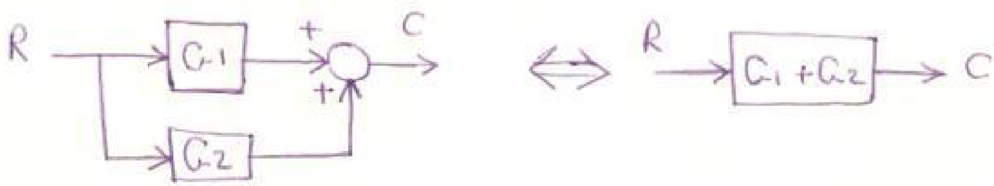
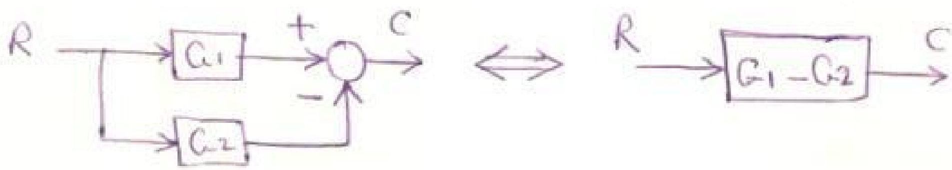
Block Diagram Reduction

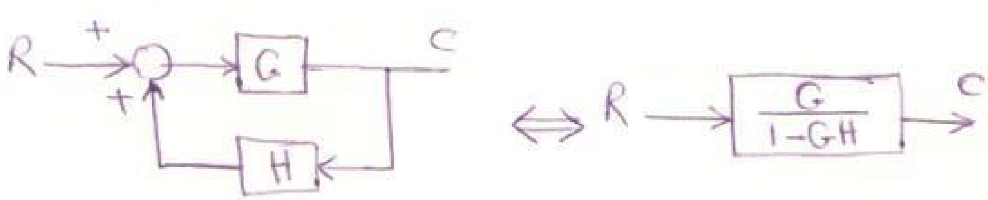
Rule ① Cascaded elements

Rule ② Addition or Subtraction

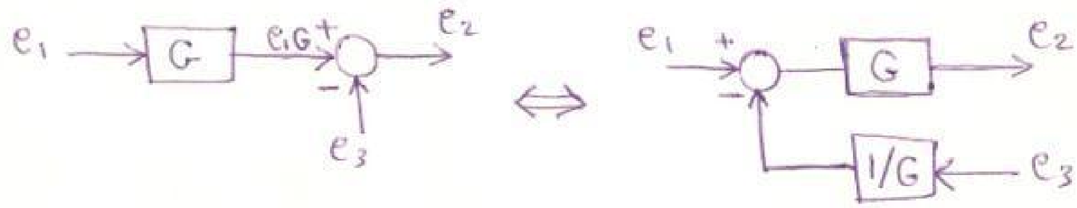




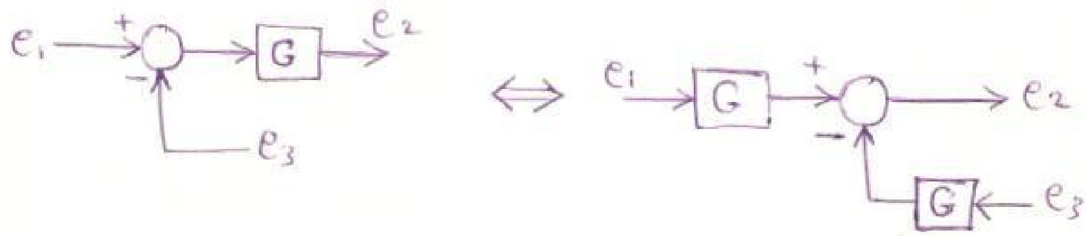
Rule ③ Closed – Loop



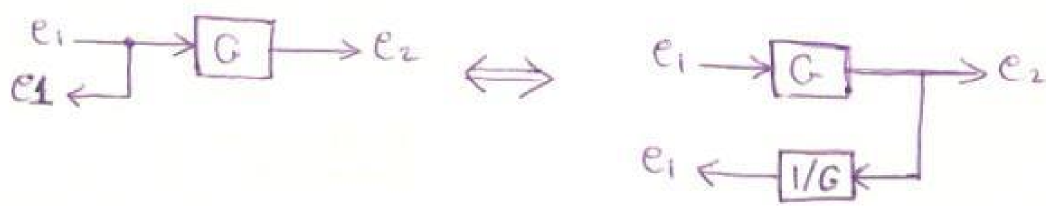
Rule ④ Moving a Summing point a head of a block



