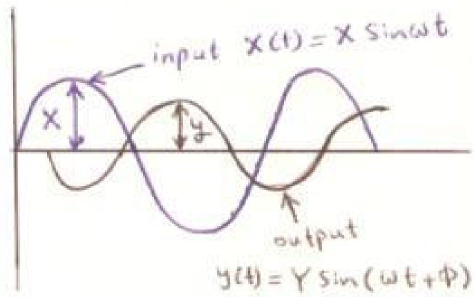
Frequency - Response Analysis

The term frequency response means the steady-state response of a system to a sinusoidal input, i.e, we vary the frequency of the input signal over a certain range and study the resulting frequency response.

Consider the following system

$$X \longrightarrow \boxed{\frac{K}{TS+1}} \longrightarrow y$$

$G(s)$



If $x(t) = X \sin\omega t$ then $y(t)$ can be found as follows:
substituting $j\omega$ for s in $G(s)$ yields

$$G(j\omega) = \frac{K}{j\omega T + 1}$$

$$|G(j\omega)| = \frac{K}{\sqrt{1+T^2\omega^2}}$$

while the phase angle ϕ is

$$\phi = \angle G(j\omega) = -\tan^{-1}\omega T$$

Thus $$y(t) = \frac{XK}{\sqrt{1+T^2\omega^2}} \sin(\omega t - \tan^{-1}\omega T)$$

This system is a phase-Lag system for Large ω.

Ex:- Consider the network given by

$$G(s) = \frac{s + \frac{1}{T_1}}{s + \frac{1}{T_2}}$$

Determine whether the network is a Lead or Lag network.

Solution: $x(t) = X \sin \omega t$

$$G(j\omega) = \frac{j\omega + \frac{1}{T_1}}{j\omega + \frac{1}{T_2}} = \frac{T_2 (1 + T_1 j\omega)}{T_1 (1 + T_2 j\omega)}$$

We have

$$|G(j\omega)| = \frac{T_2 \sqrt{1 + T_1^2 \omega^2}}{T_1 \sqrt{1 + T_2^2 \omega^2}}$$

and

$$\phi = \angle G(j\omega) = \tan^{-1} T_1 \omega - \tan^{-1} T_2 \omega$$

$$y(t) = \frac{X T_2 \sqrt{1 + T_1^2 \omega^2}}{T_1 \sqrt{1 + T_2^2 \omega^2}} \sin(\omega t + \tan^{-1} T_1 \omega - \tan^{-1} T_2 \omega)$$

if $T_1 > T_2$, then $\tan^{-1} T_1 \omega - \tan^{-1} T_2 \omega > 0$

Thus if $T_1 > T_2$

then the network is a Lead network.

If $T_1 < T_2$

then the network is a Lag network.

Frequency-Response characteristics in graphical Forms

1. Nyquist plot or polar plot.
2. Bode diagram or Logarithmic plot.
3. Log-magnitude-versus phase plot (Nichols plots).

Polar plot (Nyquist plot)

It is a plot of magnitude of $G(j\omega)$ varies with the Phase angle of $G(j\omega)$ on a polar Coordinate as ω varied from $0 \to \infty$. It is often called the Nyquist plot.

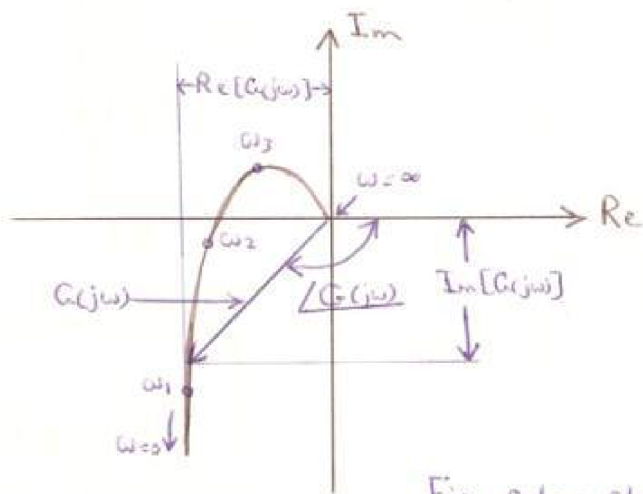


Fig. polar plot

Information about stability is available directly from a polar plot of the sine response of open Loop transfer function $G(j\omega)H(j\omega)$.

