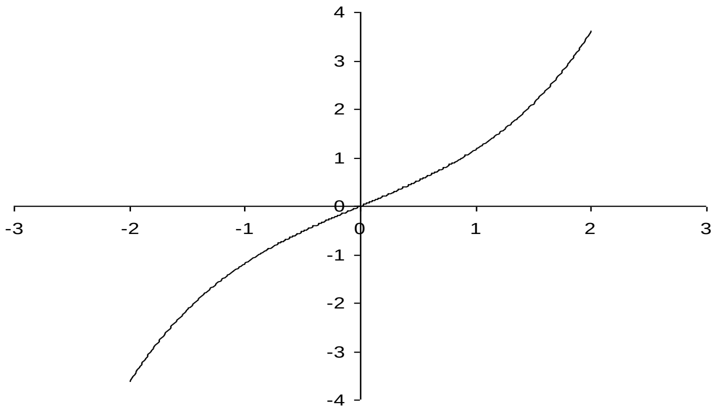
Hyperbolic and Inverse hyperbolic Functions

A) Hyperbolic Functions

1- Definitions

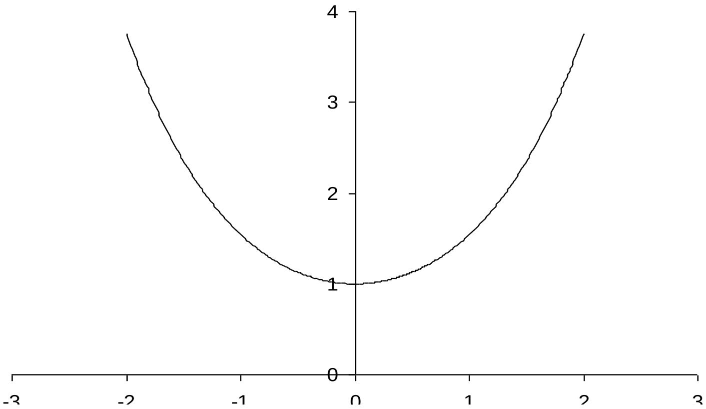
$(1)\ y = \sinh x = \dfrac{e^{x} - e^{-x}}{2}$

$D: \text{All } x;\ R: \text{All } y$



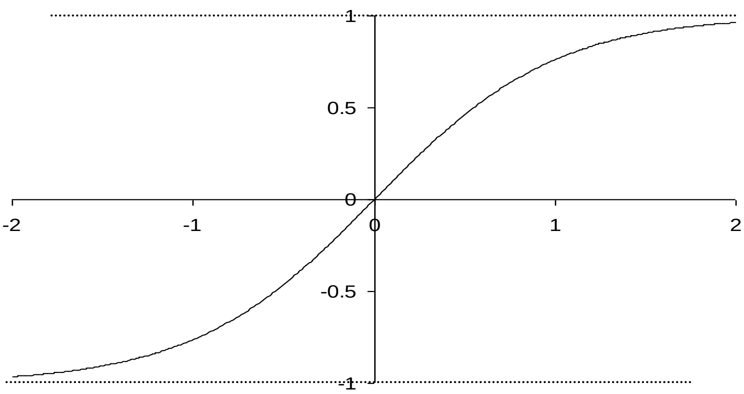
$(2)\ y = \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$D: \text{All } x;\ R: y \geq 1$



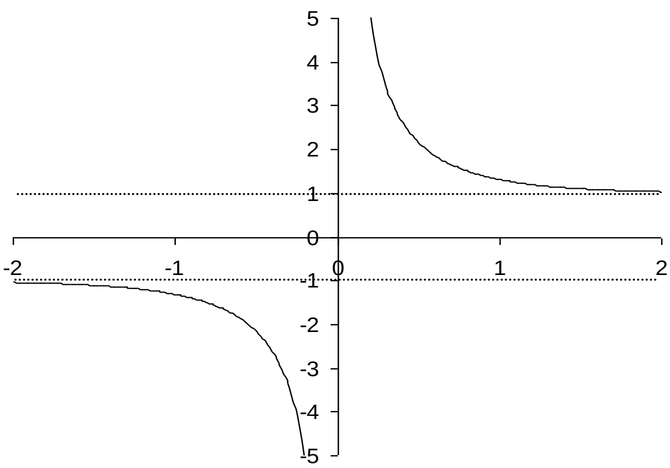
$(3)\ y = \tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

