$\int u\,dv = uv - \int v\,du$

for $\int u\,dv = \int x^2 \sin(x)\,dx$

$u = x^2$, $\frac{du}{dx} = 2x$, $du = 2x\,dx$

$dv = \sin(x)\,dx \longrightarrow \int dv = \int \sin(x)\,dx$

$v = -\cos(x)$

$\int x^2 \sin(x)\,dx = -x^2 \cos(x) + 2\int x \cos x\,dx$

$u = x$, $\frac{du}{dx} = 1$, $du = dx$

$dv = \cos(x)\,dx \Rightarrow \int dv = \int \cos(x)\,dx \Rightarrow V = \sin$

$\int x \cos(x)\,dx = x\sin(x) - \int \sin(x)\,dx$

$\int \sin(x)\,dx = -\cos(x)$

$$\therefore \int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

$$\int x^2\sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$$

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

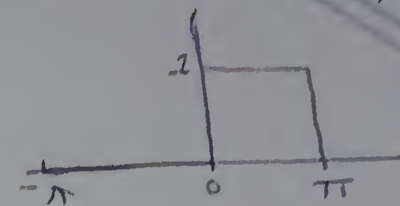
تعويض

$$\int x^2\sin(x)\,dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$$

$$\int x^2\sin(x)\,dx = -x^2\cos(x) + 2x\sin x + 2\cos x + C$$

H.W (2)

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$



$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{2\pi}\int_{0}^{\pi} 1\,dx = \frac{1}{2\pi}\pi = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{0}^{\pi}\cos(nx)\,dx$$

$$= \frac{1}{\pi}\,\frac{\sin nx}{n}\Big|_0^{\pi} = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_{0}^{\pi}\sin(nx)\,dx$$

$$= \frac{1}{\pi}\,\frac{-\cos nx}{n}\Big|_0^{\pi} = \frac{1}{n\pi}(-\cos n\pi + 1)$$

$$= \frac{1}{n\pi}\left((-1)^{n+1} + 1\right)$$

$$b_n = \begin{cases} \frac{2}{n\pi} & n: \text{odd} \\ 0 & n: \text{even} \end{cases}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\sin nx\right)$$

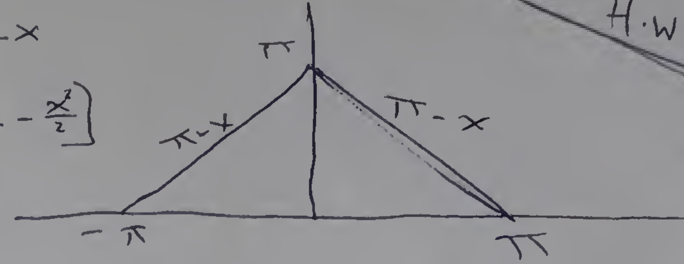
n is odd
$n = 2k-1$
$k = 1, 2, 3, \ldots$

$$\therefore\ f(x) = \frac{1}{2} + \sum_{k=1}^{\infty}\left(\frac{2}{(2k-1)\pi}\sin((2k-1)x)\right)$$

H.W (3)

$\int_{-\pi}^{0} x+\pi + \int_{0}^{\pi} \pi - x$

$\left[\frac{x^2}{2} + \pi x\right] + \left[\pi x - \frac{x^2}{2}\right]$



Solution:

$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$

$= \frac{1}{2\pi} \left\{ \left[\frac{x^2}{2} + \pi x\right]_{-\pi}^{0} + \left[\pi x - \frac{x^2}{2}\right]_{0}^{\pi} \right\}$

$= \frac{\pi}{2}$

Ans: $\frac{\pi}{2} + \frac{4}{\pi}\left(\cos x + \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x\right.$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\, dx$.

$= \frac{1}{\pi} \left\{ \left[\frac{x+\pi}{n} \sin(nx) + \frac{1}{n^2}\cos(nx)\right]_{-\pi}^{0} + \left[\frac{-1}{n^2}\cos nx + \frac{1}{n^2}\right]_{0}^{\pi} \right\}$

