Department of Communications Engineering

Communication Systems

Third Year Class

Dr. Montadar Abas Taher

montadar@ieee.org

Lecture 16

Pulse Modulation, Pulse Amplitude Modulation (PAM) I

Pulse Modulation

Review of Sampling Theorem

Dr. Montadar Abas Taher
Department of Communications Engineering,
College of Engineering, University of Diyala. 32001,
Ba'aqubah, Diyala, Iraq.
Email: montadar@engineering.uodiyala.edu.iq
montadar@ieee.org
HP: +964 772 459 8470

* Discrete-time systems are :
  - Inexpensive
  - Light weight,
  - programmable, and
  - easily reproducible.

* processing discrete-time signals are more flexible and more preferable than continuous-time signals.

Sampling operation is the tool that used to convert the continuous-time signals to discrete-time signals.

A band-limited signal of bandwidth W Hz sampled at frequency f_s, can be reconstructed from the sampled version if $f_s \geqslant 2W$

The reconstruction filter must has a bandwidth B in the range $W < B < (f_s - W)$

NOTE: $2W$ frequency is called the Nyquist frequency. In other words, Nyquist frequency is $f_s = f_N = 2W$

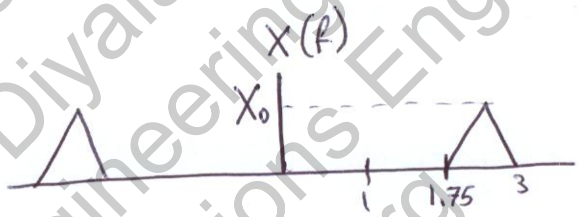
Sampling a Bandpass Signal:

The signal with bandwidth W Hz and upper frequency f_u Hz can be sampled successfully with a sampling frequency of:-

$$\boxed{f_s = \frac{2 f_u}{m}}$$

where $\boxed{m \le \left\lceil \frac{f_u}{W} \right\rceil}$

EX. for the signal shown



the upper frequency $f_u = 3$ Hz.

the signal's Bandwidth $W = 3 - 1.75 = 1.25$ Hz

$$m \le \left\lceil \frac{f_u}{W} \right\rceil = \frac{3}{1.25} = 2.4 = 2$$

$$\therefore f_s = \frac{2 f_u}{m} = \frac{2 \times 3}{2} = 3 \text{ Hz}$$

NOTE: if the signal is a lowpass signal, $f_s = 6$ Hz.

Ex. An analog signal is expressed by the equation $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t)$ ~~+10 sin(300πt)~~ $- \cos(100\pi t)$. Calculate the Nyquist rate for this signal.

Solu.

$$x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$$

$$x(t) = 3\cos(\omega_1 t) + 10\sin(\omega_2 t) - \cos(\omega_3 t)$$

$$\omega_1 = 2\pi f_1 = 50\pi \rightarrow f_1 = 25\ Hz$$

$$\omega_2 = 2\pi f_2 = 300\pi f_2 \rightarrow f_2 = 150\ Hz$$

$$\omega_3 = 2\pi f_3 = 100\pi f_3 \rightarrow f_3 = 50\ Hz$$

Largest frequency is $f_2 = 150\ Hz$

$$\therefore f_s = 2 f_2 = 300\ Hz.$$

Ex. Find the Nyquist rate and the Nyquist interval for the signal

$$x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t).$$

Solu. since $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$\therefore x(t) = \frac{1}{2\pi}\cdot\frac{2}{2}\cos(4000\pi t)\cos(1000\pi t)$$

$$= \frac{1}{4\pi}\left[2\cos(4000\pi t)\cos(1000\pi t)\right]$$

$$= \frac{1}{4\pi}\left[\cos(4000\pi t + 1000\pi t) + \cos(4000\pi t - 1000\pi t)\right]$$

$$= \frac{1}{4\pi}\left[\cos(5000\pi t) + \cos(3000\pi t)\right]$$

$$\therefore \omega_1 = 2\pi f_1 = 5000\pi f_1 \longrightarrow f_1 = 2500\ Hz$$

$$\omega_2 = 2\pi f_2 = 3000\pi f_2 \longrightarrow f_2 = 1500\ Hz$$

$$\therefore f_s = 2 f_1 = 5000\ Hz$$

$$T_s = \frac{1}{f_s} = \frac{1}{5000} = 0.2 * 10^{-3}\ sec.$$

EX. A continuous time signal $x(t) = 8\cos(200\pi t)$. Determine:-

① Minimum sampling rate, (Nyquist rate to avoid aliasing).