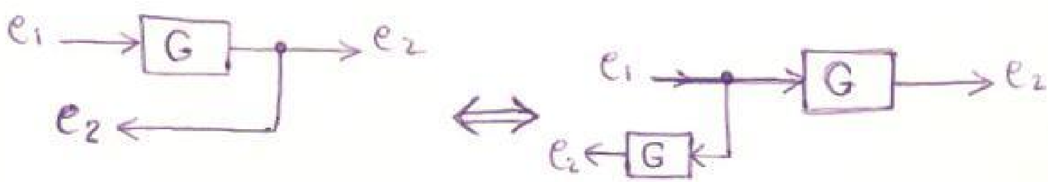
Rule ⑤ Moving a Summing point behind a block



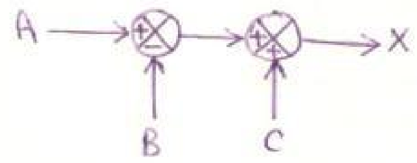
Rule ⑥ Moving a pick off point behind a block

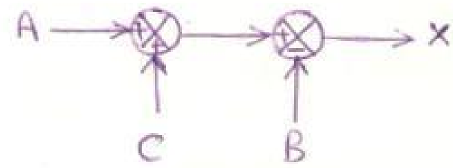


Rule ⑦ Moving a pick off point a head of a block

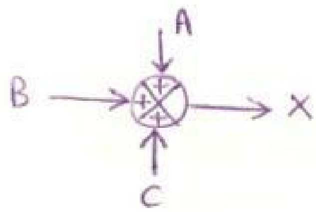


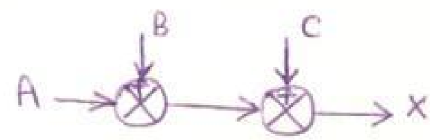
Rule ⑧





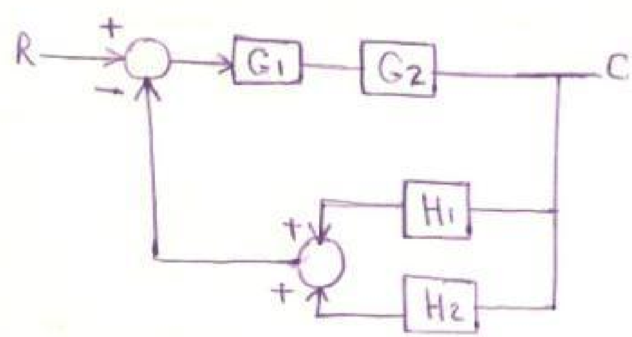
$X = A - B + C$ $\qquad$ $X = A + C - B$



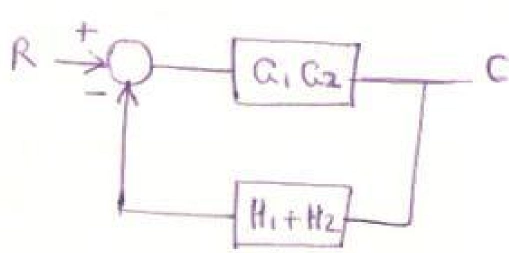


$X = A + B + C$ $\qquad$ $X = A + B + C$

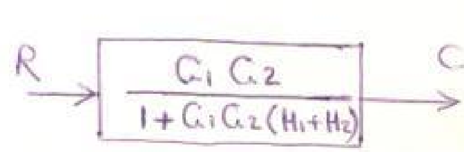
Ex1:



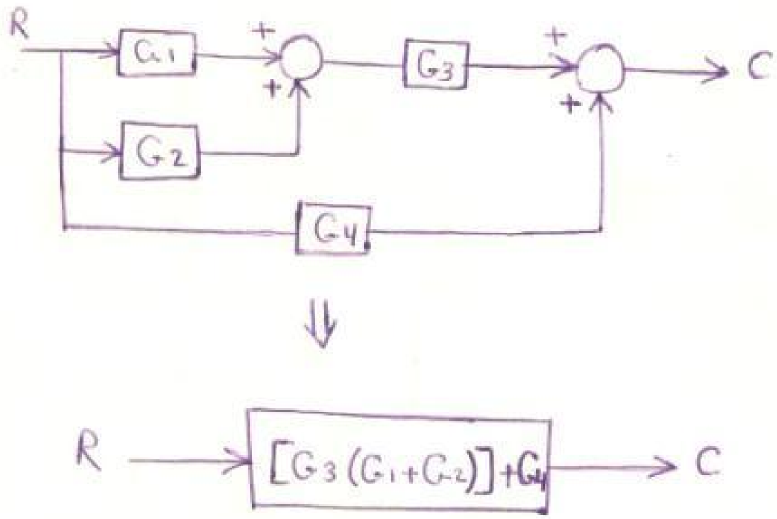
$\Downarrow$



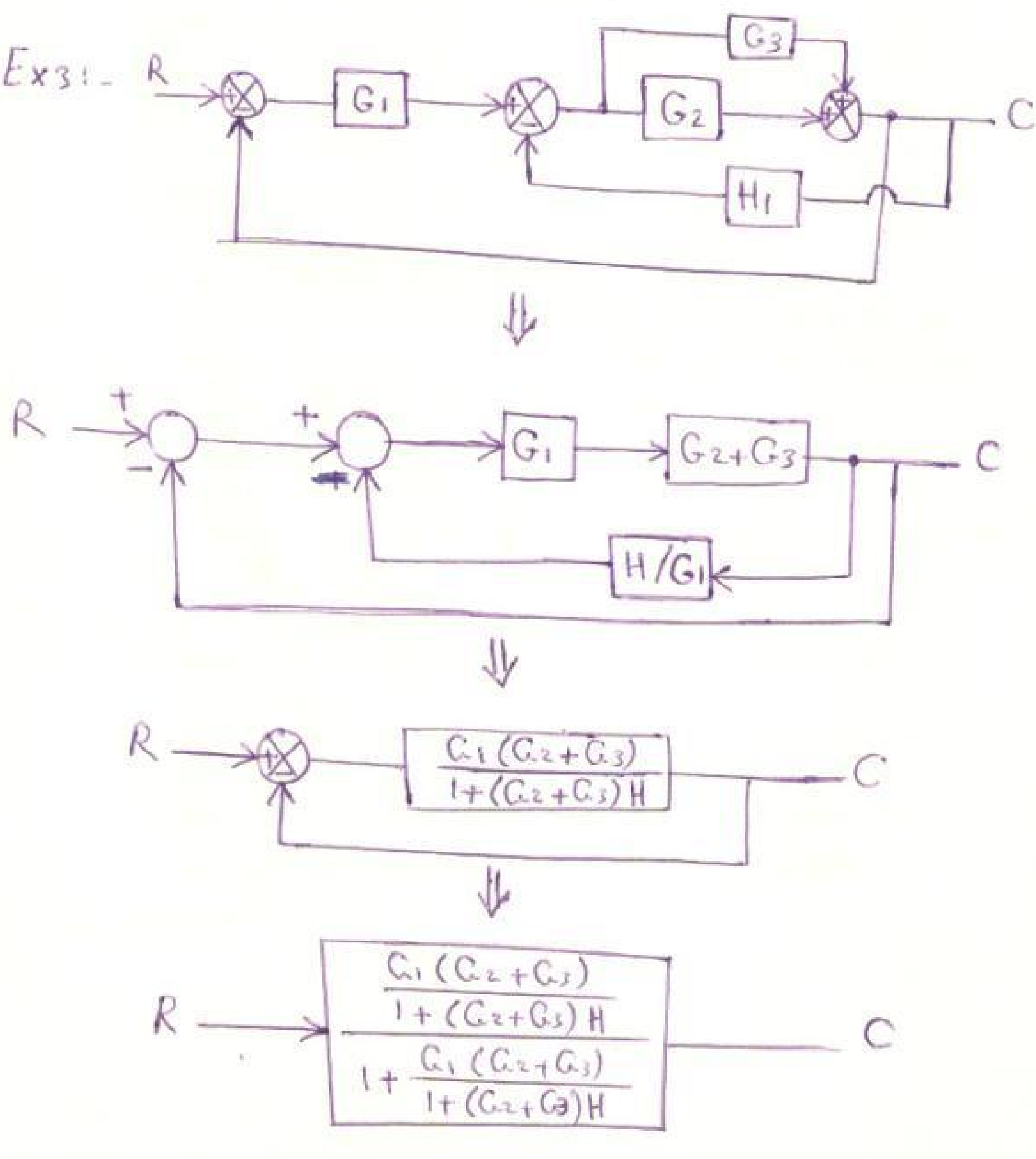