$D: \text{All } x;\ R: -1 < y < 1$



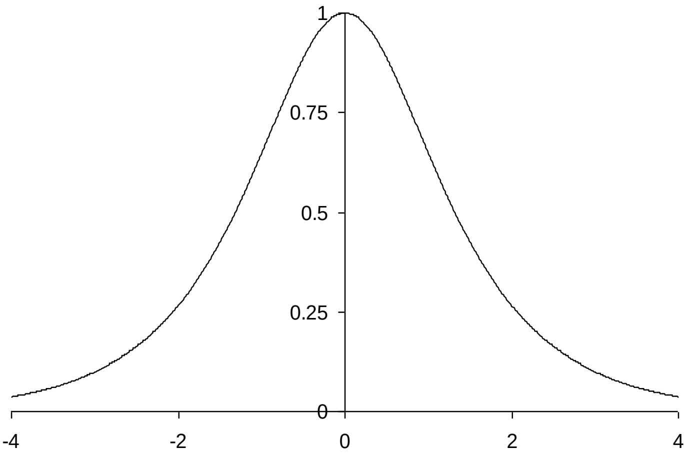
$(4)\ y = \coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

$D: \text{All } x \text{ Except } x = 0;\ R: y > 1 \text{ and } y < -1$



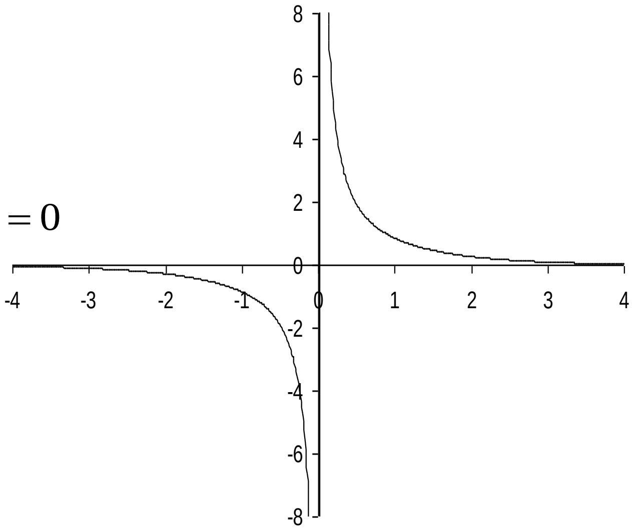
$(5)\ y = \operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^{x} + e^{-x}}$

$D:\ \text{All } x;\ R:\ 0 < y \le 1$



$(6)\ y = \operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^{x} - e^{-x}}$

$D:\ \text{All } x \text{ Except } x = 0;\ R:\ \text{All } y \text{ Except } y = 0$



2- Identities

$(1)\ \cosh x + \sinh x = e^{x}$

$\therefore\ \cosh x + \sinh x = \dfrac{e^{x} + e^{-x}}{2} + \dfrac{e^{x} - e^{-x}}{2} = 2\dfrac{e^{x}}{2} = e^{x}$

$(2)\ \cosh x - \sinh x = e^{-x}$

$\therefore\ \cosh x - \sinh x = \dfrac{e^{x} + e^{-x}}{2} - \dfrac{e^{x} - e^{-x}}{2} = 2\dfrac{e^{-x}}{2} = e^{-x}$

$(3)\ \cosh^{2} x - \sinh^{2} x = 1$

$\because\ (\cosh x + \sinh x)(\cosh x - \sinh x) = e^{x} \times e^{-x}\ \Rightarrow\ \cosh^{2} x - \sinh^{2} x = 1$

$(4)\ 1 - \tanh^{2} x = \operatorname{sech}^{2} x$

$\because\ \cosh^{2} x - \sinh^{2} x = 1,\ \ \div \cosh^{2} x\ \ \Rightarrow 1 - \tanh^{2} x = \operatorname{sech}^{2} x$

$(5)\ \coth^2 x - 1 = \operatorname{csc}h^2 x$

$\because\ \cosh^2 x - \sinh^2 x = 1,\ \div \sinh^2 x\ \Rightarrow\ \coth^2 x - 1 = \operatorname{csc}h^2 x$

$(6)\ \sinh 2x = 2\sinh x \cosh x$

$(7)\ \cosh 2x = \cosh^2 x + \sinh^2 x$

$$(8)\ \cosh^2 x = \frac{1+\cosh 2x}{2}$$

$$(9)\ \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$(10)\ \sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

