1.1 Ordinary Differential Equations (ODEs)

D.E: A differential equ. is an equ. that involves one or more derivatives, or differentials.

D.E. are classified by:

1- Type: Ordinary or partial

2- Order: The order of D.E. is the highest order derivative that occurs in the equ.

3- Degree: The exponent of the highest power of the highest order derivative.

* Ordinary D.E.: Depends on only one independent variable.

* Partial D.E.: depends on two or more indenpendent Variables.

Ex.1 $\dfrac{dy}{dx} = 5x + 3$ 1st. order - 1st. degree

Ex.2 $\left(\dfrac{d^3y}{dx^3}\right)^2 + \left(\dfrac{d^2y}{dx^2}\right)$ 3rd. order - 2nd. degree

Ex.3

$y\dfrac{d^3y}{dx^3} + \sin x \dfrac{d^2y}{dx^2} + 5xy = 0$ 3rd. order - 1st. degree

1.1.1 Separable ODEs. :

A first order D.Eq. can be solved by integration if it is possible to collect All (y) terms with (dy) and all (x) terms with (dx) if it possible to write the D.Eq. in the form :

$$g(y)\frac{dy}{dx} = f(x)$$

then the general Solution is

$$\int g(y)\,dy = \int f(x)\,dx + C$$

$$\boxed{y' = \frac{dy}{dx}}$$

Ex.1 $\left(y' = 1 + y^2\right)$

Solution:

$$\frac{dy}{dx} = 1 + y^2 \implies \frac{dy}{1+y^2} = dx$$

By integration, $\displaystyle\int \frac{dy}{1+y^2} = \int dx$

$$\arctan y = x + C \quad \text{or} \quad y = \tan(x+C)$$

Extended method: Reduction to Separable form

is called a (homogeneous) ODE.

for equations

$$y' = f\left(\frac{y}{x}\right)$$

$$\boxed{\begin{array}{l} \text{or } \sin\left(\frac{y}{x}\right) \\ \text{or } \left(\frac{y}{x}\right)^4 \end{array}}$$

The form of such an ODE suggests that we set $\frac{y}{x} = u$; thus,

$$y = ux \xrightarrow{\text{نشتق الطرفين}} y' = u'x + u$$

Substitution into $\dot{y} = f(y/x)$

then gives $\dot{u}x + u = f(u)$

or $\dot{u}x = f(u) - u$ we see that

this can be Seperated:

$$\boxed{\frac{du}{f(u)-u} = \frac{dx}{x}}$$

Ex: Solve: $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{xy}$

Solution:

$$\frac{dy}{dx} = \frac{1 + \frac{y^2}{x^2}}{\frac{y}{x}} \quad \text{(homo.)} \qquad (1)$$

Put $\left(\dfrac{y}{x} = u\right) \longrightarrow y = ux$

نشتق

$$\frac{dy}{dx} = x \cdot \frac{du}{dx} + u \qquad (2)$$

$$\Rightarrow x \cdot \frac{du}{dx} + u = \frac{1+u^2}{u} \Longrightarrow$$

$$x \cdot \frac{du}{dx} = \frac{1 \cancel{+u^2} \cancel{-u^2}}{u}$$

$$x \cdot \frac{du}{dx} = \frac{1}{u} \Longrightarrow \int u \, du = \int \frac{1}{x} \, dx$$

$$\frac{u^2}{2} = \ln x + C \qquad \boxed{u = \frac{y}{x}}$$

$$\therefore \quad \frac{y^2}{2x^2} = \ln x + C$$