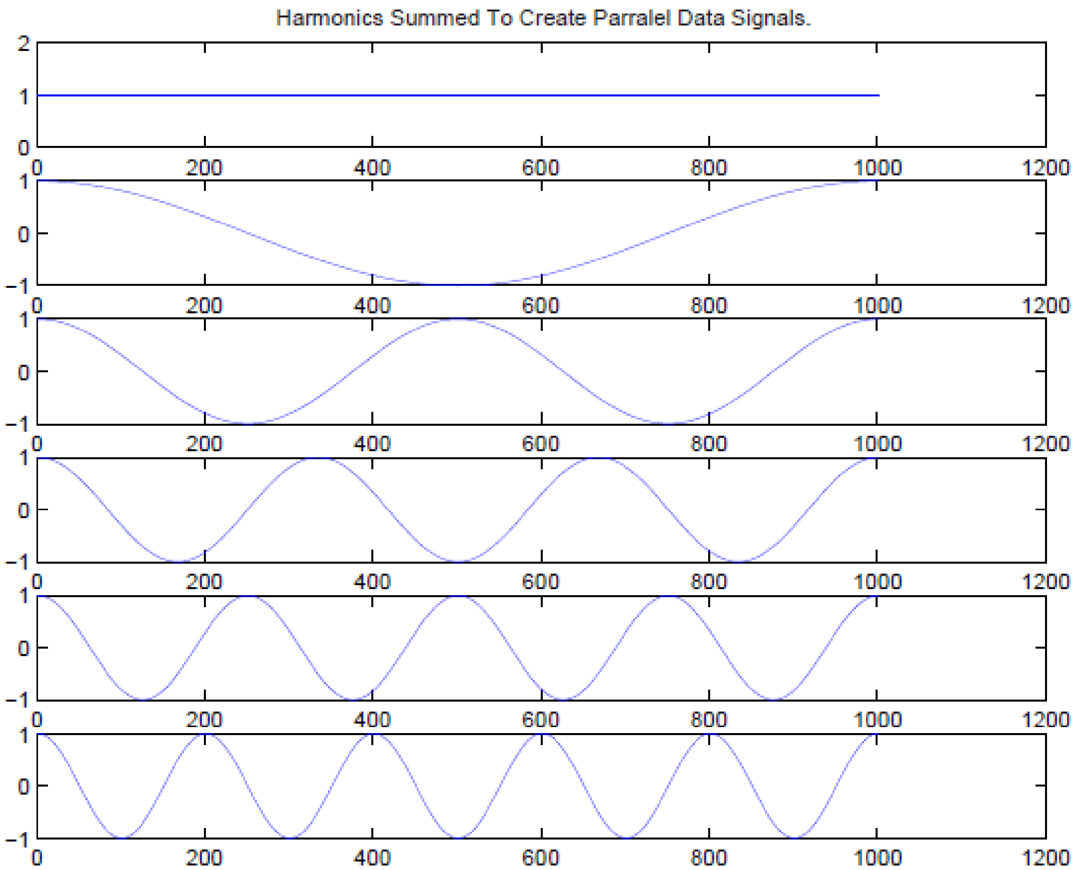

Fig 2.6: Fourier series frequency harmonic

2.3.2 OFDM Spectrum

Our OFDM symbol is a sum of sinusoids of a fundamental frequency and its harmonics in the time domain. The rectangular windowing of the transmitted OFDM symbol results in a sinc function at each sub-carrier frequency in the frequency response. Thus, the frequency spectrum of an OFDM symbol is as shown below: The Figure (2.2) is not the actual spectrum of OFDM. The spectrum of each sub-carrier has been superimposed to illustrate the orthogonally of the sub-carriers. Although overlapping, the sub-carriers do not interfere with each other since each sub-carrier peak corresponds to a zero crossing for all other sub-carriers.

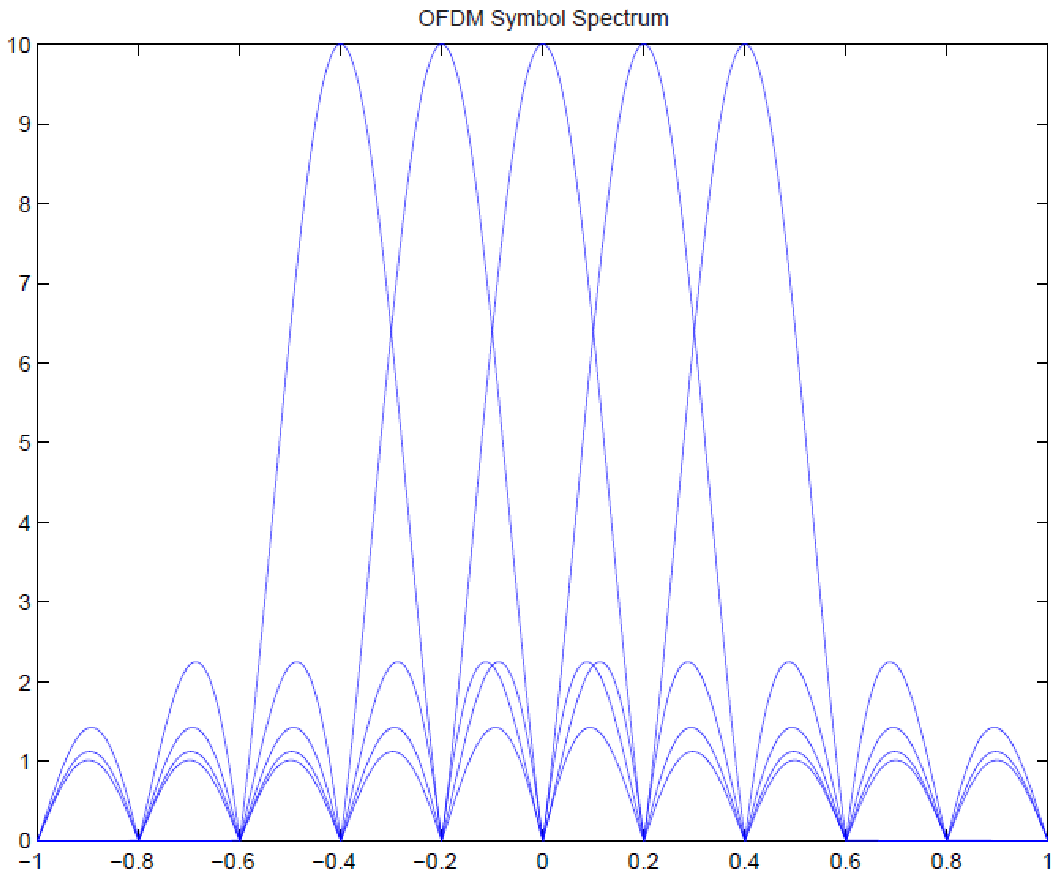

Fig 2.7 : OFDM time domain spectrum

2.4 Inter Symbol Interference

Inter symbol interference (ISI) is when energy from one symbol spills over to the next symbol. This is usually caused by time dispersion in multi-path when reflections of the previous symbol interfere with the current symbol. In OFDM, because each sub-carrier is transmitting at a lower data rate (longer symbol duration), this will negate the effects of time dispersion, which results in ISI.

2.5 Inter Carrier Interference

Inter carrier interference (ICI) is occurs when the sub carriers lose their orthogonally, causing the sub carriers to interfere with each other. This can arise due to Doppler shifts and frequency and phase offsets.

2.6 Cyclic Prefix (Guard Interval)

The Cyclic Prefix is a periodic extension of the last part of an OFDM symbol that is added to the front of the symbol in the transmitter, and is removed at the receiver before demodulation.

The Cyclic Prefix has two important benefits:

- The Cyclic Prefix acts as a guard space between successive OFDM symbols and therefore prevents Inter-symbol Interference (ISI), as long as the length of the CP is longer than the impulse response of the channel.