$(11)\ \cosh(x \pm y) = \cosh x\ \cosh y \pm \sinh x \sinh y$

<u>3- Derivatives</u>

$$(1)\ y = \sinh x = \frac{e^x - e^{-x}}{2} \Rightarrow \frac{dy}{dx} = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$(2)\ y = \cosh x = \frac{e^x + e^{-x}}{2} \Rightarrow \frac{dy}{dx} = \frac{e^x - e^{-x}}{2} = \sinh x$$

$$(3)\ y = \tanh x = \frac{\sinh x}{\cosh x} \Rightarrow \frac{dy}{dx} = \frac{\cosh x \cosh x - \sinh x \sinh x}{\cosh^2 x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

$$\therefore\ \frac{dy}{dx} = \frac{1}{\cosh^2 x} = \sec h^2 x$$

$$(4)\ y = \coth x = \frac{\cosh x}{\sinh x} \Rightarrow \frac{dy}{dx} = \frac{\sinh x \sinh x - \cosh x \cosh x}{\sinh^2 x} = \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$$

$$\therefore\ \frac{dy}{dx} = \frac{-1}{\sinh^2 x} = -\operatorname{csc}h^2 x$$

$$(5)\ y = \sec hx = \frac{1}{\cosh x} \Rightarrow \frac{dy}{dx} = \frac{\cosh x \times (0) - 1 \times \sinh x}{\cosh^2 x} = -\frac{1}{\cosh x} \cdot \frac{\sinh x}{\cosh x}$$

$$\therefore\ \frac{dy}{dx} = -\sec hx \tanh x$$

$$(6)\ y = \operatorname{csc}hx = \frac{1}{\sinh x} \Rightarrow \frac{dy}{dx} = \frac{\sinh x \times (0) - 1 \times \cosh x}{\sinh^2 x} = -\frac{1}{\sinh x}\frac{\cosh x}{\sinh x}$$

$$\therefore \frac{dy}{dx} = -\operatorname{csc} hx \coth x$$

4- Integrals

$(1) \int \sinh x\, dx = \cosh x + c$

$(2) \int \cosh x\, dx = \sinh x + c$

$(3) \int \operatorname{sec} h^2 x\, dx = \tanh x + c$

$(4) \int \operatorname{csc} h^2 x\, dx = -\coth x + c$

$(5) \int \operatorname{sec} hx \tanh x\, dx = -\operatorname{sec} hx + c$

$(6) \int \operatorname{csc} hx \coth x\, dx = -\operatorname{csc} hx + c$

Ex1: Find the derivative of:

$(1)\ y = \sinh^2(\sqrt{x})$

$$\therefore \frac{dy}{dx} = 2\sinh(\sqrt{x})\cosh(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) = \frac{\sinh\sqrt{x}\cosh\sqrt{x}}{\sqrt{x}}$$

$$(2)\ y = \cosh(\ln x) \Rightarrow y = \frac{e^{\ln x} + e^{-\ln x}}{2} = \frac{1}{2}\left(e^{\ln x} + \frac{1}{e^{\ln x}}\right) = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\therefore y = 0.5x + 0.5x^{-1} \Rightarrow \frac{dy}{dx} = 0.5 + 0.5(-x^{-2}) = 0.5\left(1 - \frac{1}{x^2}\right)$$

$$(3)\ y = \sin^{-1}(\tanh x) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\tanh^2 x}} \operatorname{sec} h^2 x$$

$$\because 1 - \tanh^2 x = \operatorname{sec} h^2 x \Rightarrow \frac{dy}{dx} = \frac{\operatorname{sec} h^2 x}{\sqrt{\operatorname{sec} h^2 x}} = \operatorname{sec} hx$$

$(4)\ x \sinh y = \operatorname{sec} hx$

$$\therefore x\cosh y\, y' + \sinh y = -\operatorname{sec} hx \tanh x \Rightarrow x\cosh y\, y' = -\sinh y - \operatorname{sec} hx \tanh x$$

$$\therefore \frac{dy}{dx} = \frac{-\sinh y - \operatorname{sec} hx \tanh x}{x\cosh y}$$