$\Longrightarrow$



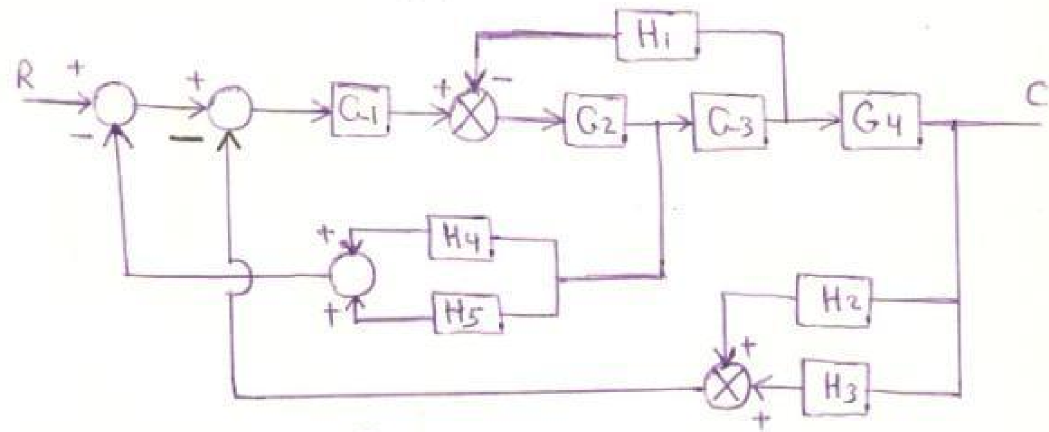
Ex2:



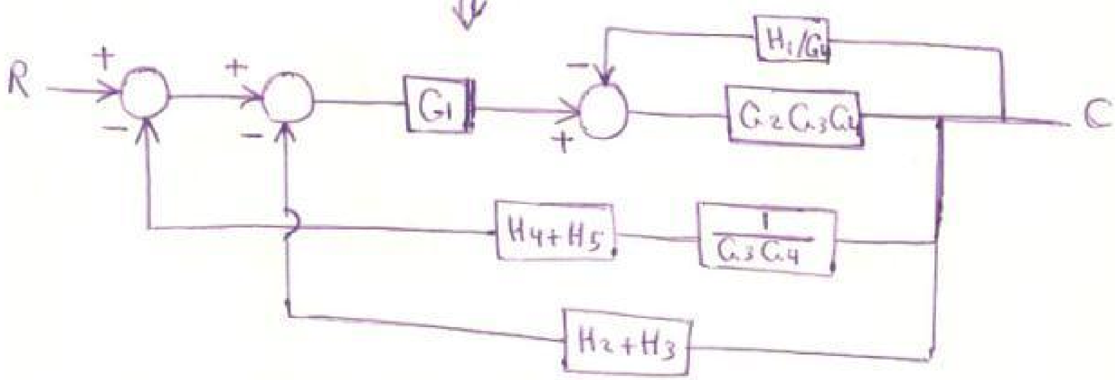
Ex3:-



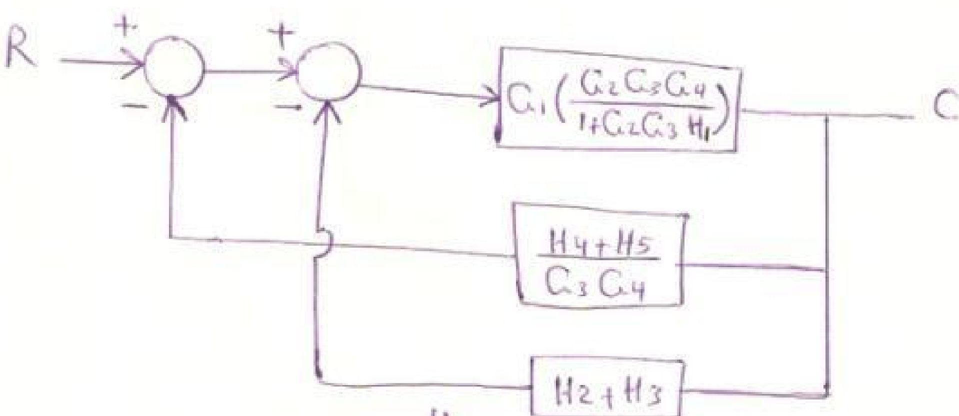
Ex4:


