Control Systems

Definitions

1- Systems: A system is a combination of components that act together and perform a certain objective.

2. Reference Input: It is the actual Signal input to the control system.

3. output (controlled variable): The quantity that must be maintained at a prescribed value.

4. Open-Loop Control System: A system in which the output has no effect upon the input signal.

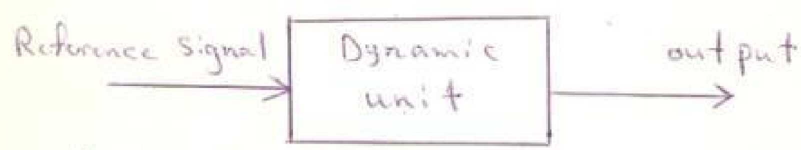


Fig. Open-Loop Control system.

5. Closed-Loop Control system: A system in which the output has an effect upon the input quantity in such a manner as to maintain the desired output value.

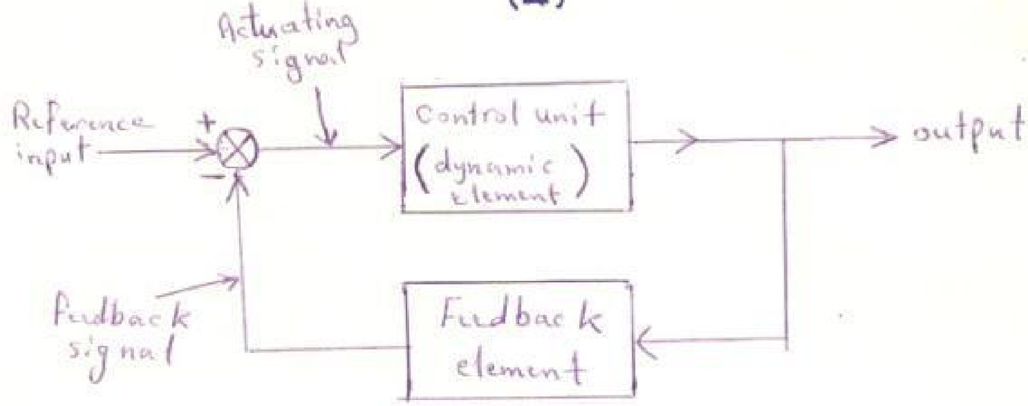


Fig. Closed-Loop Control System

6. plants : A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.

7. Control unit (dynamic element) : The unit that reacts to an actuating signal to produce a desired output. This unit does the work of Controlling the output and thus may be a power amplifier.

8. Feedback Element : The unit that provides the means for feeding back the output quantity, or a function of the output, in order to Compare it with the reference input.

9. Actuating Signal : The signal that is the difference between the reference input and the feedback signal. It actuates the control unit in order to maintain the output at the desired value.

Transfer function

The transfer function of a Linear time-invariant differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial Conditions are zero.

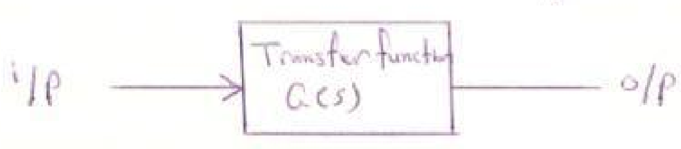
$$a_0 y^{n} + a_1 y^{n-1} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 x^{n} + b_1 x^{n-1} + \cdots + b_{m-1}\dot{x} + b_m x \qquad (n \geq m)$$

Where y ---- output of the system
x ---- input of the system

$$\text{transfer function} = G(s) = \left.\frac{\mathcal{L}\,\text{output}}{\mathcal{L}\,\text{input}}\right|_{\substack{\text{zero initial}\\ \text{conditions}}}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 s^{m} + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^{n} + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Block diagram: A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.