Ex2: Evaluate the following integrals:

$$(1)\ \int \cosh(2-3x)dx = \frac{1}{-3}\int -3\cosh(2-3x)dx = \frac{-1}{3}\sinh(2-3x)+c$$

$$(2)\ \int \tanh(5x)dx = \int \frac{\sinh(5x)}{\cosh(5x)}dx = \frac{1}{5}\int \frac{5\sinh(5x)}{\cosh(5x)}dx = \frac{1}{5}\ln|\cosh(5x)|+c$$

$$(3)\ \int_0^1 \sinh^2 x\,dx \ \Rightarrow\ \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$= \int_0^1 \frac{\cosh 2x - 1}{2}dx = \left[\frac{\sinh 2x}{4} - \frac{x}{2}\right]_0^1 = \left[\frac{\sinh 2}{4} - \frac{1}{2}\right] - \left[\frac{\sinh 0}{4} - \frac{0}{2}\right]$$

$$= \left[\frac{3.626}{4} - \frac{1}{2}\right] = \left[\frac{0}{4} - 0\right] = 0.406$$

$$(4)\ \int_0^2 \frac{\sinh x}{\sinh x + \cosh x}dx$$

$$\because\ \sinh x + \cosh x = e^x \ \text{ and }\ \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\therefore\ \int_0^2 \frac{\sinh x}{\sinh x + \cosh x}dx = \int_0^2 \frac{e^x - e^{-x}}{2e^x}dx = \int_0^2 \frac{1 - e^{-2x}}{2}dx = \int_0^2 0.5\,dx - \int_0^2 0.5\,e^{-2x}\,dx$$

$$= \left[0.5x - \frac{0.5}{-2}e^{-2x}\right]_0^2 = \left[0.5x + 0.25e^{-2x}\right]_0^2 = \left[0.5(2) + 0.25e^{-4}\right] - \left[0.5(0) + 0.25e^{0}\right]$$

$$= [1 + 0.00457] - [0 + 0.25] = 0.7545$$

$$(5)\ \int \sinh^3 x\,dx = \int \sinh x \sinh^2 x\,dx$$

$$\because\ \cosh^2 x - \sinh^2 x = 1 \Rightarrow \sinh^2 x = \cosh^2 x - 1$$

$$= \int \sinh x(\cosh^2 x - 1)dx = \int \cosh^2 x \sinh x\,dx - \int \sinh x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c$$

$$(6)\ \int \frac{\cosh(\ln x)}{x^2+1}dx = \int \frac{e^{\ln x} + e^{-\ln x}}{2(x^2+1)}dx = \int \frac{x + \frac{1}{x}}{2(x^2+1)}dx = \int \frac{x^2+1}{2x(x^2+1)}dx$$