R
C
G1
G2
G3
G4
H1
H2
H3
H4
H5

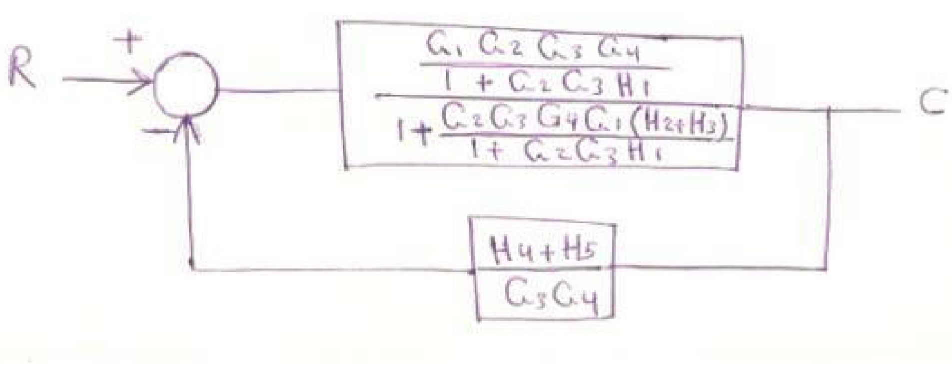
⇓

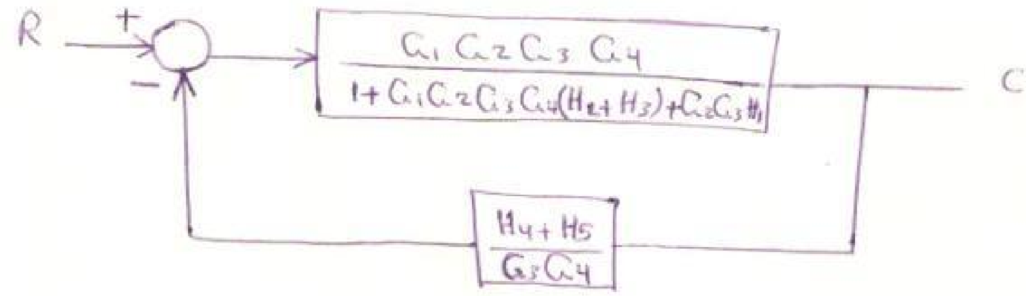

R
C
G1
H1/G4
G2G3G4
H4+H5
1/(G3G4)
H2+H3

⇓

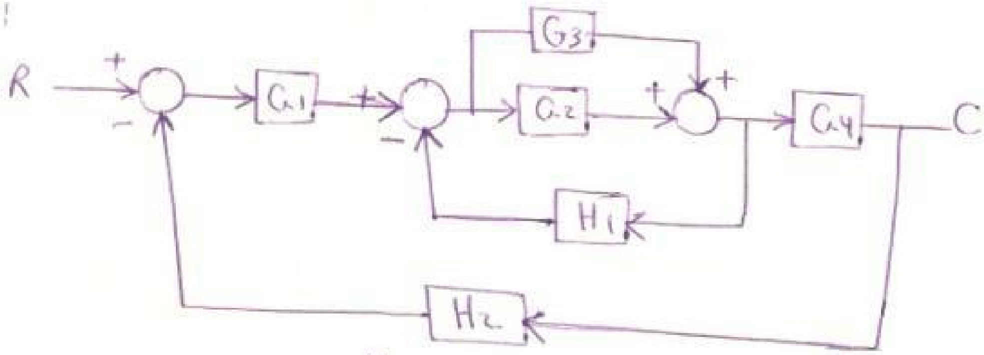

R
C
G1(G2G3G4/(1+G2G3H1))
(H4+H5)/(G3G4)
H2+H3

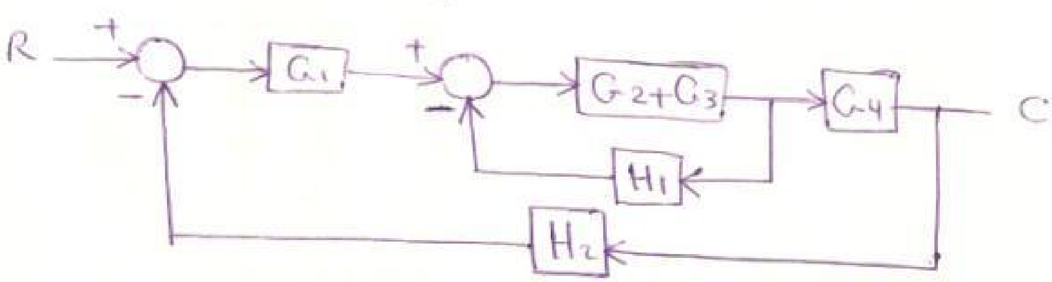
⇓

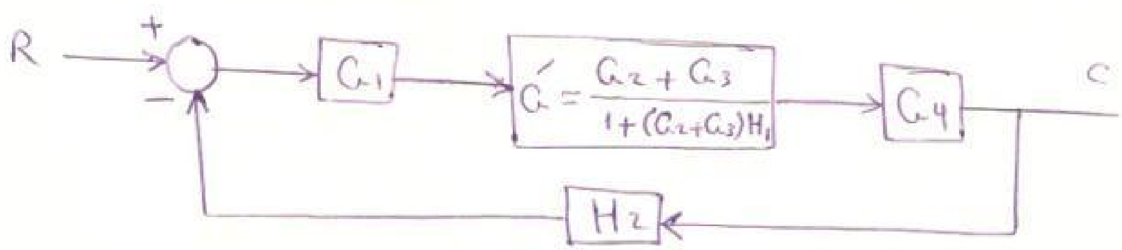

R
C
(G1G2G3G4/(1+G2G3H1)) / (1+G2G3G4G1(H2+H3)/(1+G2G3H1))
(H4+H5)/(G3G4)

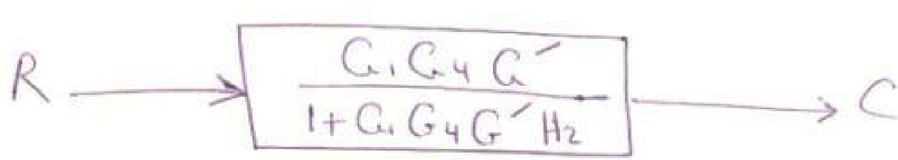

R
+
−
$\dfrac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 G_3 G_4 (H_2 + H_3) + G_2 G_3 H_1}$
C
$\dfrac{H_4 + H_5}{G_3 G_4}$

Ex5:


R
+
−
G_1
+
−
G_3
G_2
+
+
G_4
C
H_1
H_2

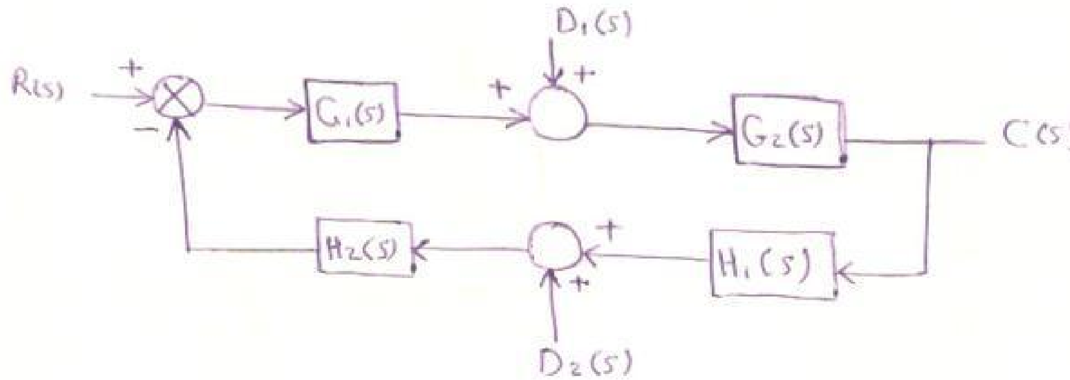

R
+
−
G_1
+
−
$G_2 + G_3$
G_4
C
H_1
H_2


R
+
−
G_1
$G' = \dfrac{G_2 + G_3}{1 + (G_2 + G_3) H_1}$
G_4
C
H_2


R
$\dfrac{G_1 G_4 G'}{1 + G_1 G_4 G' H_2}$
C

The principles of superposition

Ex1: Determine the output for the system shown below:

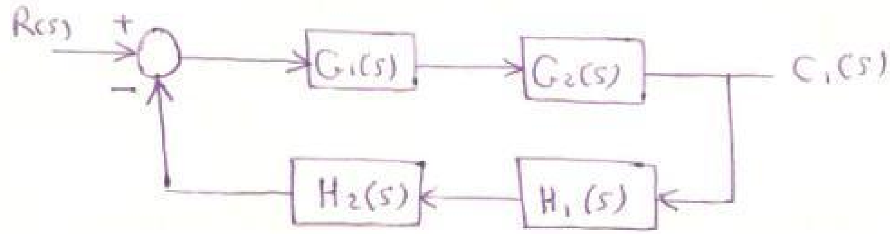


Solution: According to the principle of superposition, then, we must try to find the output considering one input at a time.

① - output due to input R(s)

Let $D_1(s) = 0$ & $D_2(s) = 0$

Hence the system becomes:

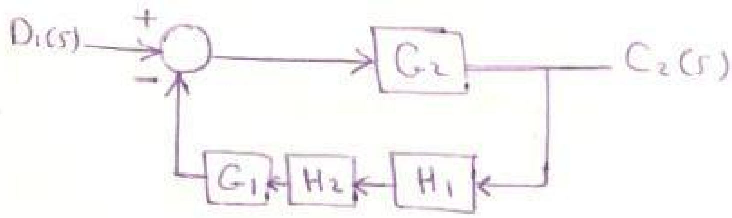


which gives $$\frac{C_1(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1+G_1(s)\,G_2(s)\,H_1(s)\,H_2(s)} \quad \text{......} \quad ①$$

② . output due to input $D_1(s)$

Let $R(s) = 0$ & $D_2(s) = 0$

We expect the system to reduce to

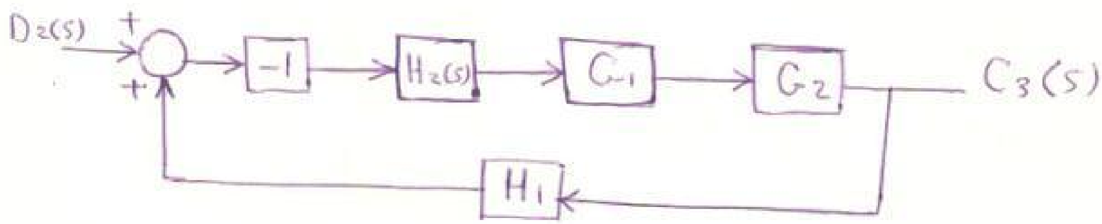


$$\therefore \frac{C_2(s)}{D_1(s)} = \frac{G_2}{1+G_1G_2H_1H_2} \quad \text{------ ②}$$