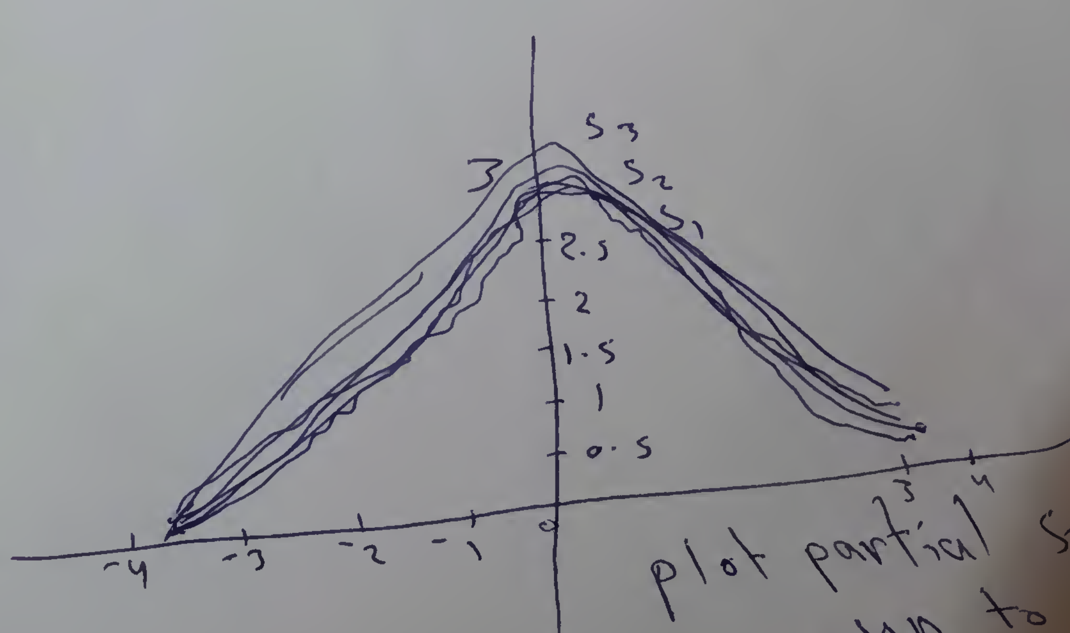
$= \frac{2}{\pi n^2}(1 - \cos(\pi n))$ (2 : odd, 0 : even)

$a_1 = \frac{4}{\pi}$, $a_2 = 0$; $a_3 = \frac{4}{9\pi}$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx$

$= \frac{1}{\pi} \left\{ \left[\frac{-(x+\pi)}{n}\cos(nx) + \frac{1}{n^2}\sin(nx)\right]_{-\pi}^{0} + \left[\frac{\pi}{n}\right] \right\} = 0$

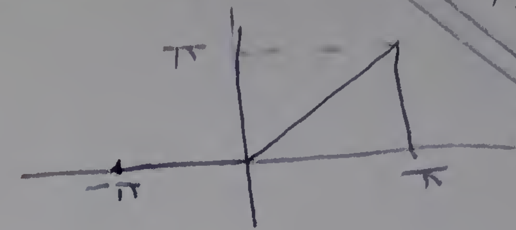
$\therefore f(x) = \frac{\pi}{2} + \frac{4}{\pi}\left(\cos(x) + \frac{1}{9}\cos(3x) + \frac{1}{25}\cos(5x)\right)$



plot partial sums up to n=5

H.W (4)

$$f(x) = \begin{cases} 0 & (-\pi < x < 0) \\ x & (0 < x < \pi) \end{cases}$$



① $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{2\pi}\int_{0}^{\pi} x\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\Big|_0^{\pi}\right] = \frac{1}{2}\cdot\frac{\pi^2}{2} = \frac{\pi}{4}$

② $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$

$= \frac{1}{\pi}\cdot\frac{1}{n^2}\left(nx\sin(nx) + \cos(nx)\right)\Big|_0^{\pi}$

$= \frac{1}{\pi}\cdot\frac{1}{n^2}\left(\cos(n\pi) - 1\right) = \frac{1}{n^2\pi}\left((-1)^n - 1\right)$

$$a_n = \begin{cases} \frac{-2}{n^2\pi} & n: \text{odd} \\ 0 & n: \text{even} \end{cases}$$

③ $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_{0}^{\pi} x\sin nx\,dx$

$= \frac{1}{\pi}\left(\left[-\frac{1}{n}x\cos(nx) + \frac{1}{n^2}\sin(nx)\right]_0^{\pi}\right)$

$= \frac{1}{\pi}\left(\frac{-\pi\cos n\pi}{n}\right) = \frac{(-1)^{n+1}}{n}$

$$b_n = \begin{cases} \frac{1}{n} & n: \text{odd} \\ -\frac{1}{n} & n: \text{even} \end{cases}$$

$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{1}{n^2\pi}\left((-1)^n - 1\right)\cos nx + \frac{(-1)^{n+1}}{n}\sin nx\right)$

$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \cdots\right) + \left(\sin x - \frac{\sin 2x}{2} + \right.$