$$= \int \frac{0.5}{x}dx = 0.5\ln|x| + c$$

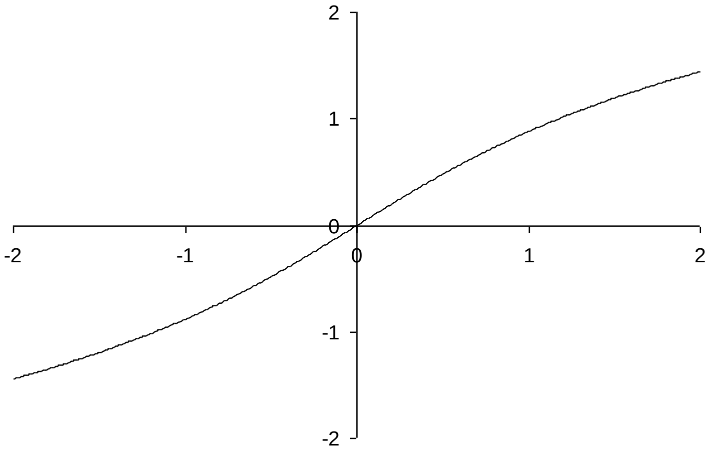
B) Inverse Hyperbolic Functions

1- Definitions

$(1)\ y = \sinh^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right)$

$\therefore\ x = \sinh y$

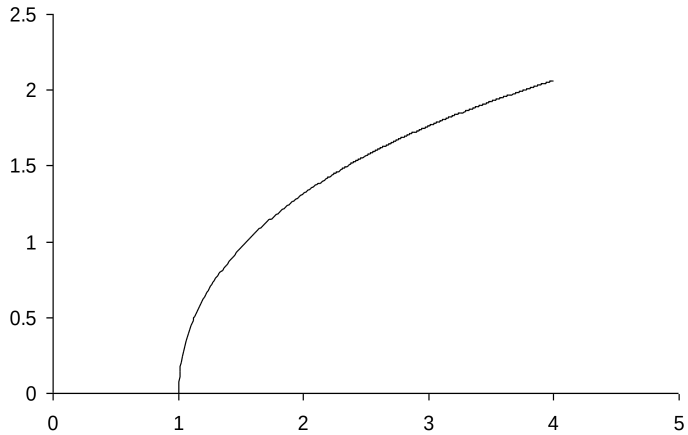
$D: \text{All } x;\ R: \text{All } y$



$(2)\ y = \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$

$\therefore\ x = \cosh y$

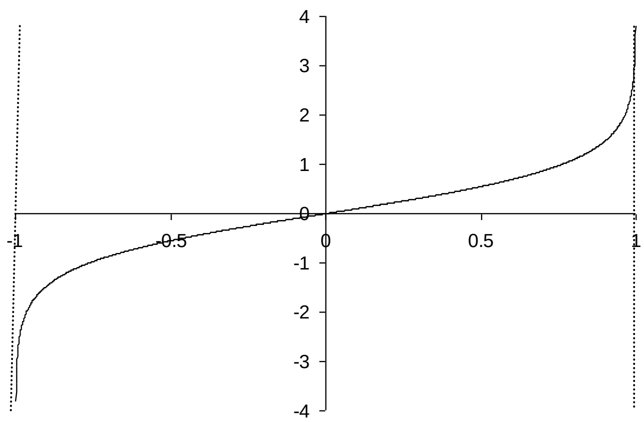
$D: x \geq 1;\ R: y \geq 0$



$(3)\ y = \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right)$

$\therefore\ x = \tanh y$

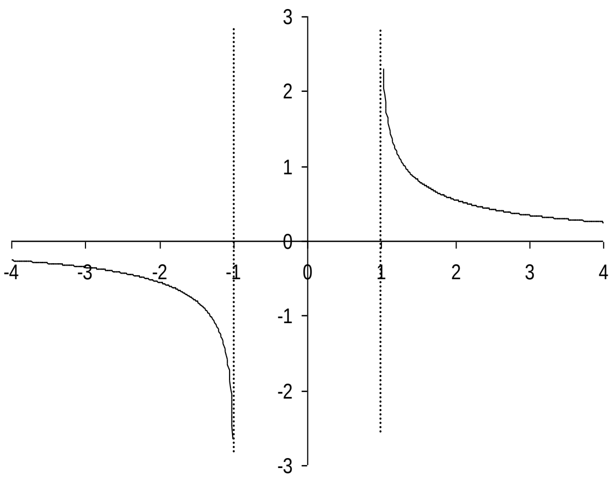
$D: -1 < x < 1;\ R: \text{All } y$



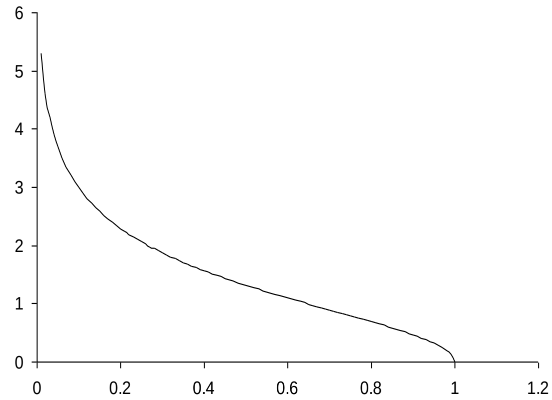
$(4)\ y = \coth^{-1} x = \frac{1}{2} \ln\left(\frac{x + 1}{x - 1}\right)$

