Department of Communications Engineering

Communication Systems

Third Year Class

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Lecture 3

Modulation Index, Demodulation, Power, and Efficiency

Modulation Index

modulation index is also called the modulation depth.
or degree of modulation
or modulation factor

$$m_a = \frac{|m(t)|_{max}}{\text{Maximum carrier Frequency}}$$

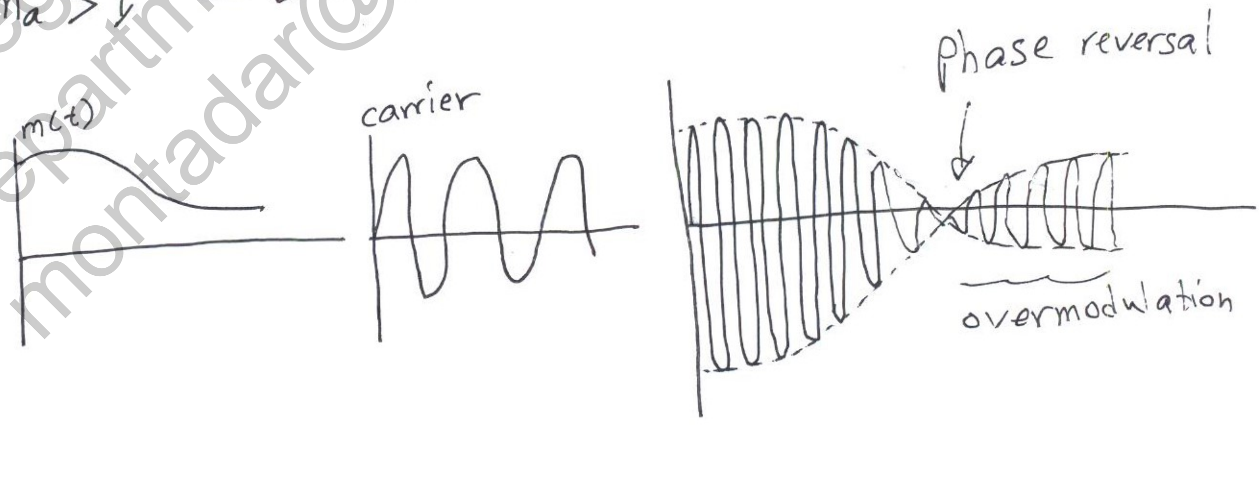
$$\therefore \quad m_a = \frac{|m(t)|_{max}}{A_c}$$

* Note: the baseband or the modulating signal will be **preserved** in the envelope of the AM signal only if we have

$$\boxed{|m(t)|_{max} \leqslant A_c}$$

In other words, $\boxed{m_a \leqslant 1}$

* if $m_a > 1 \longrightarrow$ overmodulation [signal distortion]



Single Tone Amplitude Modulation

* So far, we have considered that the message or the baseband or the modulating signal is random.

* Lets be more specific: Assume the message is single-tone, in other words, let the message be a single carrier sinusoid signal:

$$m(t) = V_m \cos(\omega_m t) \qquad \boxed{\omega_m = 2\pi f_m}$$

if the carrier signal is $c(t) = A_c \cos(\omega_c t)$ $\boxed{\omega_c = 2\pi f_c}$

$$\therefore \quad \boxed{x_{AM}(t) = \underbrace{A_c\left[1 + \frac{V_m}{A_c}\cos\omega_m t\right]}_{\text{Envelope part}} \underbrace{\cos\omega_c t}_{\text{carrier}}}$$

Hence, the modulation index $\boxed{m_a = \frac{V_m}{A_c}}$

Therefore,

$$\boxed{x_{AM}(t) = A_c\left[1 + m_a\cos\omega_m t\right]\cos\omega_c t}$$

* Lets see the frequency components of the AM-signal, which is a single-tone in the current case.