- The Cyclic Prefix ensures orthogonality between the sub-carriers by keeping the OFDM symbol periodic over the extended symbol duration, and therefore avoiding Inter-carrier Interference (ICI).

Mathematically, the Cyclic Prefix / Guard Interval converts the linear convolution with the channel impulse response into a cyclic convolution. This results in a diagonalised channel, which is free of ISI and ICI interference.

The disadvantage of the Cyclic Prefix is that there is a reduction in the Signal to Noise Ratio due to a lower efficiency by duplicating the symbol. This SNR loss is given by:

$$\boldsymbol{SNRloss} = -10\, log_{10}\left(1 - \frac{T_{cp}}{T}\right) \tag{2.8}$$

where *Tcp* is the length of the Cyclic Prefix and *T* = *Tcp* +*Ts* is the length of the transmitted symbol and *Ts* is the signal transmission.

To minimize the loss of SNR, the CP should not be made longer than necessary to avoid ISI and ICI.

2.7 Inverse Discrete Fourier Transform

OFDM modulation is applied in the frequency domain. The complex QAM data symbols are modulated onto orthogonal sub-carriers. But in order to transmit over a channel, we need a signal in the time domain. To do this, we apply the Inverse Discrete Fourier Transform (IDFT) in the transmitter to convert the signal from frequency domain to an OFDM symbol in the time domain. Because the IDFT is a linear transformation, the DFT can be applied at the receiver to convert the data back into the frequency domain. This section will provide an explanation of the IDFT / DFT and why it is a key component of an OFDM system.

To implement the multi-carrier system using a bank of parallel oscillators and modulators would not be very efficient in analog hardware. However, in the digital domain, multicarrier-modulation can be done very efficiently with the current DSP hardware and software.

To do this modulation we exploit the properties of the Discrete Fourier Transform (DFT) and the Inverse DFT (IDFT). From Fourier, we know that when the DFT of a sampled time signal is taken, the frequency domain results are the components of the signal with respect to the Fourier Basis which are multiples of a fundamental frequency as a function of the sampling period and the number of samples. The IDFT performs the opposite the DFT. It takes the signal defined by the frequency components and converts them to a time signal.

In OFDM, we compose the signal in the frequency domain. Since the Fourier Basis is orthogonal, we just take the IDFT of the N inputs which are our frequency components to convert to a time domain equivalent for transmission over the channel.

In practice, the Fast Fourier Transform (FFT) and IFFT are used instead of the DFT and IDFT because of their lower hardware complexity. All further references will be to FFT and IFFT.

3.1 Introduction

In a wireless environment, the channel is much more unpredictable than a wire channel because of a combination of factors such as multi-path, frequency offset, timing offset, and noise. This results in random distortions in amplitude and phase of the received signal as it passes through the channel. These distortions change with time since the wireless channel response is time varying. Channel estimation attempts to track the channel response by periodically sending known pilot symbols, which enables it to characterize the channel at that time. This pilot information is used as a reference for channel estimation. The channel estimate can then be used by an equalizer to correct the received constellation data so that they can be correctly demodulated to binary data.

Modulation can be classified as differential or coherent. For differential, information is encoded in the difference between two consecutive symbols so no channel estimate is required. However, this limits the number of bits per symbol and results in a 3-dB loss in SNR. Coherent modulation allows the use of arbitrary signaling constellations, allowing for a much higher bit rate than differential modulation and better efficiency. This chapter will give a description of our system environment, the channel model and present several channel estimation techniques that are required for the case of coherent Modulation.

3.2 System Environment

3.2.1 Wireless

The system environment we will be considering in this thesis will be wireless indoor and urban areas, where the path between transmitter and receiver is blocked by various objects and obstacles. For example, an indoor environment has walls and furniture, while the outdoor environment contains buildings and trees. This can be characterized by the impulse response in a wireless environment.

3.2.2 Multipath Fading

Most indoor and urban areas do not have direct line of sight propagation between the transmitter and receiver. Multi-path occurs as a result of reflections and diffractions by objects of the transmitted signal in a wireless environment. These objects can be such things as buildings and trees. The reflected signals arrive with random phase offsets as each reflection follows a different path to the receiver. The signal power of the waves also decreases as the distance increases. The result is random signal fading as these reflections destructively and constructively superimpose on each other. The degree of fading will depend on the delay spread (or phase offset) and their relative signal power.

3.2.3 Fading Effects due to Multi-path Fading

Time dispersion due to multi-path leads to either flat fading or frequency selective fading:

- Flat fading occurs when the delay is less than the symbol period and affects all frequencies equally. This type of fading changes the gain of the signal but not the spectrum. This is known as amplitude varying channels or narrowband channels, since the bandwidth of the applied signal is narrow compared to the channel bandwidth.

- Frequency selective fading occurs when the delay is larger than the symbol period. In the frequency domain, certain frequencies will have greater gain than others frequencies.

3.2.4 White noise

In wireless environments, random changes in the physical environment resulting in thermal noise and unwanted interference from many other sources can cause the signal to be corrupted. Since it is not possible to take into account all of these sources, we assume that they produce a single random signal with uniform distributions across all frequencies. This is known as white noise.

3.3 Channel Model

This section will show how the channel will become diagonalised from a cyclic convolution due to the insertion of the Cyclic Prefix. If the Cyclic Prefix is longer than the impulse response of the channel, we can show that the OFDM channel can be viewed as a set of parallel Gaussian channel (a complex gain followed by Additive White Gaussian noise) that is free of ISI and ICI.

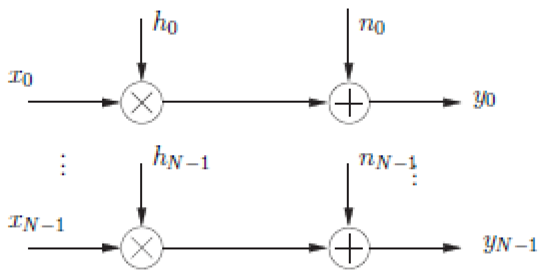

Fig 3.1: Parallel Gaussian channels

First, let our QAM signalling symbols be expressed as

$$x = \begin{bmatrix} x_o \\ \vdots \\ x_{N-1} \end{bmatrix} \tag{3.1}$$

After we apply the IFFT to s, our OFDM symbol becomes

$$x = F^H x \begin{bmatrix} X \\ \vdots \\ X_{N-1} \end{bmatrix} \tag{3.2}$$

where the matrix F is the DFT matrix. For the channel impulse response

$$\mathbf{h} = \begin{bmatrix} h_o \\ \vdots \\ h_{m-1} \end{bmatrix} \tag{3.3}$$

where m is less than the length of the cyclic prefix.

To simplify our derivation, we will choose N = 4 sub-carriers and m = 2 tap impulse response, but this proof will generally apply as long as m satisfies the above condition. So after passing through the multi-path channel, the received OFDM symbol can be expressed as a convolution, h* x. In matrix form, this becomes

$$\mathbf{y} = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 \\ 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{3.4}$$

If we insert the cyclic prefix before sending across the channel, this convolution becomes

$$\mathbf{y} = \begin{bmatrix} Y_{CP} \\ Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_3 \\ X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{3.5}$$

And after removing the cyclic prefix at the receiver, we can express this as

$$\mathbf{y} = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & h_1 \\ h_1 & h_0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 \\ 0 & 0 & h_1 & h_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} \iff \mathrm{Y} = H\mathrm{X}' = H_c\mathrm{X} \tag{3.6}$$

This is equivalent to a circular convolution. The channel matrix H_c is now a circulant matrix and X’ is the cyclically extended symbol.