③ - output due to input $D_2(s)$

Let $R(s) = 0$ & $D_1(s) = 0$

Hence the system becomes

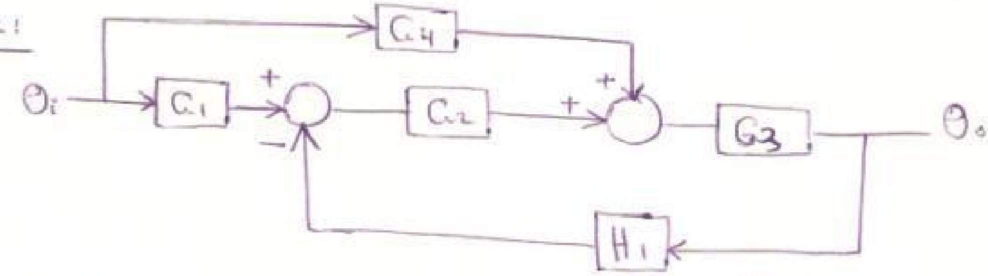


$$\therefore \frac{C_3(s)}{D_2(s)} = \frac{-G_1G_2H_2}{1+G_1G_2H_1H_2} \quad \text{------ ③}$$

$\therefore$ Total output $C = C_1 + C_2 + C_3$

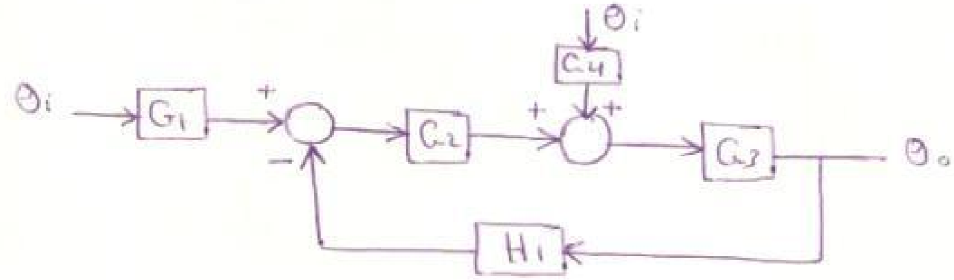
$$\text{i.e } C = \frac{R\,G_1G_2}{1+G_1G_2H_1H_2} + \frac{D_1\,G_2}{1+G_1G_2H_1H_2} - \frac{D_2\,G_1G_2H_2}{1+G_1G_2H_1H_2}$$

Ex 2:

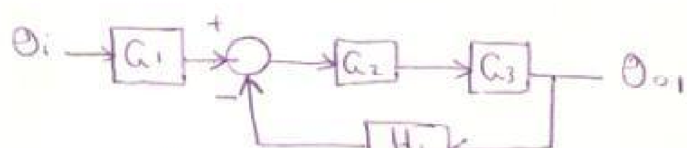


Find the relationship between θ_o & θ_i

Solution: Redraw the diagram as follows:



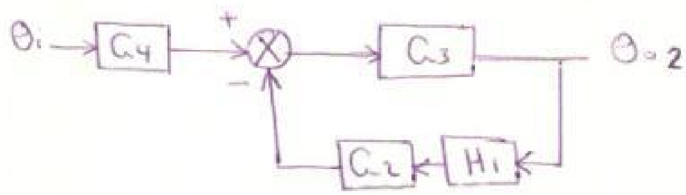
①. Let $\theta_i G_4 = 0$



$$\frac{\theta_{o1}}{G_1\theta_i} = \frac{G_2G_3}{1+G_2G_3H_1}$$

$$\therefore \frac{\theta_{o1}}{\theta_i} = \frac{G_1G_2G_3}{1+G_2G_3H_1} \quad \text{---} \ (1)$$

②. Let $\theta_i G_1 = 0$



$$\frac{\theta_{o2}}{\theta_i G_4} = \frac{G_3}{1+G_3G_2H_1}$$

$$\frac{\theta_{o2}}{\theta_i} = \frac{G_3G_4}{1+G_2G_3H_1} \quad \text{---} \ (2)$$

$$\therefore \text{total } \frac{\theta_o}{\theta_i} = \frac{\theta_{o1}}{\theta_i} + \frac{\theta_{o2}}{\theta_i} = \frac{G_1G_2G_3}{1+G_2G_3H_1} + \frac{G_3G_4}{1+G_2G_3H_1} = \frac{G_3(G_1G_2+G_4)}{1+G_2G_3H_1}$$