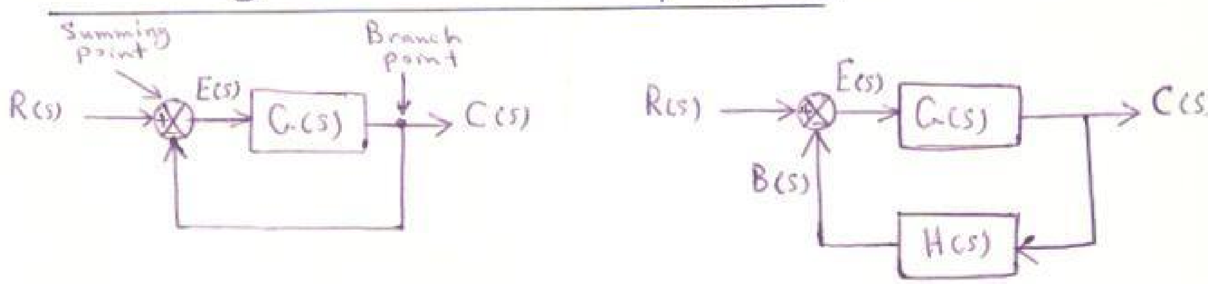


Summing point: A circle with a cross is the symbol that indicates a summing operation.

Fig. Summing point

Branch point: A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.

Block diagram of a closed Loop System:



* open-Loop transfer function $= \frac{B(s)}{E(s)} = G(s)\,H(s)$

* feed forward transfer function $= \frac{C(s)}{E(s)} = G(s)$

* closed-Loop transfer function:

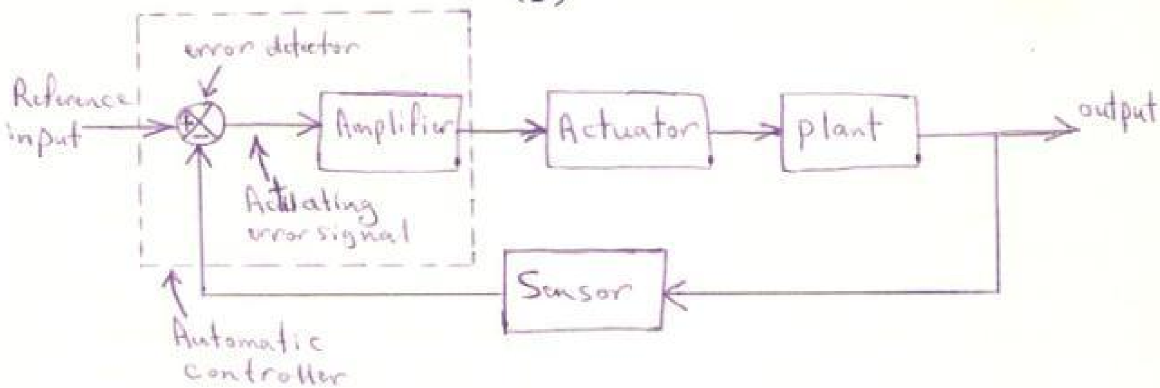
$$C(s) = G(s)\,E(s)$$

$$E(s) = R(s) - B(s) = R(s) - H(s)\,C(s)$$

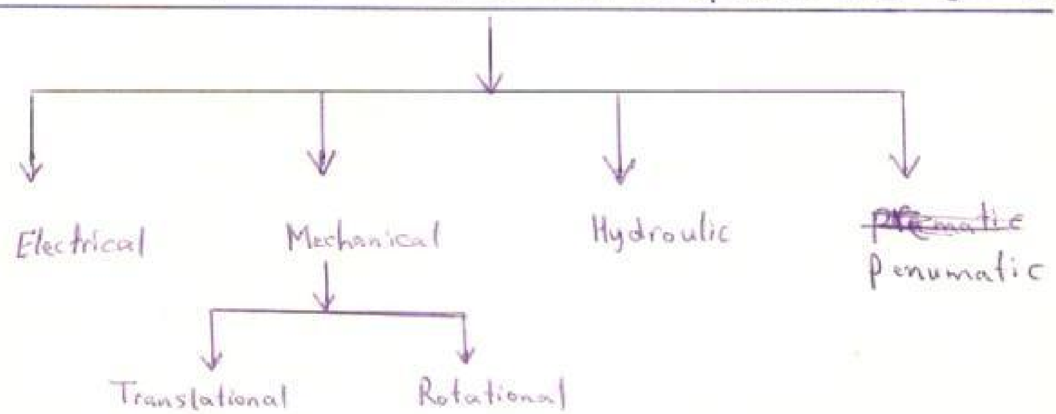
$$C(s) = G(s)\left[R(s) - H(s)\,C(s)\right]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \text{closed-Loop transfer function}$$

Automatic Controllers: An automatic Controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value.