② If sampling frequency $f_s = 400$ Hz, what is the discrete-time signal $x(n)$ or $x(nT_s)$ obtained after sampling?

③ If sampling frequency $f_s = 150$ Hz, what is the discrete-time signal $x(n)$ or $x(nT_s)$ obtained after sampling?

④ what is the frequency $0 < f < \frac{f_s}{2}$ that yields samples identical to those obtained in part ③?

Solution: $x(t) = 8\cos(2\pi 100 t) \rightarrow f = 100$ Hz

① $f_s = 2f = 2 * 100 = 200$ Hz.

② $f_s = 400$ Hz $\rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos\left(2\pi n \frac{100}{400}\right) = 8\cos\left(\frac{2\pi n}{4}\right) = 8\cos\left(\frac{\pi n}{2}\right)$.

$\therefore x(n) = x(nT_s) = 8\cos\left(\frac{n\pi}{2}\right)$.

③ $f_s = 150$ Hz $\rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos\left(2\pi n \frac{100}{150}\right) = 8\cos\left(\frac{4\pi n}{3}\right) = 8\cos\left(\frac{6\pi n}{3} - \frac{2\pi n}{3}\right)$

$x(n) = 8\cos\left[\left(2\pi - \frac{2\pi}{3}\right)n\right] = 8\cos\left(\frac{2\pi n}{3}\right)$

④ $f_s = 150$ Hz $\rightarrow x(nT_s) = x\left(\frac{n}{f_s}\right) = 8\cos(2\pi f t) = 8\cos(2\pi f n/150)$

~~Since in~~ from part ③ $\frac{2\pi n}{3} = \frac{2\pi f n}{150} \rightarrow f = \frac{150}{3} = 50$ Hz

$\therefore x_4(t) = 8\cos(100\pi t)$.

NOTE: $x(t) = 8\cos(2\pi 100 t)$, if $f_s = 150$, then alis frequency

$f_a = 150 - 100 = 50$ Hz will appears.

Pulse Amplitude Modulation (PAM)

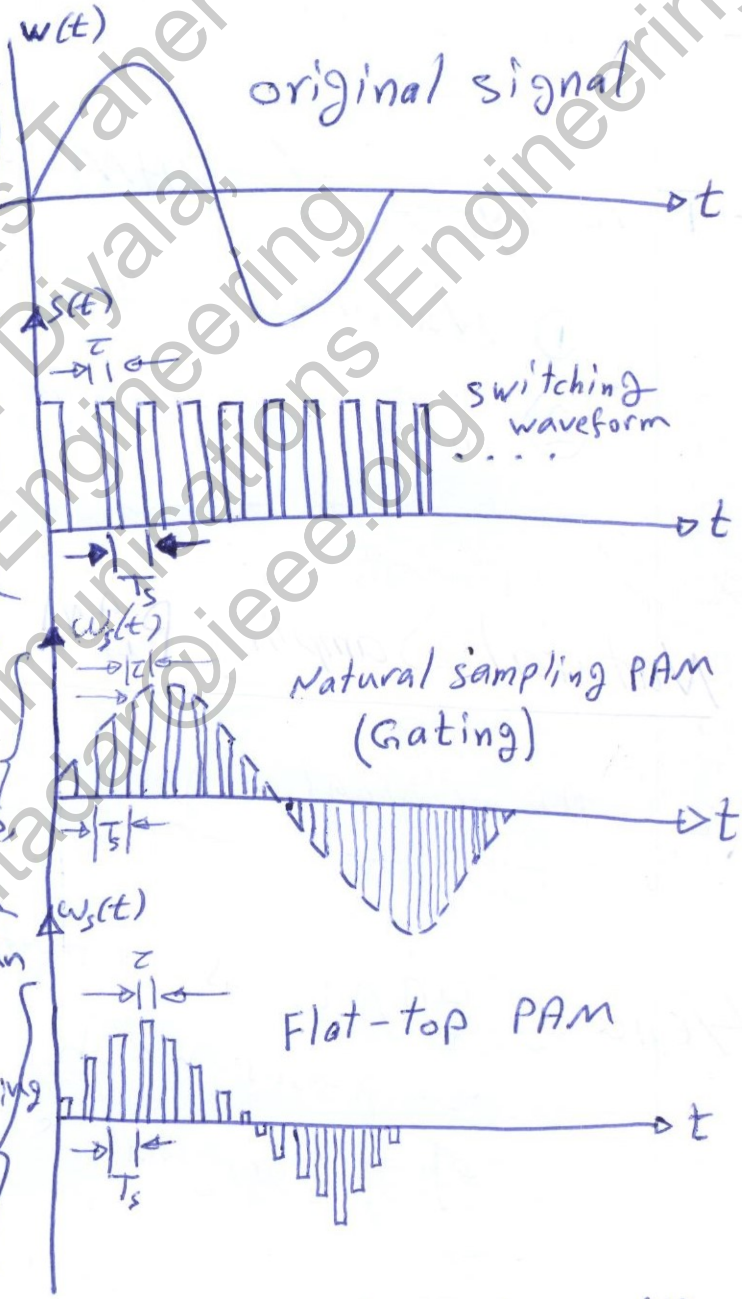
- The pre-request for PAM is the Sampling theorem.
- PAM is one sort of analog pulse modulation.
- Two types of PAM according to the sampling way:-
  1. Natural Sampling (Gating).
  2. Instantaneous Sampling

