Department of Communications Engineering

Communication Systems

Third Year Class

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Lecture 7

Types of FM and PM Modulations and Generations

Types of FM modulation :

* There are two types of FM signals :-

① Narrow band FM ② wide band FM.

Narrowband FM : we know

$$S(t)_{FM} = A_c \cos\left[\omega_c t + k_f \int_0^t x(t)\,dt\right]$$

let $y(t) = \int_0^t x(t)\,dt$

$$\therefore\; S(t)_{FM} = A_c \cos\left[\omega_c t + k_f\, y(t)\right]$$

now in phasor form

$$S(t)_{FM} = \mathrm{Re}\left[A_c\, e^{j[\omega_c t + k_f y(t)]}\right]$$

or

$$C(t)_{FM} = A_c\, e^{j[\omega_c t + k_f y(t)]}$$

* The condition of narrowband is $\boxed{k_f\, y(t) \ll 1}$

$$\therefore\; e^{j k_f y(t)} \simeq 1 + j k_f\, y(t)$$

$$\therefore C_{FM}(t) = A_c e^{j\omega_c t} \cdot e^{jk_f y(t)}$$

$$= A_c \left[1 + j k_f y(t)\right] e^{j\omega_c t}$$

But $s(t)_{FM}$ = Real part of $C_{FM}(t)$

$$\therefore \boxed{s(t)_{FM} = A_c \cos\omega_c t - A_c k_f y(t) \sin\omega_c t}$$

narrowband PM :

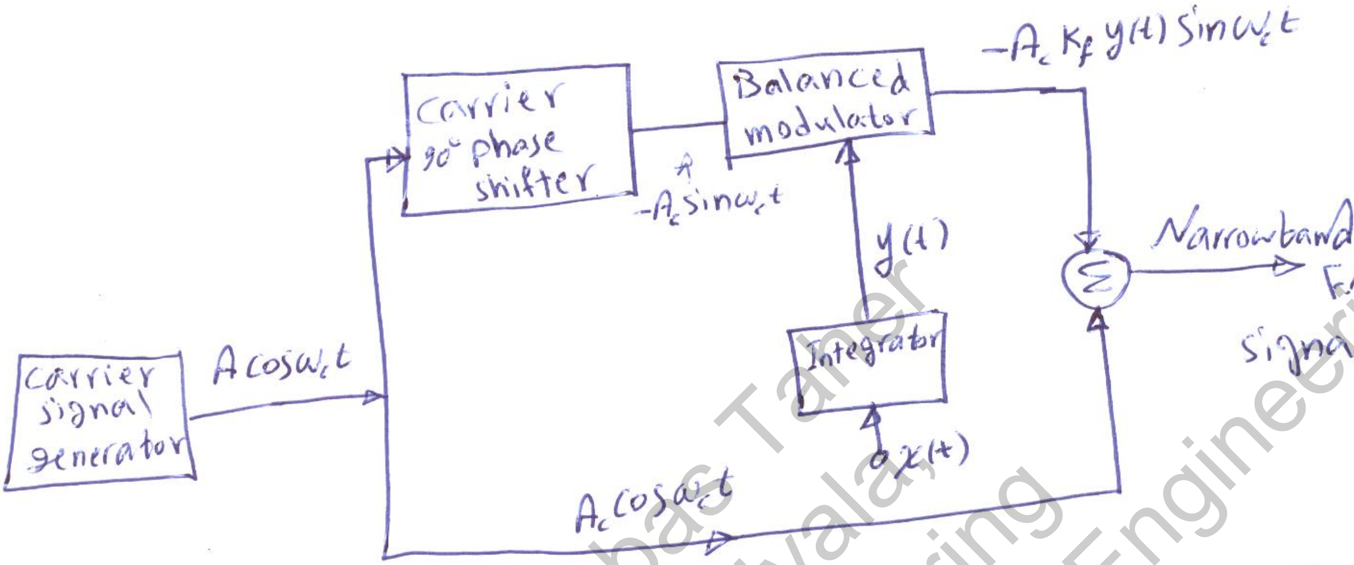
using the same procedure ~~for~~ of the narrowband FM, the PM signal (narrowband) will be