We can now use the property of circular convolution on finite length sequences

$$\mathbf{y} = \mathrm{h} \otimes \mathrm{x} \;\Leftrightarrow\; Y[k]X[k], \qquad k = (0,\ldots,N\text{-}1) \tag{3.7}$$

This property means every circulant matrix H_c is diagonalised by the DFT matrix F.

$$\boldsymbol{x} = FY = FH_cX = FH_cF^Hx = \Lambda x, \quad \Lambda = \left[\begin{bmatrix} H[0] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H[N-1] \end{bmatrix}\right] \tag{3.8}$$

So our multi-path fading channel can be written as:

$$\boldsymbol{y} = H_c + n \tag{3.9}$$

where y is the received vector of signaling points, x is the transmitted signaling points, H_c is the diagonalised channel attenuation vector, and n is a vector of complex, zero mean, Gaussian noise with variance σ_n^2

The attenuation on each tone is given by

$$\boldsymbol{H}[\boldsymbol{k}] = G\left(\frac{k}{NT_s}\right), \qquad k = 0, \ldots, N-1 \tag{3.10}$$

where G(.) is the frequency response of the channel during the current OFDM symbol and T_s is the sampling period of the system.

The impulse response of the channel can be expressed as

$$\boldsymbol{g}(\boldsymbol{\tau}) = \sum_{k=0}^{M-1} \alpha_k \, \delta(\tau - \tau_k T_s) \tag{3.11}$$

where α_k are independent zero mean, complex Gaussian random variables, and τ_k is the delay of the *k*th impulse. The next few sections will talk about our considerations regarding some issues on the wireless channel and the justifications for our channel model.

3.3.1 Rayleigh Distribution

The Rayleigh distribution is a statistical distribution that is used to model amplitude variations of the impulse response in a wireless multi-path channel. A Rayleigh distribution assumes:

- There is no direct line of sight (LOS) component in the received signal.
- There are many indirect components from reflected and scattered signals, each taking different paths to the receiver.

Because these assumptions are valid for the wireless environment described above, we will use a Rayleigh distributed model for our channel response.

It can be shown amplitude of two quadrature Gaussian noise sources follows a Rayleigh distribution. If we let *s*(*t*) be the signal transmitted through a Rayleigh channel, then r(t) can be expressed as :

$$\boldsymbol{r(t)} = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \qquad (3.12)$$

where x(t) and y(t) are normalized random processes, with zero mean and variance σ. Then the combination probability density function is

$$\boldsymbol{p(x,y)} = \frac{\mathbf{1}}{\boldsymbol{2\pi\sigma^2}}\exp\left(\frac{x^2+y^2}{2\sigma^2}\right) \qquad (3.13)$$

We can express *r*(*t*) in polar form in terms of amplitude and phase of the received Signal

$$\boldsymbol{r(t)} = R(t)\cos(2\pi f_c t + \theta(t)) \qquad (3.14)$$

where $R(t)$ and $\theta(t)$ are given by

$$\boldsymbol{R(t)} = R = \sqrt{(x^2 + y^2)}$$
$$\boldsymbol{\theta(t)} = \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{3.15}$$

By using the polar transformation, the probability density function now becomes:

$$\boldsymbol{p(R,\theta)} = \frac{R}{2\pi\sigma^2}\exp\left(\frac{-R^2}{2\sigma^2}\right) \tag{3.16}$$

Integrating $p(R, \theta)$ over θ from 0 to 2π, we obtain the probability density function $p(R)$:

$$\boldsymbol{p(R)} = \frac{R}{\sigma^2}\exp\left(\frac{-R^2}{2\sigma^2}\right) \tag{3.17}$$

which follows a Rayleigh distribution.

3.3.2 Power Delay Profile

The Power Delay Profile (PDP) describes the envelope of the impulse response as a function of the delay. The PDP of an urban and indoor environment is generally described by an exponential, since each delayed impulse usually has less power than the previous ones. Thus, our PDP is described by the following equation

$$\boldsymbol{\varphi(\tau)} \sim \boldsymbol{exp}\ \left(\frac{-\tau}{\tau_{rms}}\right) \tag{3.18}$$

3.3.3 AWGN

When the signal passes through the channel, it is corrupted by white noise. This is modelled by the addition of white Gaussian noise (AWGN). AWGN is a random process with power spectral density as follows:

$$\emptyset(\boldsymbol{f}) = \frac{1}{2} N_O [{}^{W}/_{Hz}] \tag{3.19}$$

where N_O is a constant and often called the noise power density.

3.3.4 Channel Synchronization

There are two types of channel synchronisation models - Sample spaced and Non sample spaced synchronization[5].

- Sample spaced synchronisation assumes all delayed impulses of its channel impulse response are at integer multiples of the sampling period T.
- In non sample spaced channels, the delayed impulses are not at periods of the sampling period T, thus the most of the impulse power is spread locally among the closest sampling intervals at the receiver, and leads to a larger impulse response at the receiver as shown in figure 3.2.

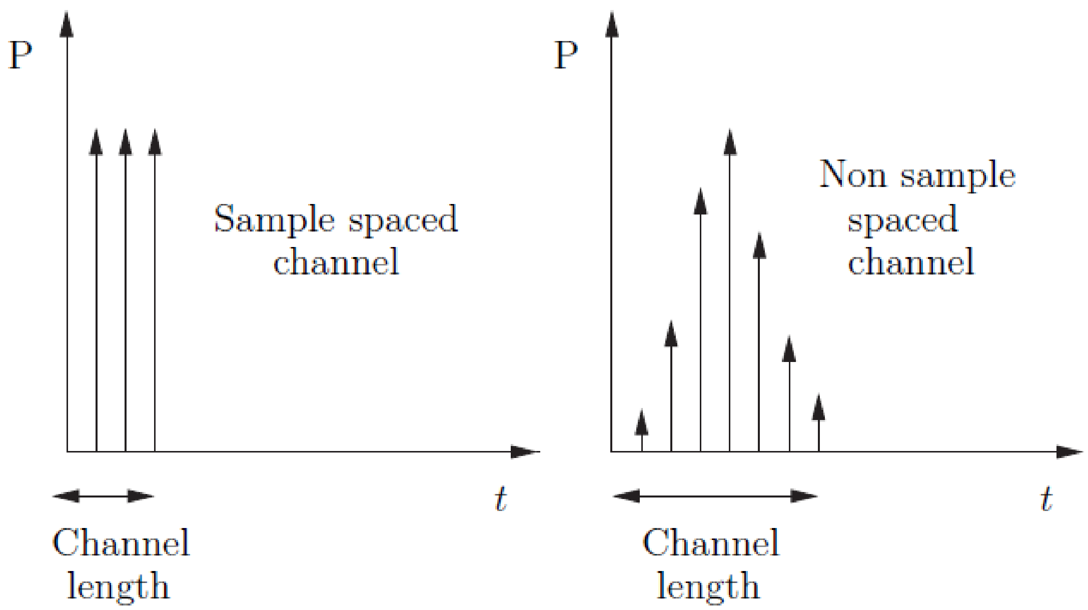

Fig 3.2: Resampling a non-sample-spaced channel extends the channel length.

For simplicity, we will only consider synchronised sample spaced channels for channel estimation in this thesis.

3.3.5 Assumptions on channel

To simplify our simulated channel, the following are assumed to hold:

- The impulse response is shorter than the Cyclic Prefix. Therefore, there is no ISI and ICI and the channel is therefore diagonal.

- The channel is a synchronized, sample spaced channel.

- Channel noise is additive, white and complex Gaussian.

- The fading on the channel is slow enough to be considered constant during one OFDM frame.

3.4 Pilot Based Channel Estimation

The following estimators use on pilot data that is known to both transmitter and receiver as a reference in order to track the fading channel. The estimators use block based pilot symbols, meaning that pilot symbols are sent across all sub-carriers periodically during channel estimation. This estimate is then valid for one OFDM frame before a new channel estimate will be required.

Since the channel is assumed to be slow fading, our system will assume a frame format, transmitting one channel estimation pilot symbol, followed by five data symbols, as indicated in the time frequency lattice shown in figure3.3. Thus each channel estimate will be used for the following five data symbols

3.5 Least Squares Estimator

The simplest channel estimator is to divide the received signal by the input signals, which should be known pilot symbols. This is known as the Least Squares (LS) Estimator and can simply be expressed as:

$$\boldsymbol{H_{LS}} = \frac{y}{x} \tag{3.20}$$

This is the most naive channel estimator as it works best when no noise is present in the channel. When there is no noise the channel can be estimated perfectly. This estimator is equivalent to a zero-forcing estimator.