* Integral and derivative factors $(j\omega)^{\mp 1}$

$$G(j\omega) = \frac{1}{j\omega} \quad \text{is the negative imaginary axis.}$$

$$G(j\omega) = \frac{1}{j\omega} = -j\frac{1}{\omega} = \frac{1}{\omega}\angle -90^\circ$$

The polar plot of $G(j\omega) = j\omega$ is the positive imaginary axis.

* First-order factors $(1+j\omega T)^{\mp 1}$

$$G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$$

$$G(j0) = 1\angle 0^\circ \quad \text{at } \omega = 0$$
$$G\left(j\tfrac{1}{T}\right) = \frac{1}{\sqrt{2}}\angle -45^\circ \quad \text{at } \omega = \frac{1}{T}$$

$$G(j\omega) = X + jY$$

where $X = \dfrac{1}{1+\omega^2T^2}$ = real part of $G(j\omega)$.

$Y = \dfrac{-\omega T}{1+\omega^2T^2}$ = imaginary part of $G(j\omega)$.

Then we obtain

$$\left(X-\tfrac{1}{2}\right)^2 + Y^2 = \left(\frac{1}{2}\,\frac{1-\omega^2T^2}{1+\omega^2T^2}\right)^2 + \left(\frac{-\omega T}{1+\omega^2T^2}\right)^2 = \left(\frac{1}{2}\right)^2$$

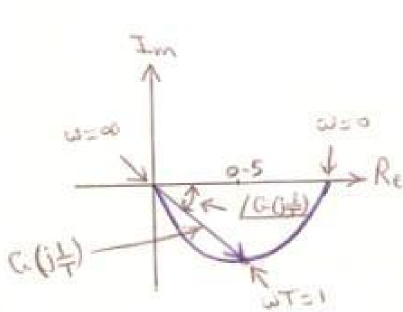


Fig. Polar plot of $\dfrac{1}{1+j\omega T}$

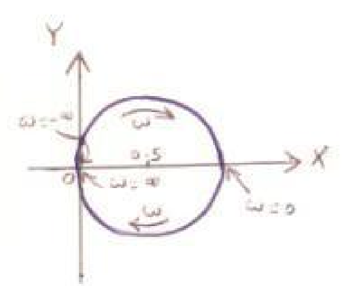


Fig. plot of $G(j\omega)$ in X-Y plane.

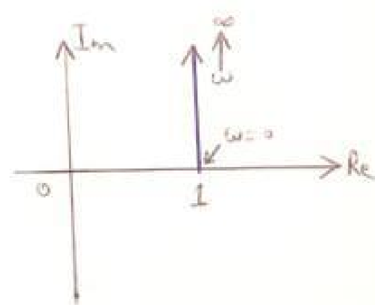


Fig. polar plot of $1+j\omega T$

* Quadratic Factors $[1+2\zeta(j\omega/\omega_n)+(j\omega/\omega_n)^2]^{\mp 1}$

$$G(j\omega)=\frac{1}{1+2\zeta(j\frac{\omega}{\omega_n})+(j\frac{\omega}{\omega_n})^2} \quad \text{for } \zeta>0$$

$$\lim_{\omega\to 0} G(j\omega) = 1\angle 0^\circ \quad ; \quad \lim_{\omega\to\infty} G(j\omega) = 0\angle 180^\circ$$

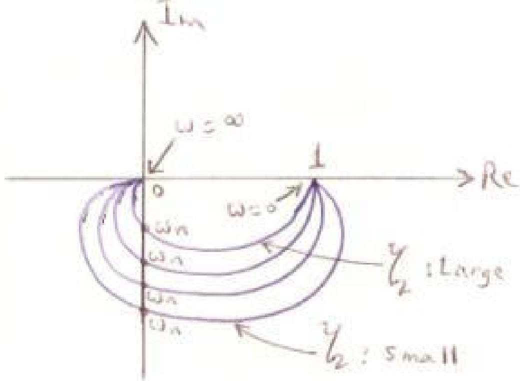


Fig. polar plot of $\frac{1}{1+2\zeta(j\frac{\omega}{\omega_n})+(j\frac{\omega}{\omega_n})^2}$; for $\zeta>0$