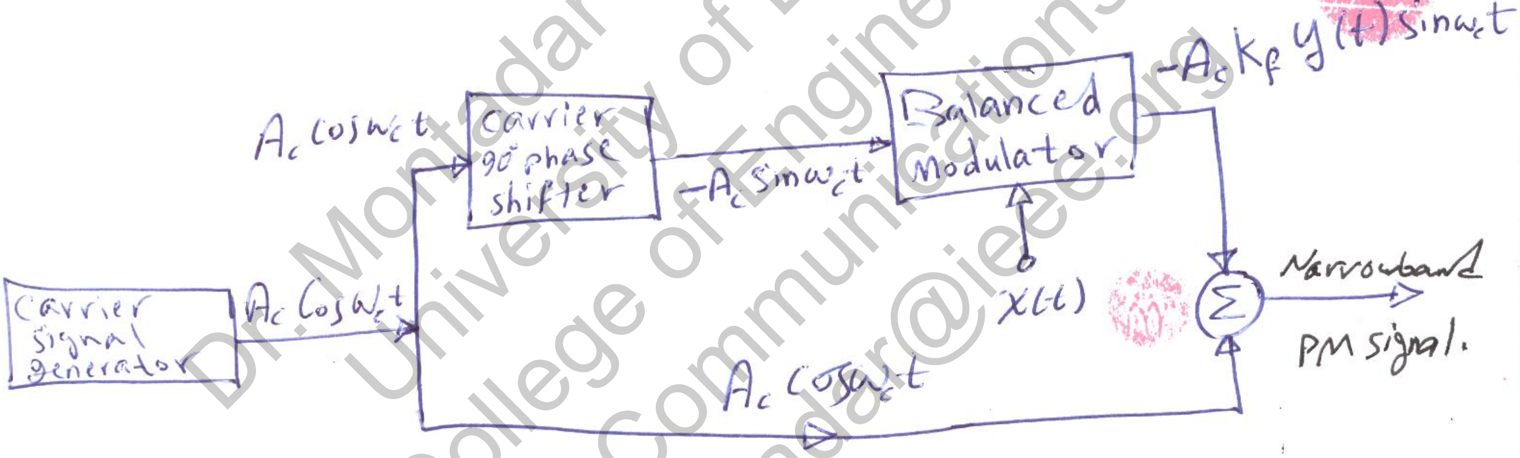
$$\boxed{s(t)_{PM} = A_c \cos\omega_c t - A k_p x(t) \sin\omega_c t}$$

Conclusion for narrowband FM & PM :

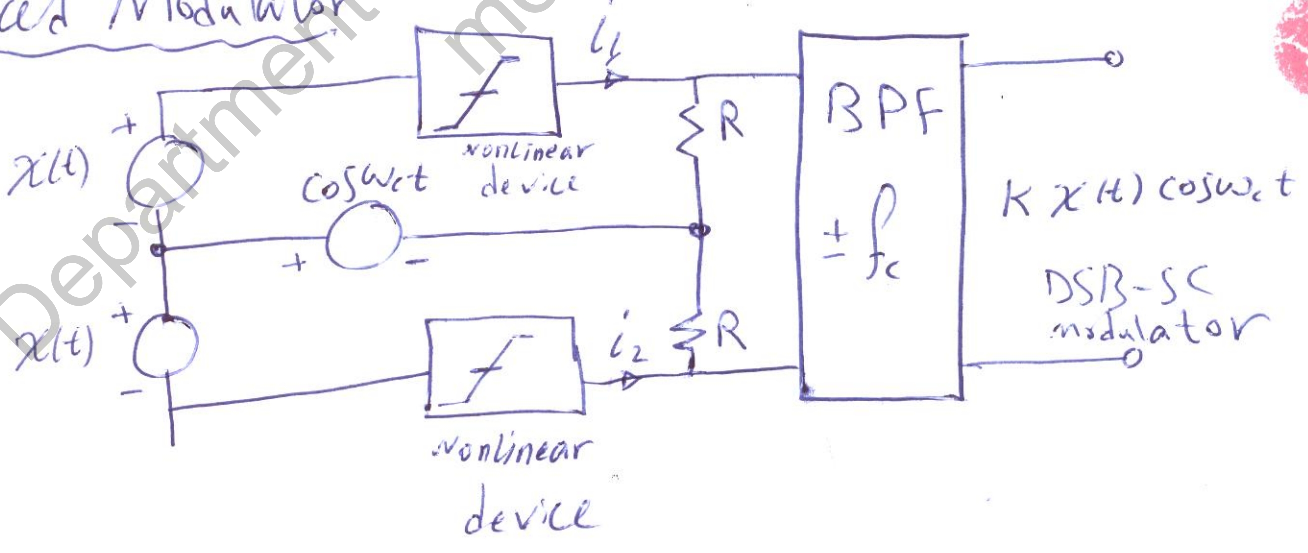
- It can be seen that the expressions for narrowband FM & PM are similar to the AM signal, thus, the bandwidth is almost similar or same as that of AM.

Generation of Narrowband FM & PM :





Balanced Modulator



Single-Tone Narrowband FM :

$$S(t)_{NFM} = A_c \cos\omega_c t - A_c k_f \, y(t) \sin\omega_c t$$

and $y(t) = \int x(t)\, dt$

let $x(t) = V_m \cos\omega_m t$ (the message signal)

$$\therefore \; y(t) = \int V_m \cos\omega_m t \, dt = \frac{V_m}{\omega_m} \sin\omega_m t$$

$$\therefore \; S(t)_{NFM} = A_c \cos\omega_c t - A_c \underbrace{k_f \frac{V_m}{\omega_m}}_{m_f} \sin\omega_m t \, \sin\omega_c t$$

but $m_f = \dfrac{k_f V_m}{\omega_m}$

$$\therefore \; \boxed{S(t)_{NFM} = A_c \cos\omega_c t - A_c m_f \sin\omega_m t \, \sin\omega_c t}$$