Mathmatical Represntation of Control Components and Systems:



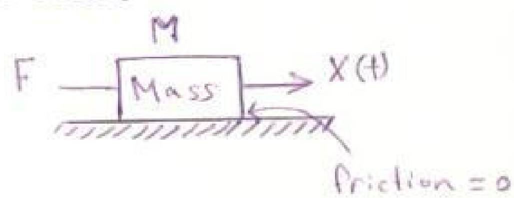
①. Mechanical Components:

a. Translational Mechanical Motion.
b. Rotational Mechanical Motion.

(a) Translational Mechanical Motion:

(1) F = Mass * acceleration

$$= M \cdot \frac{d^2x}{dt^2}$$



$$F(t) = M \frac{d^2X(t)}{dt^2}$$

$X(t)$ = Displacement at any time (t).

$$F_m(s) = M s^2 X(s)$$

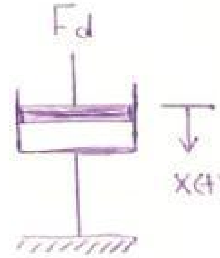
$$F_m \longrightarrow \boxed{1/MD^2} \longrightarrow X$$

$$\frac{X(s)}{F_m(s)} = \frac{1}{Ms^2} = \frac{1}{MD^2}$$

where D-operator $= \dfrac{d}{dt}$

$D = S$ under zero I.C's

(2) $$F_d(t) = B\frac{dx(t)}{dt}$$



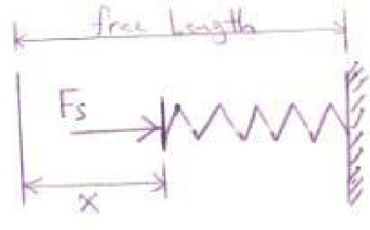
$$F_d(s) = BS\,X(s) = BD\,X(s)$$

$$\frac{X(s)}{F_d(s)} = \frac{1}{BD}$$

$$F_d \longrightarrow \boxed{1/BD} \longrightarrow X$$

where B.... Damping factor
or coefficient of the damper

(3) $$F_s = K X$$



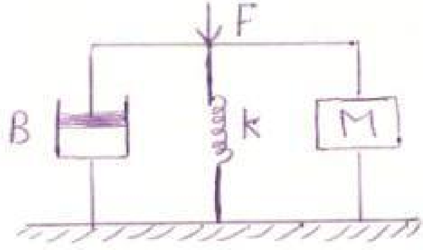
where K... elastance of spring

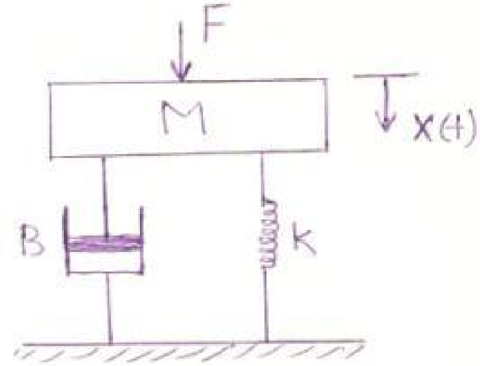
$$F_s(s) = K\,X(s)$$

$$\frac{X(s)}{F_s(s)} = 1/K$$

$$F_s \longrightarrow \boxed{1/K} \longrightarrow X$$

* Series Mechanical Combination :-





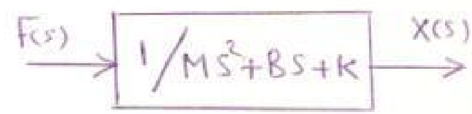
$E = Z_T I$ (in electrical circuit)