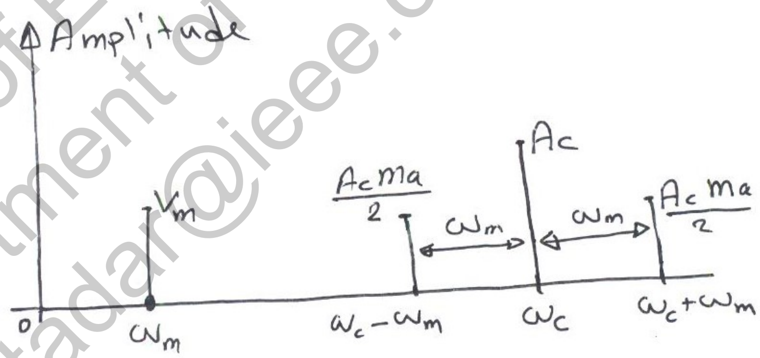
$$x_{AM}(t) = A_c\left[1 + m_a \cos\omega_m t\right]\cos\omega_c t$$

$$= A_c \cos\omega_c t + A_c m_a \cos\omega_m t \cos\omega_c t$$

$$= A_c \cos\omega_c t + \frac{A_c m_a}{2}\left[2\cos\omega_m t \cos\omega_c t\right]$$

$$= \underbrace{A_c \cos\omega_c t}_{\text{first frequency}} + \frac{A_c m_a}{2}\underbrace{\cos(\omega_c + \omega_m)t}_{\text{second frequency}} + \frac{A_c m_a}{2}\underbrace{\cos(\omega_c - \omega_m)t}_{\text{third frequency}}$$

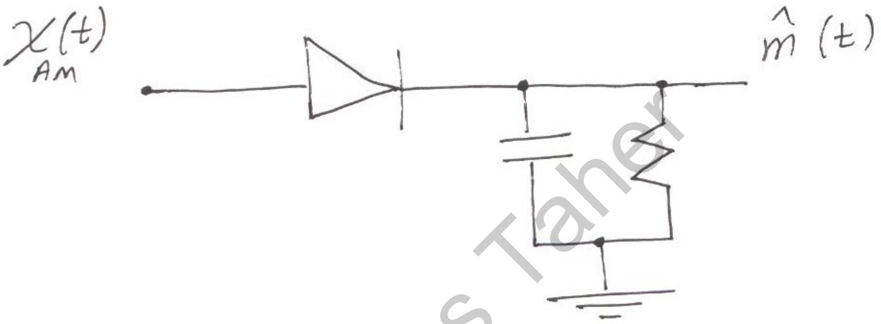
* the first frequency is a pure carrier with amplitude A_c,
* the second frequency is the upper sideband $(\omega_c + \omega_m)$ with amplitude $\frac{A_c m_a}{2}$,
* the third frequency is the lower sideband $(\omega_c - \omega_m)$ with amplitude $\frac{A_c m_a}{2}$.

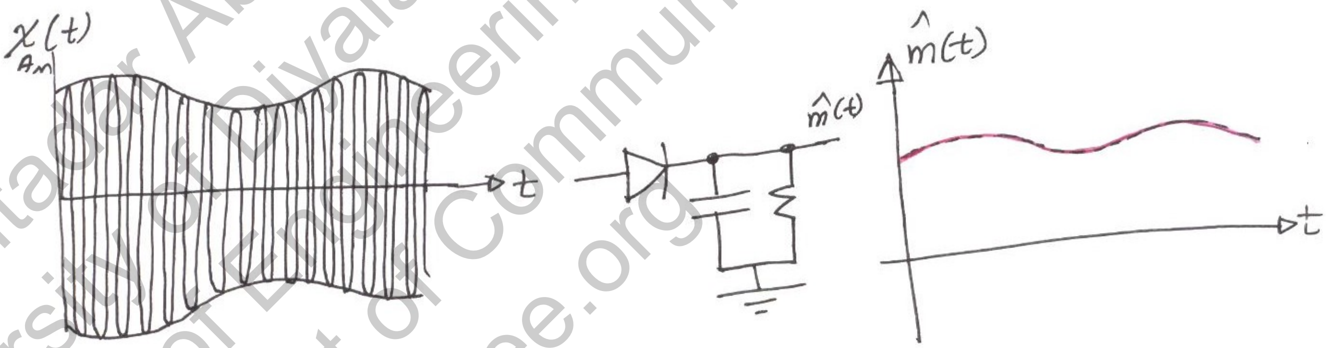


this plot is for single-sided frequency spectrum of the single-tone AM.