Wideband FM :

* if m_f is large $\longrightarrow$ large number of sidebands produced.
* To understand those complicated analysis, a message of single tone sinusoid will be used.

$$S(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$$

The phasor form of $s(t)$ is

$$C_{FM}(t) = A e^{j\omega_c t} \underbrace{e^{j m_f \sin\omega_m t}}_{\text{periodic with period } \frac{1}{f_m}}$$

* thus $e^{j m_f \sin\omega_m t}$ is periodic with period $\frac{1}{f_m}$
* $e^{j m_f \sin\omega_m t}$ can be expanded using Fourier series :

$$e^{j m_f \sin\omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{j n\omega_m t} \qquad \text{for } -\frac{1}{2f_m} \leq t \leq \frac{1}{2f_m}$$

$$C_n = f_m \int_{-\frac{\pi}{\omega_m}}^{\frac{\pi}{\omega_m}} e^{j(m_f \sin\omega_m t)} e^{-j n\omega_m t}\, dt$$

$x = \omega_m t$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(m_f \sin x - n x)}\, dx$$

$$\therefore C_n = J_n(m_f)$$

where $J_n(m_f)$ is the Bessel function of n^{th} order of the first kind.

$$\therefore e^{jm_f \sin\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{jn\omega_m t}$$

Thus

$$C_{FM}(t) = A e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{jn\omega_m t}$$

$$C_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(m_f)\, e^{j(\omega_c + n\omega_m)t}$$

from which

$$\boxed{S(t)_{FM} = A \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_c t + n\omega_m t)}$$

*Properties of Bessel function :-

1. $J_n(m_f) = J_{-n}(m_f)$ n is even
   $J_n(m_f) = -J_{-n}(m_f)$ n is odd
2. For small values of m_f :- $J_0(m_f) \simeq 1$, $J_1(m_f) \simeq \frac{m_f}{2}$, $J_n(m_f) \simeq 0$ $n > 1$
3. $\sum_{n=-\infty}^{\infty} J_n^2(m_f) = 1$

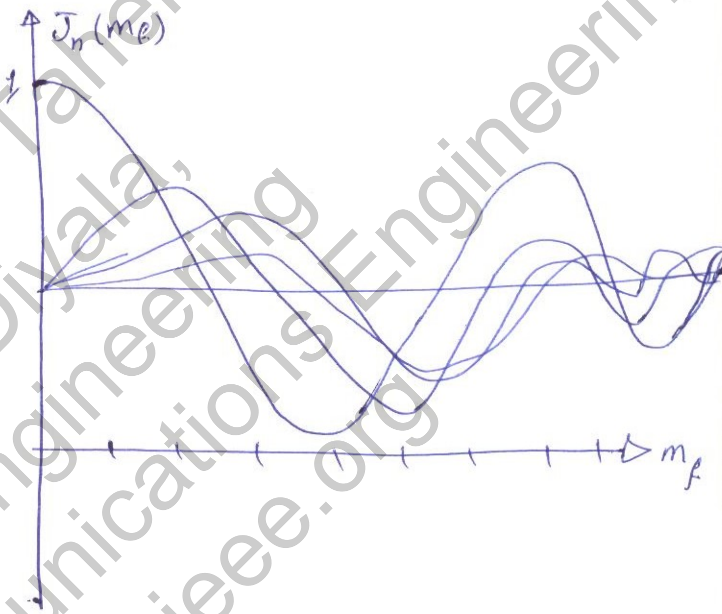
using the first property:

$$s_{FM}(t) = A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right]$$

$$+ A J_2(m_f)\left[\cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t\right]$$

$$+ A J_3(m_f)\left[\cos(\omega_c+3\omega_m)t - \cos(\omega_c-3\omega_m)t\right]$$

$$+ \cdots$$



* we conclude from the last expression of $s_{FM}(t)$

① infinite Bandwidth (theoretically)

because of the infinite number of sidebands

② For small m_f (less than 0.6)

- thus there is only the carrier term and one pair of sidebands.
- this case equivalent to Narrowband FM.

$$s_{FM}(t) = A J_0(m_f)\cos\omega_c t + A J_1(m_f)\left[\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\right].$$

③ If $m_f > 1$ to be ≈ 2.4 or 5.2, $J_0(m_f) = 0$,

$\therefore$ carrier power $= 0$, all power carried by the ~~modulated~~ sidebands ~~signal~~ Hence efficiency $= 100\%$

$$\therefore \underset{AM}{33} \leq \eta_{FM} \leq \underset{100\%}{DSB\text{-}SC}$$

Thus: According to CCIR (Consultative committee for International Radio), there are some regulations:-

1. Maximum modulation frequency = 15 kHz. (message)
2. Maximum frequency deviation $\Delta f = 75$ kHz,
3. Frequency stability of the carrier is ± 2 kHz,
4. Allowable bandwidth per channel = 200 kHz.