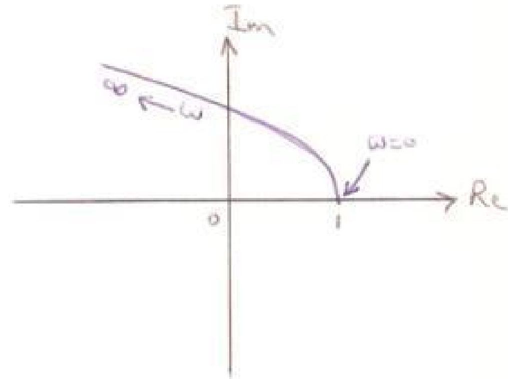


Fig. polar plot of $1+2\zeta(j\frac{\omega}{\omega_n})+(j\frac{\omega}{\omega_n})^2$; for $\zeta>0$

For $G(j\omega) = 1+2\zeta(j\frac{\omega}{\omega_n})+(j\frac{\omega}{\omega_n})^2$

$$=(1-\frac{\omega^2}{\omega_n^2})+j\left(\frac{2\zeta\omega}{\omega_n}\right)$$

The low-frequency portion of the curve is

$$\lim_{\omega\to 0} G(j\omega) = 1\angle 0^\circ$$

and the high-frequency is

$$\lim_{\omega\to\infty} G(j\omega) = \infty\angle 180^\circ$$

Ex 1: Sketch a polar plot of the T.F

$$G(s) = \frac{1}{s(Ts+1)}$$

Solution: $G(j\omega) = \dfrac{1}{j\omega(1+j\omega T)}$

$$= -\frac{T}{1+\omega^2T^2} - j\frac{1}{\omega(1+\omega^2T^2)}$$

$$\lim_{\omega \to 0} G(j\omega) = -T - j\infty = \infty \angle -90^\circ$$

$$\lim_{\omega \to \infty} G(j\omega) = 0 - j0 = 0 \angle -180^\circ$$

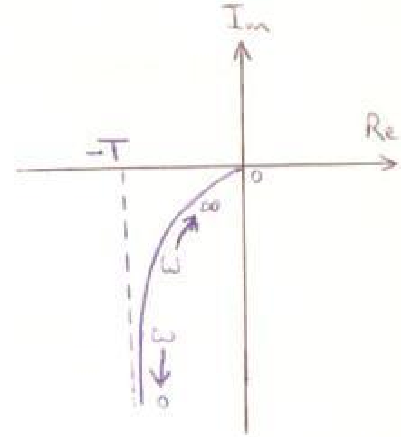


Fig. polar plot of $\dfrac{1}{j\omega(1+j\omega T)}$

Ex 2: Sketch a polar plot of the T.F

$$G(j\omega) = \frac{e^{-j\omega L}}{1+j\omega T}$$

Solution: $G(j\omega) = \left(e^{-j\omega L}\right)\left(\dfrac{1}{1+j\omega T}\right)$

$$|G(j\omega)| = |e^{-j\omega L}| \cdot \left|\frac{1}{1+j\omega T}\right|$$

$$= \frac{1}{\sqrt{1+\omega^2T^2}}$$

$$\angle G(j\omega) = \angle e^{-j\omega L} + \angle \frac{1}{1+j\omega T}$$

$$= -\omega L - \tan^{-1}\omega T$$

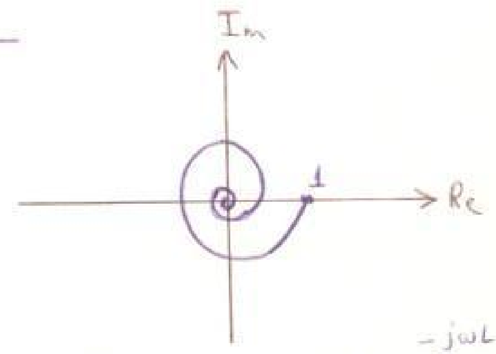


Fig. polar plot of $\dfrac{e^{-j\omega L}}{1+j\omega T}$