AM- Demodulation :

* Because of the AM-signal has a pure carrier frequency components a simple envelope detector can be employed to recover the baseband signal.





EX. 1 A modulating signal $10\sin(2\pi 10^3 t)$ is used to modulate a carrier signal $20\sin(2\pi 10^4 t)$. Determine the modulation index, percentage modulation, frequencies of the sideband components and their amplitudes. What will be the bandwidth of the modulated signal?

Solution: we have given

$$m(t) = V_m \sin(2\pi f_m t)$$
$$m(t) = 10\sin(2\pi 1000 t)$$
$$c(t) = A_c \sin(2\pi f_c t)$$
$$c(t) = 20\sin(2\pi 10{,}000 t)$$

$f_m = 1000$ Hz
$V_m = 10$
$f_c = 10{,}000$ Hz
$A_c = 20$

$\therefore$ The modulation index $m_a = \dfrac{V_m}{A_c} = \dfrac{10}{20} = 0.5$

The percentage modulation $= m_a\% = 0.5 * 100\% = 50\%$.

The upper sideband $f_{USB} = f_c + f_m = 11{,}000$ Hz $= 11$ KHz.

The Lower sideband $f_{LSB} = f_c - f_m = 9000$ Hz $= 9$ KHz.

The amplitude of the upper and lower sidebands are:

$$A_{USB} = A_{LSB} = \frac{A_c m_a}{2} = \frac{20 * 0.5}{2} = \frac{10}{2} = 5 \text{ volts.}$$

The Bandwidth $BW_{AM} = 2f_m = 2 * 1000 = 2$ KHz.

Power Contents in AM signal :-

The general mathematical expression for the AM signal is simply by summing the pure carrier with the DSB-SC signals.

$$\therefore \quad x_{AM}(t) = A_c \cos\omega_c t + m(t)\cos\omega_c t$$

* there are two power components in $x_{AM}(t)$; the carrier power (P_c) & the sideband power (P_s).

* The carrier power $P_c = \frac{1}{2\pi}\int_0^{2\pi} A_c^2 \cos^2\omega_c t\, dt = \frac{A_c^2}{2}$

* The sideband power $P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\left[2\cos^2\omega_c t\right] m^2(t)\, dt$

filter-out

$$P_s = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} m^2(t)\, dt + \cancel{\frac{1}{2\pi}\int_0^{2\pi} m^2(t)\cos(2\omega_c t)\, dt}$$

$$\therefore \quad P_s = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{1}{2} m^2(t)\right] dt$$

* However, P_s is the contribution of the USB and the LSB,

Hence,

$$P_{LSB} = P_{USB} = \frac{P_s}{2}$$

* $\therefore$ total power $P_t = P_c + P_s$

* Now, the transmission efficiency η will be

$$\boxed{\eta = \frac{P_s}{P_t}}$$

and the percentage efficiency is

$$\boxed{\eta = \frac{P_s}{P_t} * 100\,\%}$$

* For single-tone AM-signal,

$$P_s = \frac{1}{2}\frac{V_m^2}{2}$$

$$\boxed{P_s = \frac{V_m^2}{4}}$$

$$\therefore P_t = P_c + P_s = \frac{A_c^2}{2} + \frac{1}{4}V_m^2 = \frac{A_c^2}{2}\left[1 + \frac{1}{2}\left(\frac{V_m}{A_c}\right)^2\right], \quad \text{since } m_a = \frac{V_m}{A_c}$$

$$\therefore P_t = \frac{A_c^2}{2}\left[1 + \frac{1}{2}m_a^2\right]$$

or $\boxed{P_t = P_c\left[1 + \frac{m_a^2}{2}\right]}$ for single-tone AM

EX. A 400 watts carrier is modulated to a depth of 75%, find the total power in the amplitude-modulated wave. Assume the modulating signal to be a sinusoidal one.