$\therefore\ x = \coth y$

$D: -1 > x > 1;\ R: \text{All } y \text{ Except } y = 0$



$(5)\ y=\operatorname{sech}^{-1}x=\ln\left[\dfrac{1+\sqrt{1-x^2}}{x}\right]$

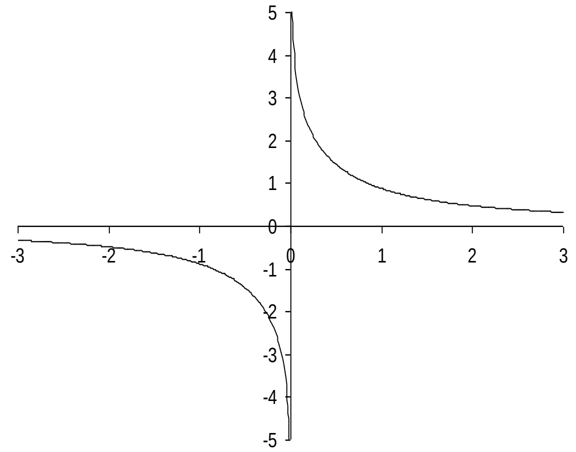


$\operatorname{sech}^{-1}x=\cosh^{-1}\left(\dfrac{1}{x}\right)$

$\because\ y=\operatorname{sech}^{-1}x\Rightarrow x=\operatorname{sech}y=\dfrac{1}{\cosh y}\Rightarrow \cosh y=\dfrac{1}{x}\Rightarrow y=\cosh^{-1}\left(\dfrac{1}{x}\right)=\operatorname{sech}^{-1}x$

$D: 0<x\le 1;\ R: y\ge 0$

$(6)\ y=\operatorname{csch}^{-1}x=\ln\left[\dfrac{1}{x}+\dfrac{\sqrt{x^2+1}}{|x|}\right]$



$\operatorname{csch}^{-1}x=\sinh^{-1}\left(\dfrac{1}{x}\right)$

$\because\ y=\operatorname{csch}^{-1}x\Rightarrow x=\operatorname{csch}y=\dfrac{1}{\sinh y}\Rightarrow \sinh y=\dfrac{1}{x}\Rightarrow y=\sinh^{-1}\left(\dfrac{1}{x}\right)=\operatorname{csch}^{-1}x$

$D:$ All x Except $x=0$; $R:$ All y Except $y=0$

2- Relations

$(1)\ \sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$

Let $y=\sinh^{-1}x\Rightarrow x=\sinh y=\dfrac{e^y-e^{-y}}{2}\Rightarrow 2x=e^y-\dfrac{1}{e^y}$

$\therefore\ 2xe^y=\left(e^y\right)^2-1\Rightarrow\left(e^y\right)^2-2xe^y-1=0$ Quadratic Equation

$\therefore\ e^y=\dfrac{2x\pm\sqrt{4x^2+4}}{2}=x\pm\sqrt{x^2+1}$

$\because\ x^2+1>x$ and $e^y=\text{positive}\ \Rightarrow\ e^y=x+\sqrt{x^2+1}$

$\therefore\ y=\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$

$(2)\ \cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$

Let $y=\cosh^{-1}x\ \Rightarrow\ x=\cosh y=\dfrac{e^y+e^{-y}}{2}\ \Rightarrow\ 2x=e^y+\dfrac{1}{e^y}$

$\therefore\ 2x\,e^y=\left(e^y\right)^2+1\Rightarrow\left(e^y\right)^2-2x\,e^y+1=0$ Quadratic Equation

$\therefore\ e^y=\dfrac{2x\pm\sqrt{4x^2-4}}{2}=x\pm\sqrt{x^2-1}\ \Rightarrow\ y=\ln\left(x\pm\sqrt{x^2-1}\right)$

Either $y=\ln\left(x+\sqrt{x^2-1}\right)$

Or $y=\ln\left(x-\sqrt{x^2-1}\right)=\ln\left[\left(x-\sqrt{x^2-1}\right)\dfrac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right]=\ln\left[\dfrac{x^2-x^2+1}{x+\sqrt{x^2-1}}\right]$

$=\ln\left[\dfrac{1}{x+\sqrt{x^2-1}}\right]=-\ln\left(x+\sqrt{x^2-1}\right)$

$\because\ \cosh^{-1}x\geq 0\ \Rightarrow\ y=\cosh^{-1}x=\ln\left(x+\sqrt{x^2-1}\right)$

$(3)\ \operatorname{sech}^{-1}x=\ln\left[\dfrac{1+\sqrt{1-x^2}}{x}\right],\ 0<x\leq 1$

$\because\ \operatorname{sech}^{-1}x=\cosh^{-1}\left[\dfrac{1}{x}\right]=\ln\left[\dfrac{1}{x}+\sqrt{\dfrac{1}{x^2}-1}\right]=\ln\left[\dfrac{1}{x}+\sqrt{\dfrac{1-x^2}{x^2}}\right]$

$\therefore\ \operatorname{sech}^{-1}x=\ln\left[\dfrac{1+\sqrt{1-x^2}}{x}\right]$