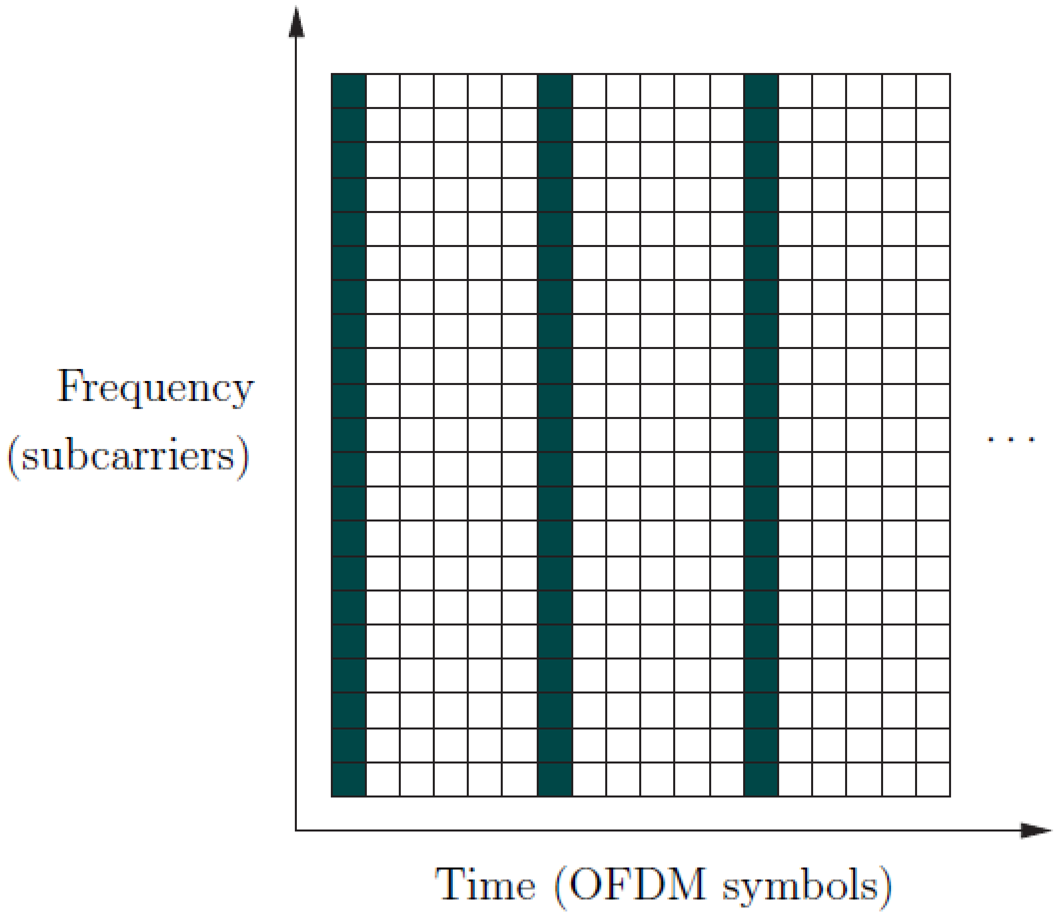

Fig 3.3: An example of block based pilot information

The main advantage is its simplicity and low complexity. It only requires a single division per sub-carrier. The main disadvantage is that it has high mean-square error. This is due to its use of an oversimplified channel and does not make use of the frequency and time correlation of the slow fading channel.

An improvement to the LS estimator would involve making use of the channel statistics. We could modify the LS estimator by tracking the average of the most recently estimated channel vectors.

3.6 Linear Minimum Mean Square Error Estimator

The Linear Minimum Mean Squares Error (LMMSE) Estimator minimizes the mean square error (MSE) between the actual and estimated channel by using the frequency correlation of the slow fading channel. This is achieved through a optimizing linear transformation applied to the LS estimator described in the previous section. From adaptive filter theory, the optimum solution in terms of the MSE is given by the Wiener-Hopf equation[4]:

$$\boldsymbol{h} = R_{hh_{ls}} R^{-1}_{h_{ls}h_{ls}} h_{ls} \tag{3.21}$$

where X is a matrix containing the transmitted signaling points on its diagonal, σ_n^2 is the additive noise variance. The matrix $R_{hh_{ls}}$ is the cross correlation between channel attenuation vector h and the LS estimate $h_{ls}$and $R_{h_{ls}h_{ls}}$is the auto correlation matrix of the LS estimate h_{ls} , given by:

$$\begin{aligned} \boldsymbol{R_{hh_{ls}}} &= E\{hh_{ls}^H\} \\ \boldsymbol{R_{h_{ls}}} &= E\{h_{ls}h_{ls}^H\} \end{aligned} \tag{3.22}$$

Since white noise is uncorrelated with the channel attenuation, the cross correlation between the channel h and noisy channel h_{ls} is the same as the autocorrelation of the channel h. Thus we can replace $R_{hh_{ls}}$with R_{hh}. Also the autocorrelation of $R_{h_{ls}h_{ls}}$ is equivalent to R_{hh} plus the noise power σ^2 and signal power. So the estimator can be expressed as:

$$\boldsymbol{h_{lmmse}} = R_{hh}(R_{hh} + \sigma_n^2(XX^H)^{-1})^{-1}h_{ls} \tag{3.23}$$

The above equation seems to pose a contradiction since we need the autocorrelation of our desired channel vector in order to estimate an optimum channel vector. But we do not know what our desired channel vector since we do not know the channel. To overcome this problem, we replace the autocorrelation with its expected value. This can be done in two ways:

- By theoretically calculating the expected value based on assumed or known channel statistics. This simplifies the complexity as the inverse only needs to be calculated once, which will be explained further on. The values will need to be recalculated each time the statistics of the channel changes.

- Through realization, the autocorrelation matrix can be averaged each time channel estimation occurs. This approach will converge slower than the above method since the expected value is calculated adaptively but is more flexible since it does not assume any fixed channel statistics.

For our OFDM channel estimation, we will use the first approach because of its lower complexity and easier implementation.

The main disadvantage of this estimator is that it has a very high complexity. The evaluation of inverse R and XX^H involves the inversion of a matrix of dimension N x N which makes this estimator computationally complex.

By using statistics that we know about the additive noise and the transmitted data, we can simplify the estimator. Since the binary data is completely random, we can assume equal probability on all constellation points, and we can replace $XX^H$by its expected value

$$\boldsymbol{E}\{\boldsymbol{XX^H}\} = E\left|\frac{1}{x_k}\right|^2 \boldsymbol{I} \tag{3.24}$$

Thus our simplified estimator can be expressed as:

$$\boldsymbol{h_{lmmse}} = R_{hh}\left(R_{hh} + \frac{\beta}{SNR} I\right)^{-1} h_{ls} \tag{3.25}$$

where I is the Identity matrix, SNR is the average signal-to-noise ratio is defined as $E|x_k|^2/\sigma_n^2$. β is a signal constellation dependent constant. For the case of 16-QAM

$$\boldsymbol{\beta} = E|x_k|^2 E\left|{}^{1}/_{x_k}\right|^2 = 17/9 \tag{3.26}$$

Thus the inverse need only be calculated once every time channel estimation occurs, or just once if set to theoretical values.

4.1 Introduction

In this chapter, the results of lest squares channel estimation will be shown. Different scenarios will be considered as will be shown later, where all scenarios will be implemented using Matlab for Quadrature Amplitude Modulation family type.