$F = Z_T X$

$Z_T = Z_m + Z_d + Z_s = MS^2 + BS + K$

$F(s) = (MS^2 + Bs + k)\, X(s)$

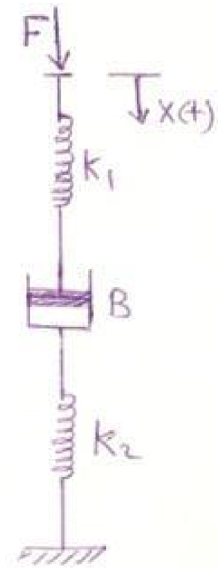
$$\therefore \frac{X(s)}{F(s)} = \frac{1}{MS^2 + BS + K}$$

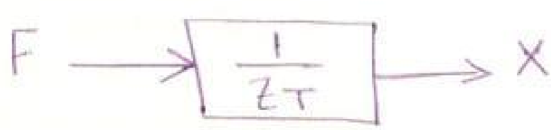


* Parallel Mechanical Combination :-

$$\frac{1}{Z_T} = \frac{1}{Z_{s_1}} + \frac{1}{Z_d} + \frac{1}{Z_{s_2}}$$

$$= \frac{1}{K_1} + \frac{1}{BS} + \frac{1}{K_2}$$

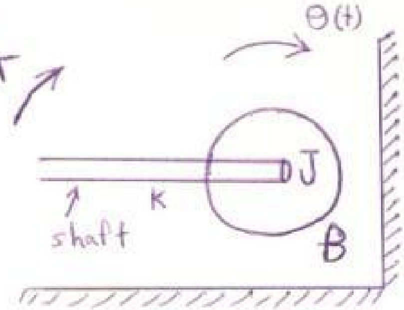




(b) Rotational Mechanical Motion:

Net Torque = change of Momentum

= moment of inertia * Angular Acceleration



$$T - T_d - T_s = J\frac{d^2\theta(t)}{dt^2}$$

$$T - B\frac{d\theta(t)}{dt} - K\theta(t) = J\frac{d^2\theta(t)}{dt^2}$$

$$T(s) = (Js^2 + Bs + K)\,\theta(s)$$

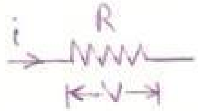
$$\frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

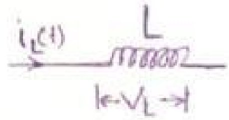
* $T_s = 0$ for riggid shaft

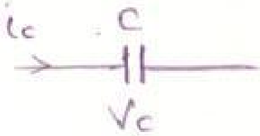
$$\therefore \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs} \quad \text{for riggid shaft}$$

$$\frac{\dot{\theta}(s)}{T(s)} = \frac{1}{Js + B}$$

②- Electrical Components:



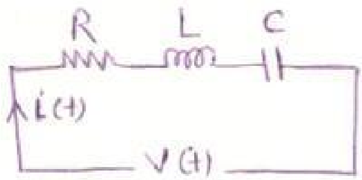




$$V = iR$$
$$V(s) = I(s)\,R$$
$$\therefore Z_R(s) = R$$

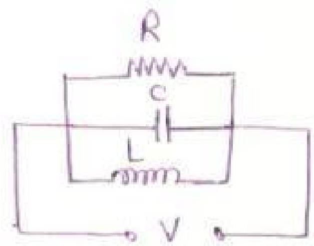
$$V_L = L\frac{di_L(t)}{dt}$$
$$V_L(S) = LS\,I(s)$$
$$Z_L(s) = LS$$

$$V_c = \frac{1}{c}\int i_c\,dt$$
$$\frac{dV_c}{dt} = \frac{i_c}{C}$$
$$S\,V_c(S) = \frac{I_c(s)}{C}$$
$$V_c(S) = \frac{1}{CS}\,I(s)$$
$$\therefore Z_c(S) = \frac{1}{CS}$$

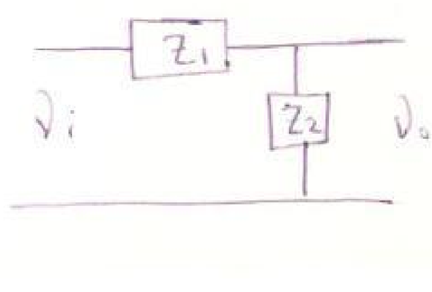


$$Z_T(s) = R + LS + \frac{1}{CS}$$
$$V(t) = Z_T\,i(t)$$
$$V(S) = Z_T(S)\,I(s) = I(s)\left(R + LS + \frac{1}{CS}\right)$$
$$\frac{I(s)}{V(s)} = \frac{1}{R + LS + \frac{1}{CS}}$$