$(4)\ \operatorname{csch}^{-1}x=\ln\left[\dfrac{1}{x}+\dfrac{\sqrt{1+x^2}}{|x|}\right],\ x\neq 0$

$\because\ \operatorname{csch}^{-1}x=\sinh^{-1}\left[\dfrac{1}{x}\right]=\ln\left[\dfrac{1}{x}+\sqrt{\dfrac{1}{x^2}+1}\right]=\ln\left[\dfrac{1}{x}+\sqrt{\dfrac{1+x^2}{x^2}}\right]$

$$\therefore \operatorname{csch}^{-1}x = \ln\left[\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right]$$

$$(5)\ \tanh^{-1} x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$$

$$\text{Let } y = \tanh^{-1} x \Rightarrow x = \tanh y = \frac{e^{y} - e^{-y}}{e^{y} + e^{-y}} \Rightarrow x = \frac{e^{2y} - 1}{e^{2y} + 1}$$

$$\therefore x e^{2y} + x = e^{2y} - 1 \Rightarrow e^{2y}(1-x) = 1+x$$

$$\therefore e^{2y} = \frac{1+x}{1-x} \Rightarrow 2y = \ln\left[\frac{1+x}{1-x}\right] \Rightarrow y = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$$

$$(6)\ \coth^{-1} x = \frac{1}{2}\ln\left[\frac{x+1}{x-1}\right]$$

$$\text{Let } y = \coth^{-1} x \Rightarrow x = \coth y = \frac{e^{y} + e^{-y}}{e^{y} - e^{-y}} \Rightarrow x = \frac{e^{2y} + 1}{e^{2y} - 1}$$

$$\therefore x e^{2y} - x = e^{2y} + 1 \Rightarrow e^{2y}(x-1) = x+1$$

$$\therefore e^{2y} = \frac{x+1}{x-1} \Rightarrow 2y = \ln\left[\frac{x+1}{x-1}\right] \Rightarrow y = \frac{1}{2}\ln\left[\frac{x+1}{x-1}\right]$$

3- Derivatives

$$(1)\ y = \sinh^{-1}(x) \Rightarrow x = \sinh y$$

$$\therefore 1 = \cosh y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{1+\sinh^2 y}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$$

$$(2)\ y = \cosh^{-1}(x) \Rightarrow x = \cosh y$$

$$\therefore 1 = \sinh y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sinh y} = \frac{1}{\sqrt{\cosh^2 y - 1}}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{x^2 - 1}}$$

$(3)\ y=\tanh^{-1}(x) \ \Rightarrow\ x=\tanh y$

$$\therefore 1=\sec h^2 y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1}{\sec h^2 y}=\frac{1}{1-\tanh^2 x}$$

$$\therefore \frac{dy}{dx}=\frac{1}{1-x^2}$$

$(4)\ y=\coth^{-1}(x) \ \Rightarrow\ x=\coth y$

$$\therefore 1=-\csc h^2 y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{-1}{\csc h^2 y}=\frac{-1}{1-\coth^2 x}$$

$$\therefore \frac{dy}{dx}=\frac{-1}{1-x^2}$$

$(5)\ y=\sec h^{-1}x \ \Rightarrow\ x=\sec hy$

$$\therefore 1=-\sec hy\tanh y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{-1}{\sec hy\tanh y}=\frac{-1}{\sec hy\sqrt{1-\sec h^2 y}}$$

$$\therefore \frac{dy}{dx}=\frac{-1}{x\sqrt{1-x^2}}$$

$(6)\ y=\csc h^{-1}x=\sinh^{-1}\left[\frac{1}{x}\right]$

$$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1+\left(\frac{1}{x}\right)^2}}\cdot\frac{-1}{x^2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1+\frac{1}{x^2}}}\cdot\frac{-1}{x^2}=\frac{-1}{x^2\sqrt{\frac{x^2+1}{x^2}}}$$

$$\therefore \frac{dy}{dx}=\frac{-1}{x^2\frac{\sqrt{x^2+1}}{|x|}} \ \Rightarrow\ \frac{dy}{dx}=\frac{-1}{|x|\sqrt{x^2+1}}$$