* In general, in all pulse modulation methods, the pulses of the sampled signal represent the information of the original signal.

Hence: PAM is a method to reshape the analog signal to pulses which carrying the same information of the original signal.

* The analog input signal will be converted to pulses, thus, the bandwidth will be much Larger than the original signal.

* In fact, the pulse rate f_s is the same as the sampling rate $f_s \geq 2B$, where B is the bandwidth of the original signal.



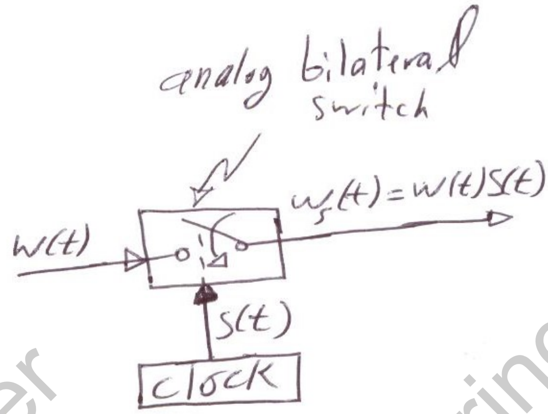
* original signal, analog, $w(t)$ with bandwidth B Hz, continuous time.

* Train of pulses with duty period of τ (aperture) and sampling period of T_s or it is the periodicity period $T_s = \frac{1}{F_s}$.

* Naturally sampled signal, the top is not flat, it is curved, thus, there is a problem in the demodulation because of the noise especially in high frequencies.

* Flat-top signal with sampling period of T_s and duty period τ. Can be generated using Sample-and-Hold circuit. performs better in noise with respect to naturally sampled PAM signals.

Natural Sampling (Gating)



Now if $w(t)$ is baseband band limited signal to B Hz'
The PAM, which is generated by gating (naturally sampled) is

$$w_s(t) = w(t)\,s(t) \quad \text{——} (1)$$

where $s(t) = \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_s}{\tau}\right)$ —— (2)

when $f_s = \frac{1}{T_s} \geq 2B$

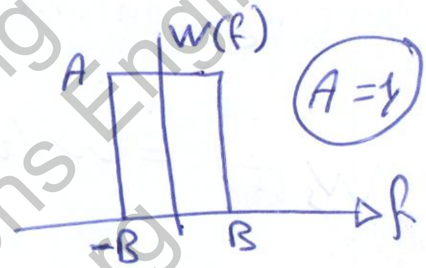
The spectrum of naturally sampled PAM is

$$W_s(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{n\tau}{T_s}\right) W(f - nf_s) \quad \text{——} (3)$$

or $$W_s(f) = \tau f_s \sum_{n=-\infty}^{\infty} \left[\operatorname{sinc}(n\tau f_s)\, W(f - nf_s)\right] \quad \text{——} (3')$$

EX. For an input waveform of rectangular spectrum of bandwidth B Hz, draw the natural sampled version of this input signal and calculate the PAM bandwidth if you know that the sampling frequency $f_s = 4B$ Hz and the duty cycle of the switching waveform is $d = \tau / T_s = \frac{1}{3}$.