4.2 Results and Discussion

In our project, we have considered four scenarios, the first scenario is an OFDM symbol length N = 256 subchannels, a constellation order M = 4, pilot frequency of 4, pilot energy = 4 times the largest point in the constellation diagram, cyclic prefix length of N/8, and last but not least the number of paths was 3.
Figure (4.1) shows the comb-type of our configuration in the Matlab simulation for the first scenario. Where the number of the simulated OFDM symbols was 10,000. However, the vertical axis represents the frequency-domain and the horizontal axis stands for the time-domain. As we have explained in chapter three, we used the comb-type pilot assisted channel estimation for fast channel varying, in other words, for fast mobility as in the fourth generation (4G).

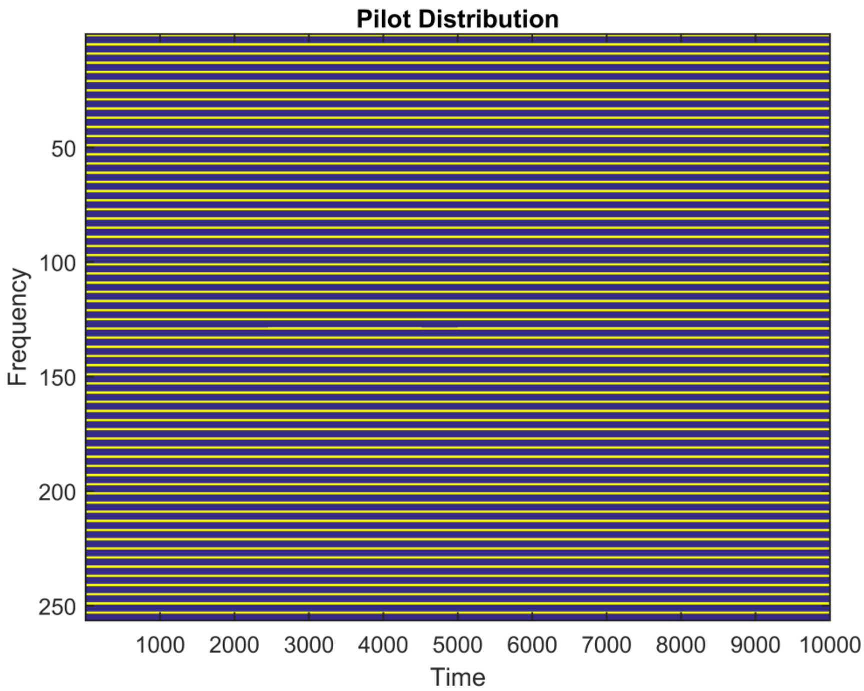

Fig 4.1: Comb-type pilot distribution configuration for 10,000 OFDM symbols.

Least squares channel estimation needs to do a mathematical operation called interpolation, see chapter 3, where we need to extend the size of the estimated channel length to the actual channel length which is N. for more information, the reader can refer to chapter 3. Figure (4.2) shows the channel parameters estimation compared with the actual channel. As aforementioned above, this scenario adopted three randomly generated paths for the simulation. However, it can be seen at the most bottom of the figure that the actual subchannel do not matches the estimated, where we have plotted only 45 points up of 256 points for clarity. This mismatch will be reflected on the BER performance behavior, as shown in Figure (4.3).

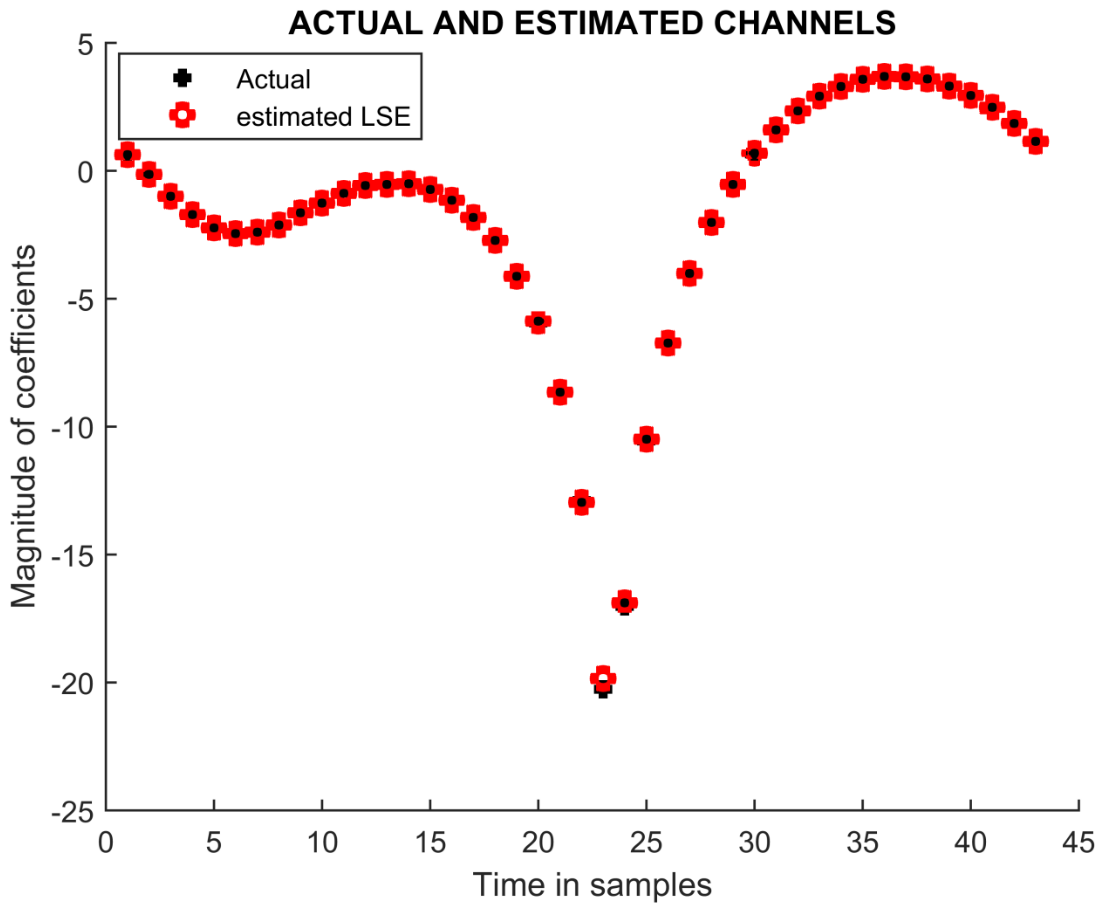

Fig 4.2: The three paths estimated and actual channels.

Figure (4.3) depicts the BER performance of the first scenario. It is shown that the required SNR for the estimated channel needs to be higher with respect to the actual, which is equivalent to as there is only AWGN channel, i.e., only one path channel.

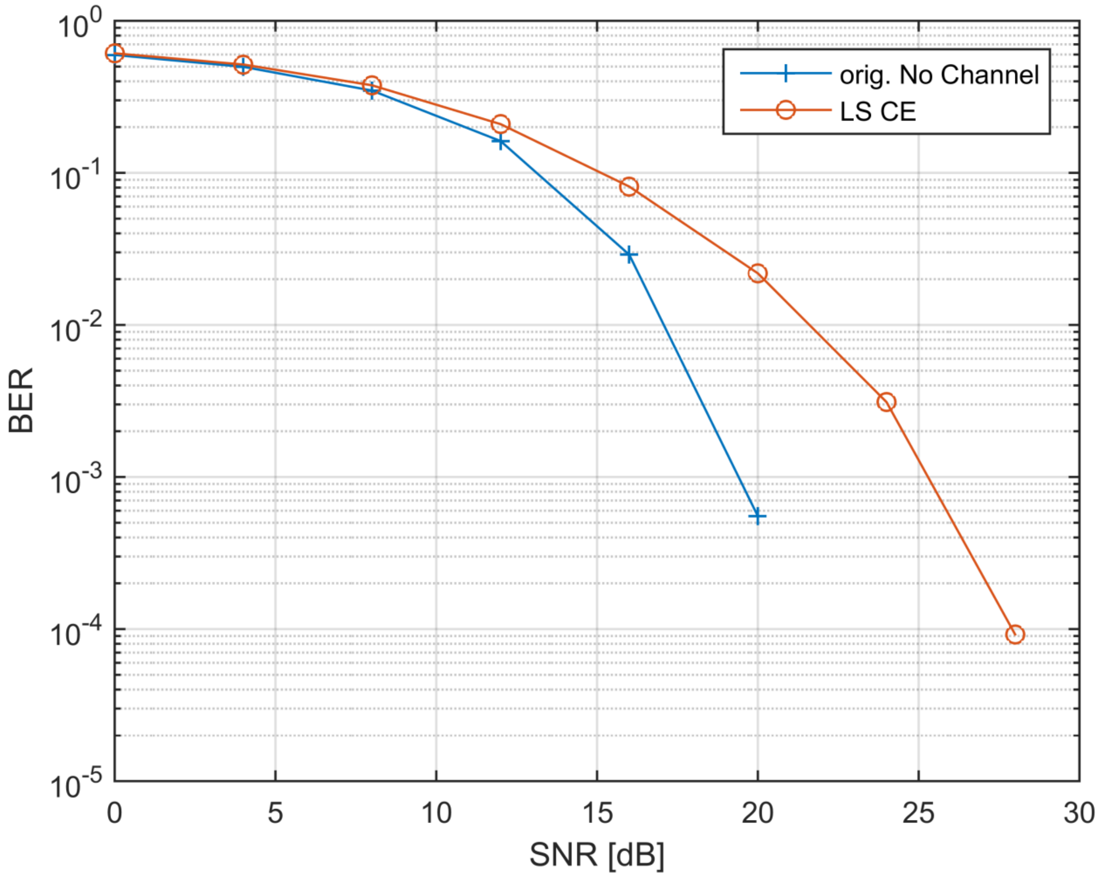

Fig 4.3: BER performance of the first scenario.

The second scenario is an OFDM symbol length N = 256 subchannels, a constellation order M = 16, pilot frequency of 4, pilot energy = 4 times the largest point in the constellation diagram, cyclic prefix length of N/8, and the number of paths still 3. Thus, only the constellation order has been changed, to see the effect of higher modulation orders on the performance of the OFDM system with multipahts.

In Figure (4.4), the pilot distribution can be seen, where it is similar to the one in Figure (4.1), but here it is higher energy and constellation order. It is shown that the time-axis shows the number 10,000, which is the total number of the randomly generated OFDM symbols for the simulation. While the Frequency axis, shows the total number of the OFDM size, which is 256 subcarrier in our simulations.

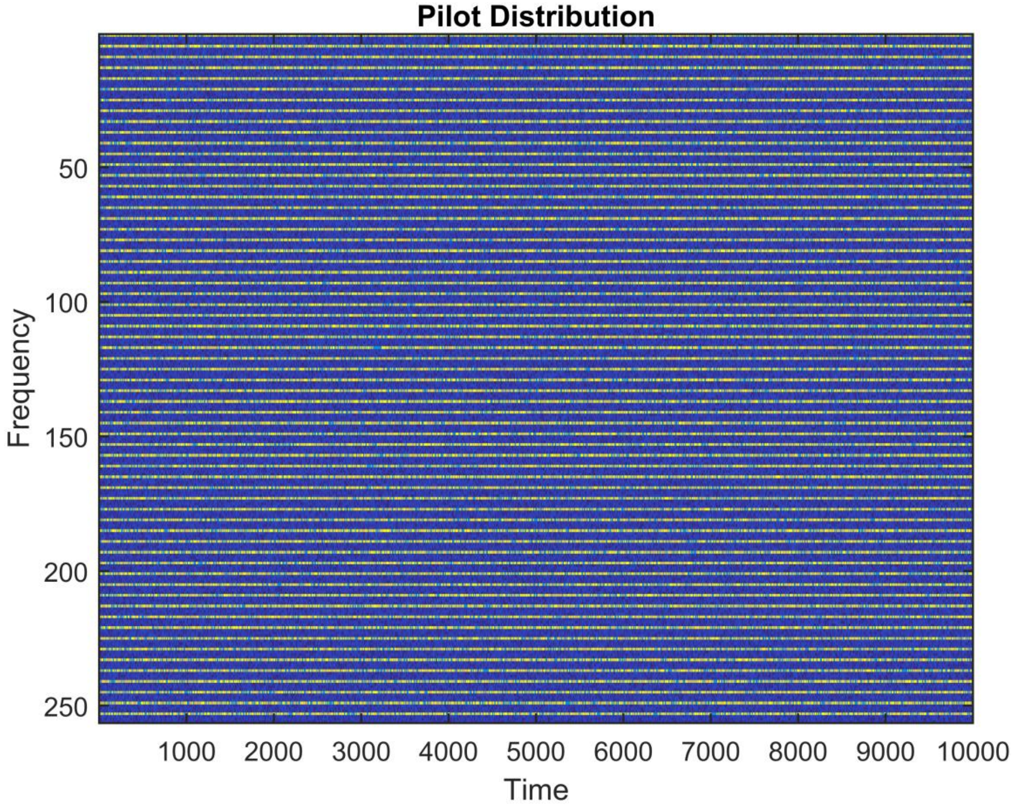

Fig 4.4: Pilot subcarriers configuration for the second scenario.

The estimated channels were shown in Figure (4.5) for this second scenario, where it can be seen that some of the black points do not match the red points, that is why the BER performance was degraded significantly, as shown in Figure (4.6). however, we have drawn only 35 points for clarity purposes.

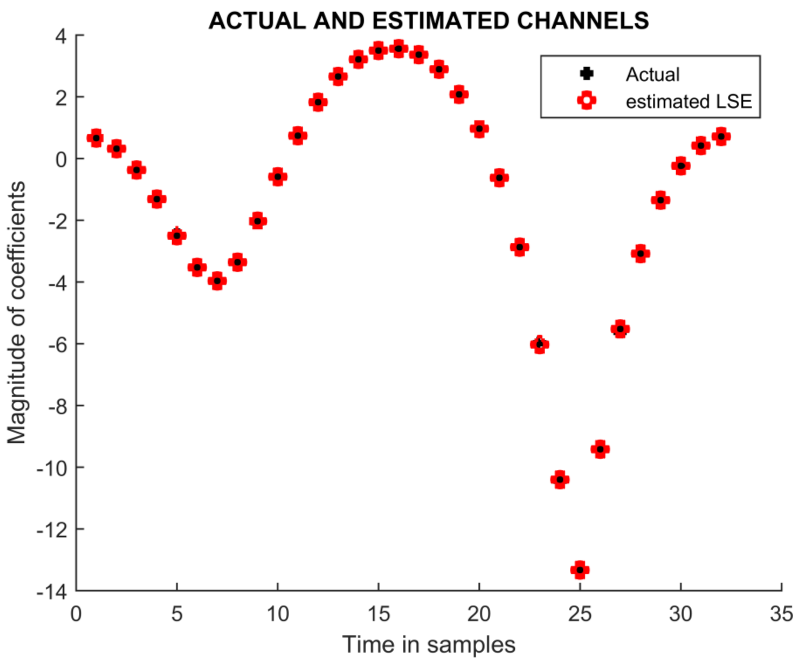

Fig 4.5: Actual and estimated channel parameters for the second scenario

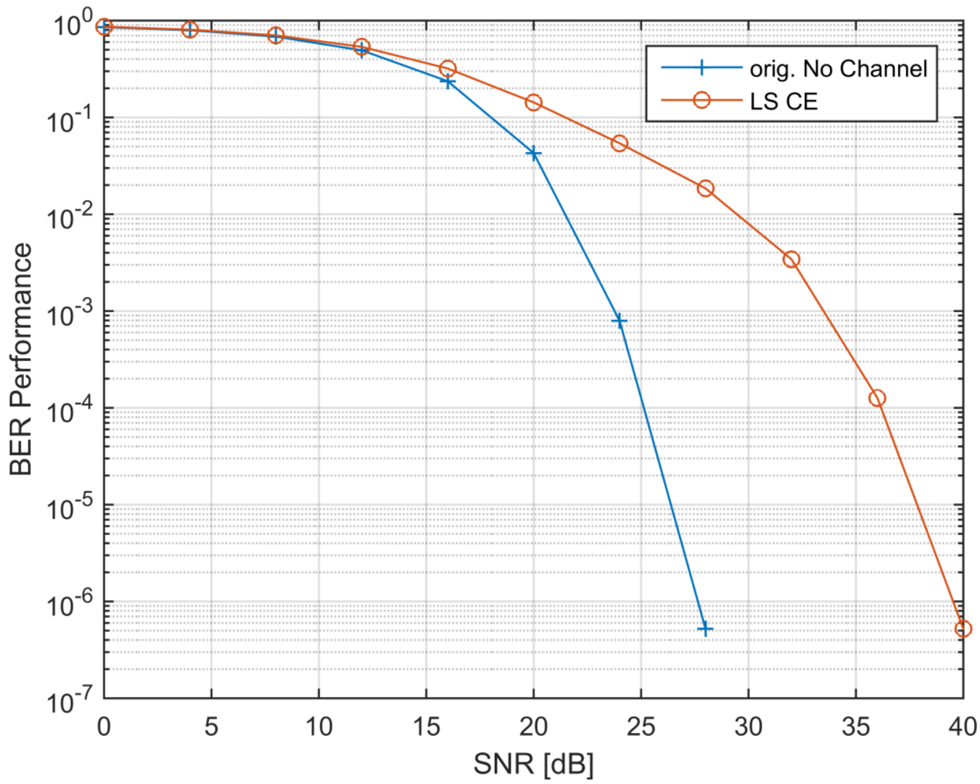

Fig 4.6: BER performance of the second scenario

On the other hand, the third and fourth scenarios has a slightly different parameters, where the channel length, or the number of multipaths has been increased by one path, thus, these scenarios will achieve four multipaths. That is - the pilot distribution will not be changed for the third scenario with respect to the first scenario, thus, there is no need to re-plot it. The same pilot distribution for the fourth scenario is similar to the second scenario, hence, it is not necessary to re-plot is also. Figure (4.7) explains the channel estimation parameters for the third scenario, while Figure (4.8) shows the channel estimation parameters for the fourth scenario. In both figures, there are only 35 subchannels where plotted for simplicity.

Figures 4.9 and 4.10 show the BER performances, respectively, of the third and fourth scenarios. It can be concluded that the number of multipaths has a recognized effect on the BER performance, where the SNR for both figures was increased to reach the required BER performance for acceptable quality.

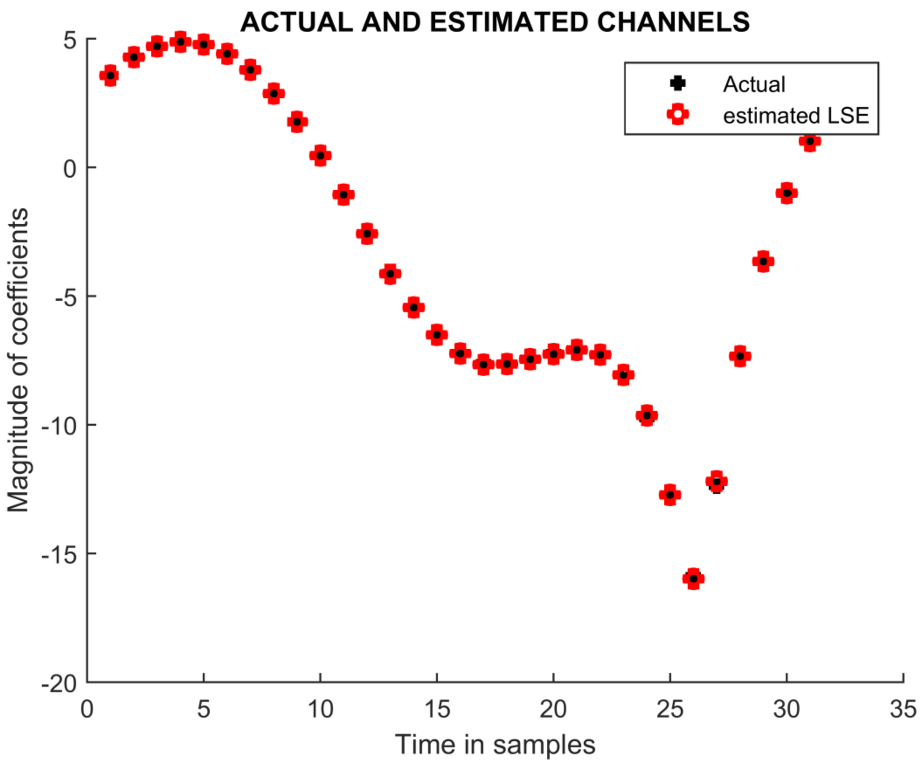

Fig 4.7: Channel estimation parameters for the third scenario.

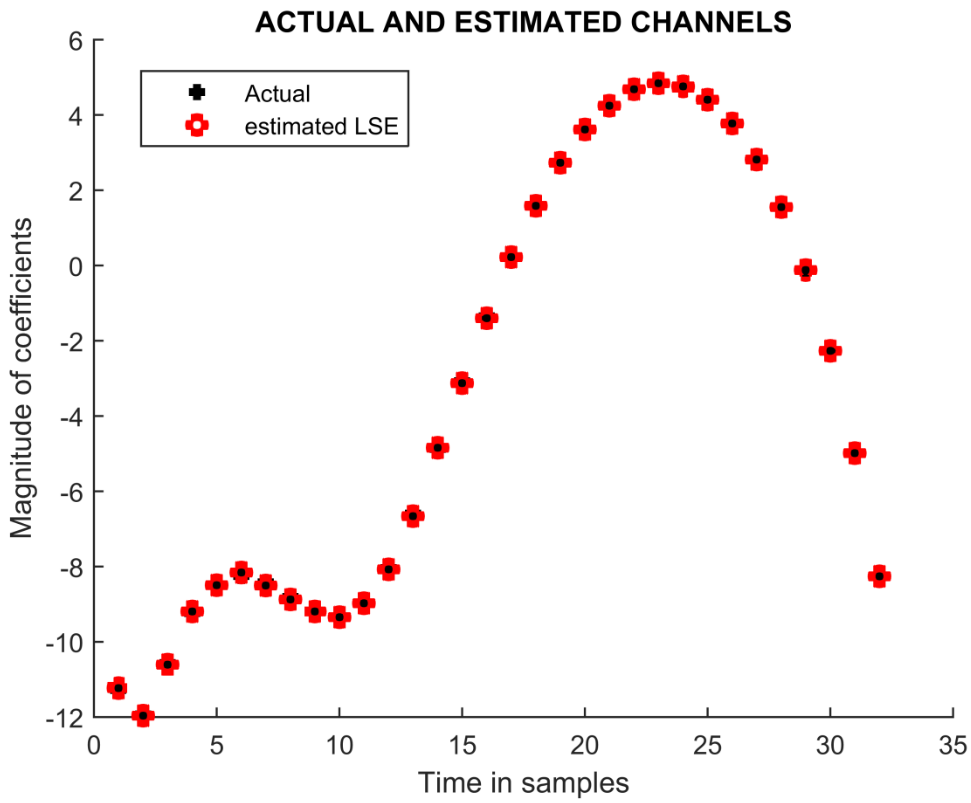

Fig 4.8: Channel estimation parameters for the fourth scenario.

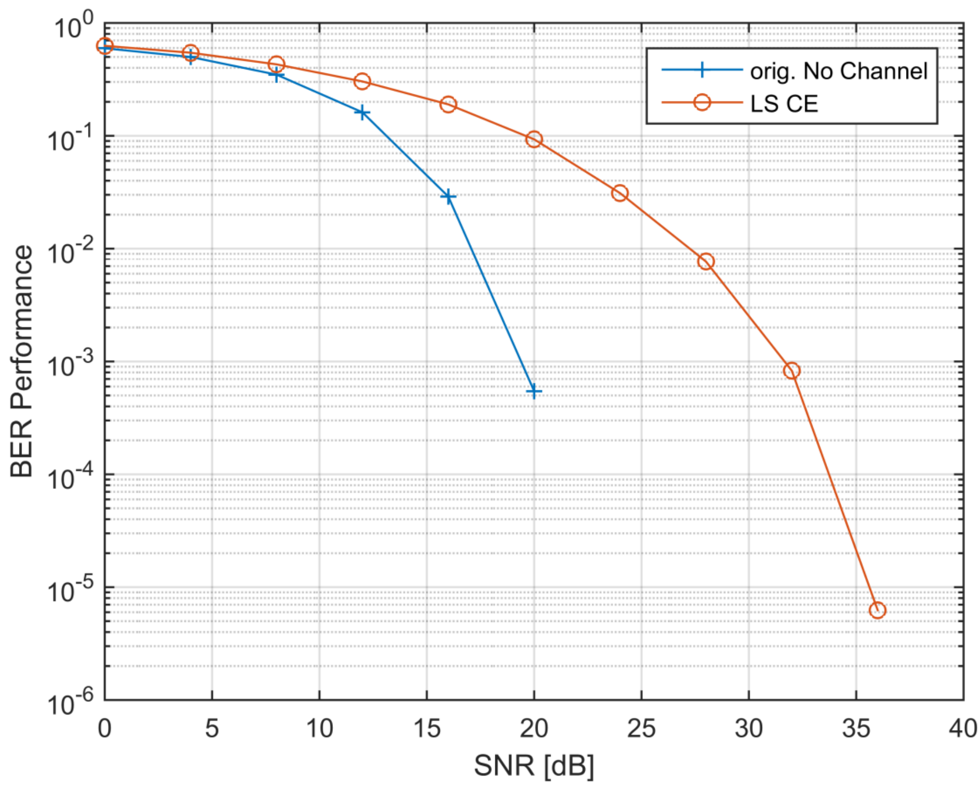

Fig 4.9: BER performance of the third scenario.

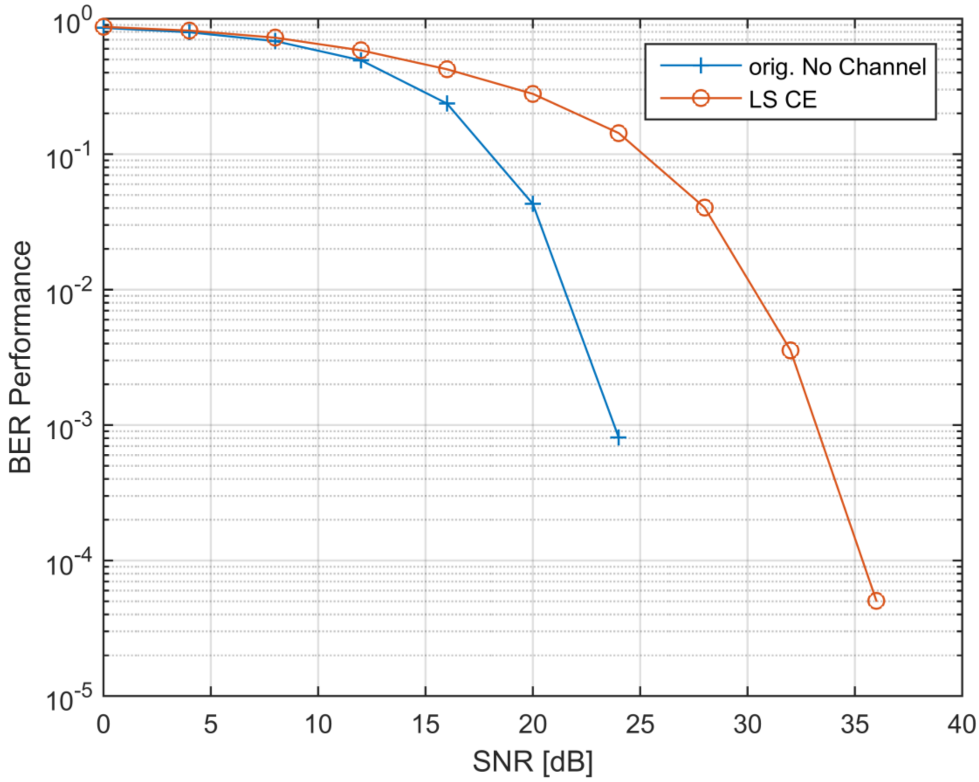

Fig 4.10: BER performance of the fourth scenario.

5.1 Conclusion

From the 1960s to today, we can see that OFDM is another tool for which the engineer can use to overcome channel effects in a wireless environment. The are many advantages in OFDM, but there are still many complex problems to solve.

We hope this thesis has provided a basic simulation tool for future students to use as a starting point in their theses. It is our motivation that by using the parameters of a working system, a much clearer and insightful explanation of the fundamentals of OFDM have been presented.

Channel Estimation is an important part of an OFDM receiver, especially in wireless environments where the channel is unpredictable and changing continuously. A good channel estimation will allow the equalizer to correct the fading effects of the channel. Of the three channel estimators studied in this thesis, the low rank approximate estimator seems to be the most practical in terms of good performance and low complexity. The LS does not perform well in low SNR environments while the LMMSE estimator complexity seems too high for a small performance improvement.

In OFDM equalization, it seems that the adaptive algorithms used in the OFDM did not add many special benefits. It's adaptive capability allowed the equalizer coefficients to change with time but it is done on the basis on resynthesized symbols for which noise and rounding errors may accumulate. These algorithm did not exploit OFDM characteristics which the zero forcing and LMMSE did. It may be wise to incorporate LMMSE design into a DFE. Because the zero forcing and LMMSE equalizers exploit the OFDM design by equalizing in the frequency domain, it is very simple, especially compared the complexities of the adaptive algorithms.

5.2 Future Work

This work can be extended to verifying the part next:

1. The cyclic prefix for OFDM can require up to 15-20% bandwidth overhead. It is desirable to develop techniques that eliminate or reduce the cyclic prefix.
2. Channel estimation techniques for space-time and space-frequency coded OFDM systems.

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الخلاصة

الهدف من هذا المشروع هو مضاعفة التردد المتعامد بالتقسيم الذي هو مفتاح تمكين التكنولوجيا لمعظم أنظمة الاتصالات اللاسلكية الحالية ذات معدل نقل البيانات العالي.الميزة الرئيسية للـ (OFDM) مقارنة بالطرق التقليدية هو أنه يحول القناة ذات النطاق العريض الى قنوات فرعية ضيقة متوازية تسمح بتقدير القناة و تكافؤ في مجال التردد بسيط نسبيا عند المستلم. تعتبر هذه التقنية فعالة للتغلب على تأثيرات القناة مثل الانتشار المتعدد والتداخل بين الرموز (ISI) من خلال الاستفادة من البادئة الدورية المناسبة. واحدة من العوائق الرئيسية في (OFDM) هو أنه أكثر حساسية لأخطاء المزامنة من نظرائه ذات الناقل الواحدة.

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مضاعفة التردد المتعامد بالتقسيم

مشروع

مقدم الى قسم هندسة الأتصالات

في جامعة ديالى – كلية الهندسة كجزء من متطلبات نيل درجة البكلوريوس

في هندسة الأتصالات

من قبل

هاجر خليل ابراهيم

نور اقبال عبدالكريم

بأشراف

د. منتظر عباس طاهر

م.م احمد محمد احمد

May/2016 **رجب/1438**