4- Integrals

$$(1)\ \int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)+c$$

$$(2)\ \int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1}(x)+c$$

$$(3)\ \int \frac{f'(x)}{1-f^2(x)}\,dx = \tanh^{-1}[f(x)]+c,\ \text{When } |f(x)|<1$$

$$\int \frac{f'(x)}{1-f^2(x)}\,dx = \coth^{-1}[f(x)]+c,\ \text{When } |f(x)|>1$$

$$\int \frac{f'(x)}{1-f^2(x)}\,dx = \frac{1}{2}\ln\left|\frac{1+f(x)}{1-f(x)}\right|+c$$

$$(4)\ \int \frac{1}{x\sqrt{1-x^2}}\,dx = -\operatorname{sec}h^{-1}(x)+c$$

$$(5)\ \int \frac{1}{x\sqrt{1+x^2}}\,dx = -\operatorname{csc}h^{-1}|f(x)|+c$$

Ex3: Evaluate the following integrals:

$$(1)\ \int \frac{1}{\sqrt{3+x^2}}\,dx = \int \frac{1/\sqrt{3}}{\sqrt{1+\left(x/\sqrt{3}\right)^2}}\,dx = \sinh^{-1}\left[\frac{x}{\sqrt{3}}\right]+c$$

$$(2)\ \int \frac{1}{\sqrt{x+x^2}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1+\left(\sqrt{x}\right)^2}} = 2\sinh^{-1}\left[\sqrt{x}\right]+c$$

$$(3)\ \int \frac{1}{x\sqrt{(\ln x)^2-9}}\,dx = \int \frac{(1/3x)}{\sqrt{\left(\frac{\ln x}{3}\right)^2-1}}\,dx = \cosh^{-1}\left[\frac{\ln x}{3}\right]+c$$

$$(4)\ \int \frac{\cos x}{\sqrt{5-\cos^2 x}}\,dx = \int \frac{\cos x}{\sqrt{5-1+\sin^2 x}}\,dx = \int \frac{\cos x}{\sqrt{4+\sin^2 x}}\,dx$$

$$= \int \frac{\cos x/2}{\sqrt{1+\left[\frac{\sin x}{2}\right]^2}}\,dx = \sinh^{-1}\left[\frac{\sin x}{2}\right]+c$$

Ex4: Prove that:

$$\tanh^{-1}\left[\frac{x^2-1}{x^2+1}\right]=\ln x$$

Solution

$$\text{Let } y=\tanh^{-1}\left[\frac{x^2-1}{x^2+1}\right]\Rightarrow \tanh y=\frac{x^2-1}{x^2+1}\Rightarrow \frac{e^y-e^{-y}}{e^y+e^{-y}}=\frac{x^2-1}{x^2+1}$$

$$\therefore \frac{e^y-e^{-y}}{e^y+e^{-y}}\times\frac{e^y}{e^y}=\frac{x^2-1}{x^2+1}\Rightarrow \frac{e^{2y}-1}{e^{2y}+1}=\frac{x^2-1}{x^2+1}$$

$$\therefore e^{2y}\left(x^2+1\right)-\left(x^2+1\right)=e^{2y}\left(x^2-1\right)+\left(x^2-1\right)\Rightarrow 2e^{2y}=2x^2\Rightarrow e^{2y}=x^2$$

$$\therefore 2y=\ln x^2=2\ln x\Rightarrow y=\ln x\Rightarrow \tanh^{-1}\left[\frac{x^2-1}{x^2+1}\right]=\ln x$$

Ex5: Prove that:

$$\frac{1+\tanh x}{1-\tanh x}=e^{2x}$$

Solution

$$\therefore \frac{1+\tanh x}{1-\tanh x}=\frac{1+\dfrac{\sinh x}{\cosh x}}{1-\dfrac{\sinh x}{\cosh x}}$$

$$=\frac{\dfrac{\cosh x+\sinh x}{\cosh x}}{\dfrac{\cosh x-\sinh x}{\cosh x}}=\frac{\cosh x+\sinh x}{\cosh x-\sinh x}=\frac{e^x}{e^{-x}}=e^{2x}$$

Ex6: If $a=\cosh x$ and $b=\sinh x$, prove that:

$$(a+b)^2e^{-2x}=a^2-b^2$$

Solution

$$\because a+b=\cosh x+\sinh x=e^x$$

$$\therefore (a+b)^2e^{-2x}=\left(e^x\right)^2e^{-2x}=e^{2x}e^{-2x}=1$$

$$\because a^2-b^2=\cosh^2 x-\sinh^2 x=1$$

$$\therefore (a+b)^2e^{-2x}=a^2-b^2$$