solution the input signal is $W(f) = A\,\mathrm{rect}\left(\frac{f}{2B}\right)$



$A = 1$

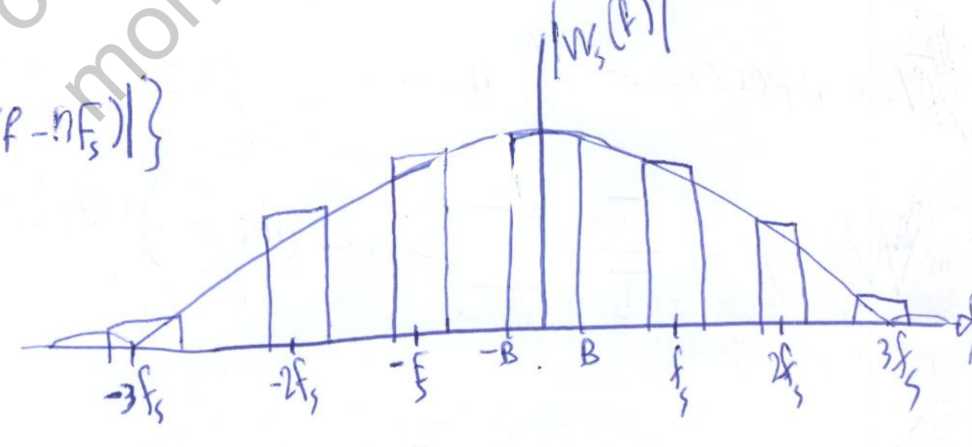
we know $W_s(f) = \tau F_s \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n \tau F_s) W(f - F_s n)$

$$d = \frac{\tau}{T_s} = \frac{1}{3}$$

$$W_s(f) = \frac{\tau}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(n \frac{\tau}{T_s}\right) W(f - nF_s)$$

$$W_s(f) = \frac{1}{3} \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{n}{3}\right) W(f - nF_s)$$

$$|W_s(f)| = \sum_{n=-\infty}^{\infty} \left\{ d\,|\mathrm{sinc}(nd)|\,|W(f - nf_s)| \right\}$$



The bandwidth is $3f_s$

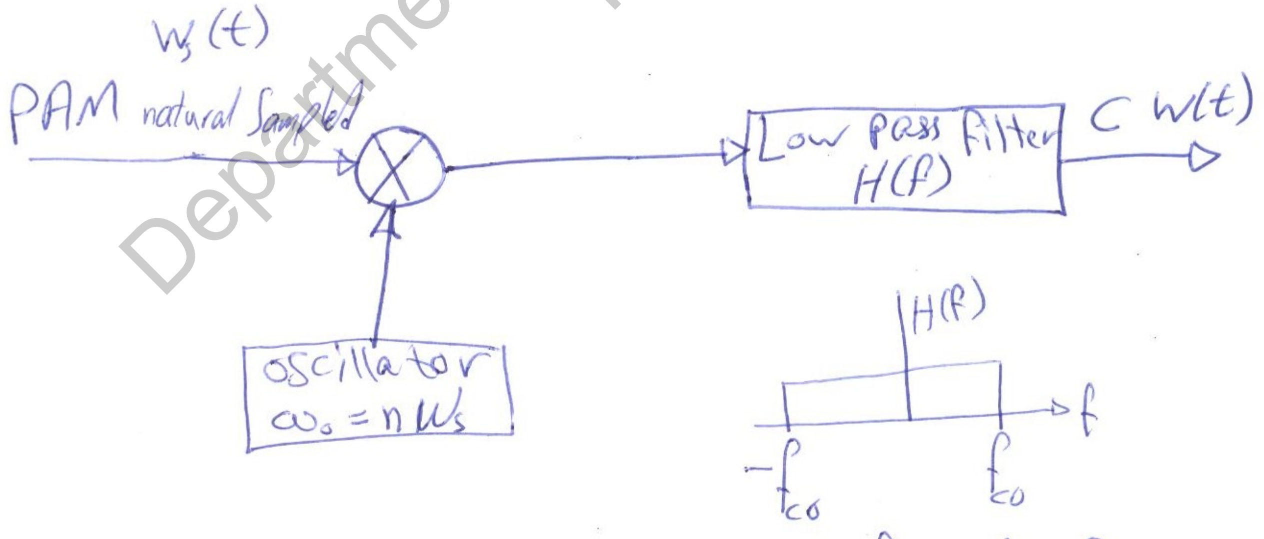
$\therefore BW = 3f_s \Rightarrow 3 * 4B = 12 B$ Hz.

Recovery of Natural sampled PAM is by Low pass filtering with a filter of bandwidth of f_{cutoff}

$$B < f_{cutoff} < f_s - B$$

which is exactly the same as the bandwidth of the recovery from the sampling theory.

Just LPF is not efficient due to the noise at high frequency, thus, another method for recovery is the product detection.



$$B < f_{co} < f_s - B$$

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