Solution we have given the carrier power $\boxed{P_c = 400\ W}$ and we have given the modulation index $\boxed{m_a = 0.75}$. we also given that the modulating signal is sinusoidal, this represents the modulated signal is single-tone AM.

$$P_t = P_c + P_s = \frac{A_c^2}{2} + \frac{1}{4} V_m^2 = \frac{A_c^2}{2}\left[1 + \frac{1}{2}\left[\frac{V_m^2}{A_c^2}\right]\right]$$

$$P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$$

$$\therefore P_t = 400\left[1 + \frac{(0.75)^2}{2}\right] = 512.5\ W.$$

EX. An AM broadcast radio transfers radiates 10k watts of power, if the modulation percentage is 60%, calculate how much of this is the carrier power.

Solution $P_t = P_c\left[1 + \frac{m_a^2}{2}\right]$, we have given $P_t = 10\ kW$, $m_a = 0.6$

$$10{,}000 = P_c + \frac{P_c}{2}(0.6)^2$$

$$20{,}000 = 2P_c + 0.36 P_c$$

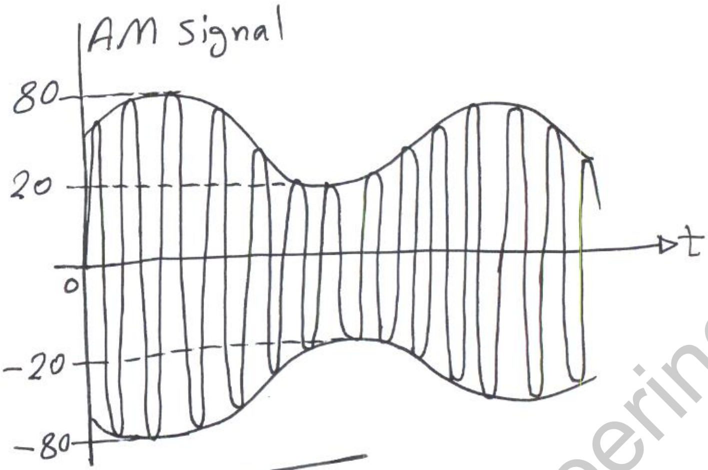
$$2.36 P_c = 20{,}000$$

$$P_c = 8474.6\ W$$

or

$$P_c = 8.475\ kW$$

Ex. Determine the modulation index of the signal shown below.



Solution

modulation Index

$$m_a = \frac{V_m}{A_c} = \frac{max[P\text{-}P] - min[P\text{-}P]}{max[P\text{-}P] + min[P\text{-}P]}$$

$max(P\text{-}P) = 2(80) = 160$

$min(P\text{-}P) = 2(20) = 40$

$$\therefore\ m_a = \frac{160-40}{160+40} = \frac{120}{200} = 0.6$$

in other words $A_c = 50$ volts

EX. A carrier wave is represented by expression $v_c(t) = 10 \sin\omega_c t$. Draw the waveform of an AM wave for $m_a = 0.5$, and 1 Hz modulating signal.

Solution Given $A_c = 10$ volts. or $v_c(t) = 10 \sin\omega_c t$

$$m_a = \frac{V_m}{A_c} \Rightarrow V_m = m_a * A_c = 10 * 0.5 = 5 \text{ Volts}$$

Hence $V_{max} = A_c + V_m = 10 + 5 = 15$ volts.

and $V_{min} = A_c - V_{min} = 10 - 5 = 5$ volts