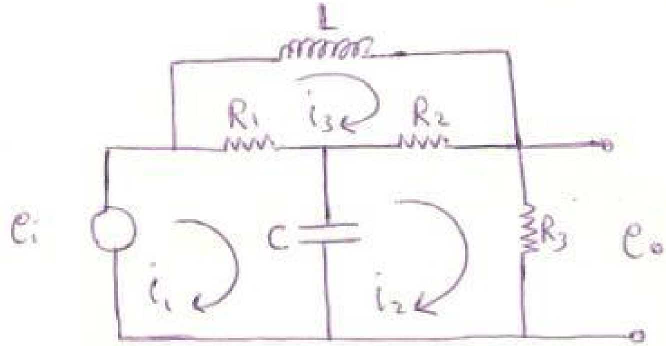
$$\frac{1}{Z_T} = \frac{1}{R} + \frac{1}{LS} + CS$$



$$\frac{v_o}{v_i} = \frac{Z_2}{Z_2 + Z_1}$$



Ex.



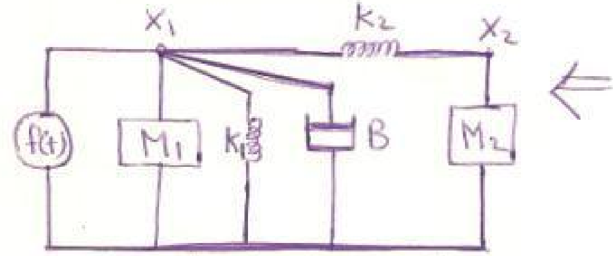
$$e_i(t) = R_1(i_1 - i_3) + \frac{1}{C}\int (i_1 - i_2)\,dt \quad \text{—— (1)}$$

$$0 = \frac{1}{C}\int (i_2 - i_1)\,dt + R_2(i_2 - i_3) + i_3R_3 \quad \text{—— (2)}$$

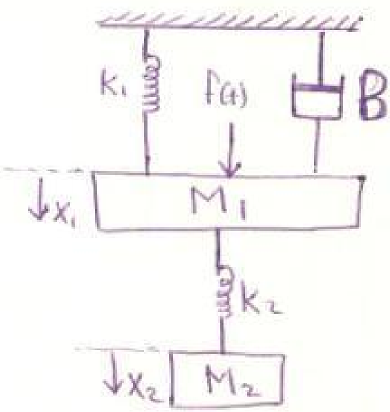
$$0 = L\frac{di_3}{dt} + R_1(i_3 - i_1) + R_2(i_3 - i_2) \quad \text{—— (3)}$$

$$e_o = i_2R_3 \quad \text{—— (4)}$$

Ex.



equivalent circuit



Node x_1:

$$f(t) = M_1 x_1'' + B x_1' + K_1 x_1 + K_2(x_1 - x_2) \quad \text{—— (1)}$$

Node x_2:

$$0 = M_2 x_2'' + K_2(x_2 - x_1) \quad \text{—— (2)}$$

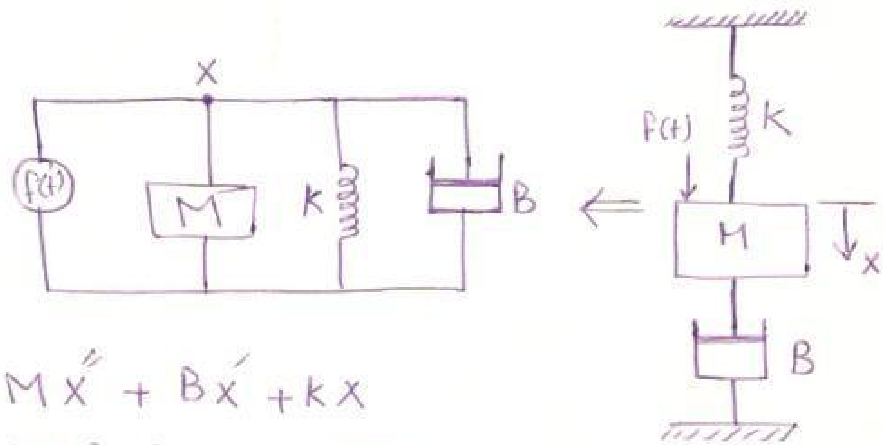
Taking Laplace Transfer

$$F(s) = (M_1 s^2 + Bs + K_1 + K_2)X_1(s) - K_2 X_2(s) \quad \text{—— ①}$$

$$0 = (M_2 s^2 + K_2)X_2(s) - K_2 X_1(s) \quad \text{—— ②}$$

Ex:

$\frac{X(s)}{F(s)}$?

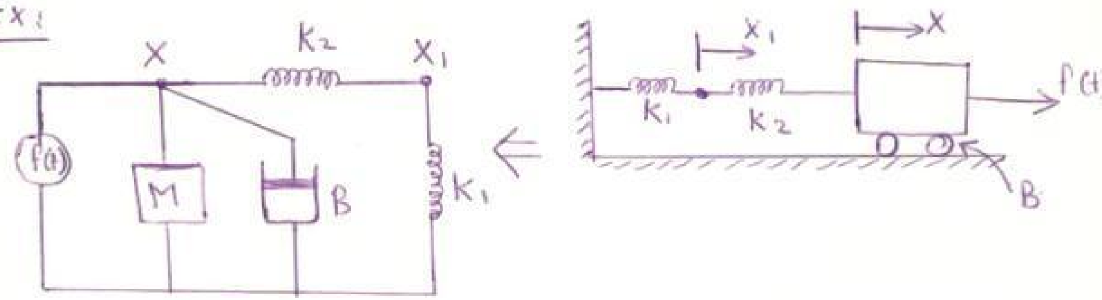


$$f(t) = MX'' + BX' + KX$$

$$F(s) = (Ms^2 + Bs + K)X(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Ex:



Node X:

$$f(t) = MX'' + BX' + K_2(X - X_1) \quad \text{—— ①}$$

$$F(s) = (Ms^2 + Bs + K_2)X(s) - K_2 X_1(s)$$

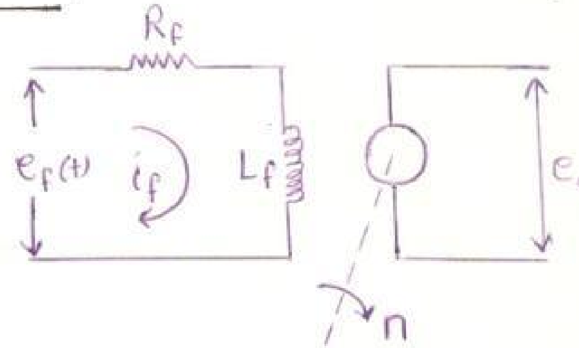
Node X1:

$$0 = K_1 X_1 + K_2(X_1 - X) \quad \text{—— ②}$$

$$0 = (K_1 + K_2)X_1(s) - K_2 X(s)$$

T.F for a D.C generator:

$$\frac{E_g(s)}{E_f(s)} = ?$$



$$e_f = i_f R_f + L_f \frac{di_f}{dt}$$

$$E_f(s) = (R_f + L_f S)\, I_f(s) \quad \text{---} \ (1)$$

$e_g \propto n\Phi$ where Φ -- flux density

$\Phi \propto i_f \Rightarrow \Phi = K_2 i_f$

$e_g = K_1 n K_2 i_f$

$e_g = K_g i_f$ (K_g = generator constant)

$$E_g(s) = K_g I_f(s) \quad \text{---} \ (2)$$

$$\frac{E_g(s)}{E_f(s)} = \frac{K_g}{R_f + L_f S} = \frac{K}{1 + TS}$$

where

$T = \dfrac{L_f}{R_f}$ = electric time constant

$K = \dfrac{K_g}{R_f}$

T.F of Tachometer generator:

It is a tacho generator. It is a d.c. generator with a permanent magnetic, therefore, compared with d.c. generator we find that:

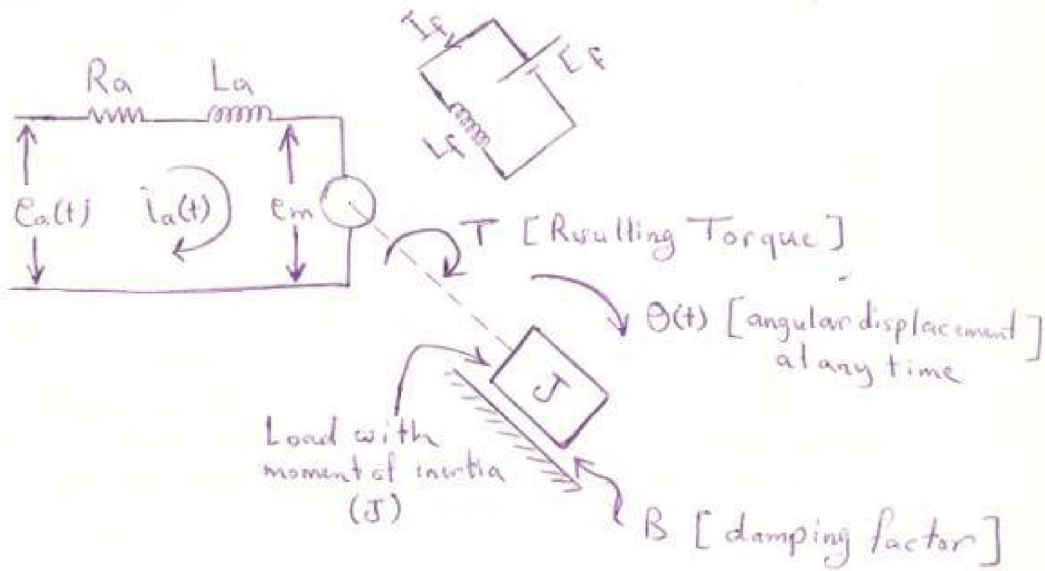
$$e_g \propto n\phi$$

where n = speed of rotating
ϕ = mag. flux density

In the case of tacho generator we have a parmanent magnetic, therefore, the flux density is constant.

Hence the (e_g) becomes $e_g = K_T n$, where K_T is tacho constant.

T.F for Armature Control D.C Servo motor (ACDSM):



e_m -- is the induced e.m.f

$$\frac{\Theta(s)}{E_a(s)} = ?$$

$$e_a(t) = i_a R_a + L_a \frac{di_a(t)}{dt} + e_m \quad \text{— ①}$$

$$E_a(s) - E_m(s) = (R_a + L_a S) I_a(s) \quad \text{— ①}$$

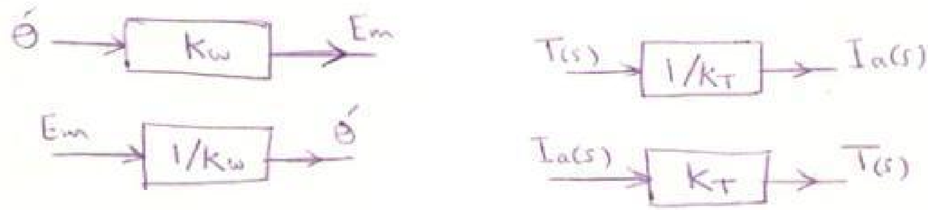
$$e_m = k_\omega \theta'$$

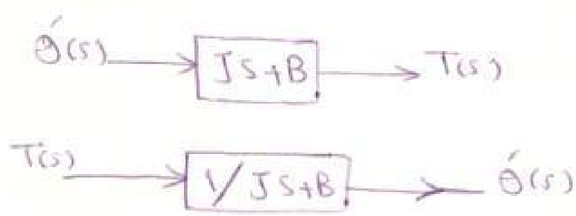
$$E_m(s) = K_\omega \theta'(s) = K_\omega S\, \theta(s) \quad \text{— ②}$$

$$T \propto i_a(t) \Rightarrow T(s) = k_T I_a(s) \quad \text{— ③}$$

$$T(t) = J \frac{d^2\theta(t)}{dt^2} + B \frac{d\theta(t)}{dt}$$

$$T(s) = (JS^2 + BS)\,\theta(s) = (JS + B)\,\theta'(s) \quad \text{— ④}$$





Closed Loop: