Signal Flow Graph

It is a method for presenting a control system, it is Like a network, it consists of: Junction points called (Nodes) and Directed Line segments Called (Branches).

Suppose we have

$C(s) = G(s)\,R(s)$

$C = GR$

The direction of the branch is from R to C.

where

- C -- is the output Node.
- R -- is the input Node.
- G -- is called as the transmittance or Gain.

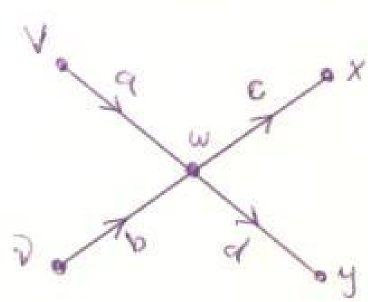
There are different types of Nodes:

- Source Nodes ---- outgoing Nodes.
- Sink Nodes ---- incomming Nodes.
- mixed Nodes ---- incomming and outgoing Nodes.

V, ν -- are Source nodes.

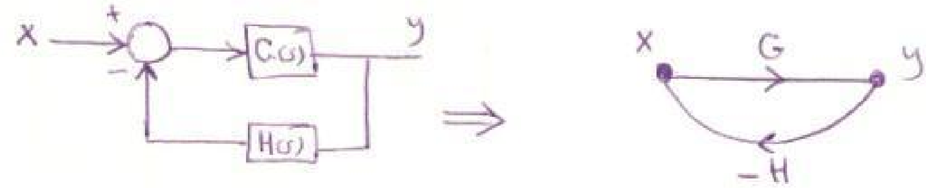
W -- mixed nodes.

X, y -- are sink nodes.

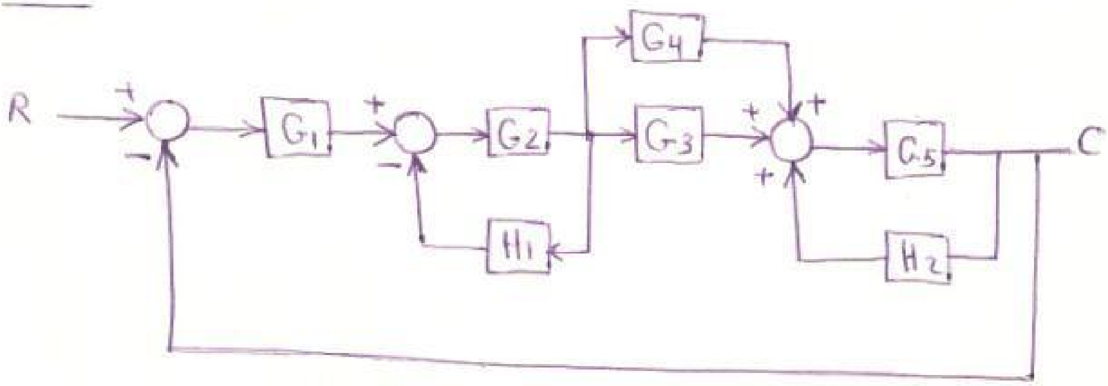


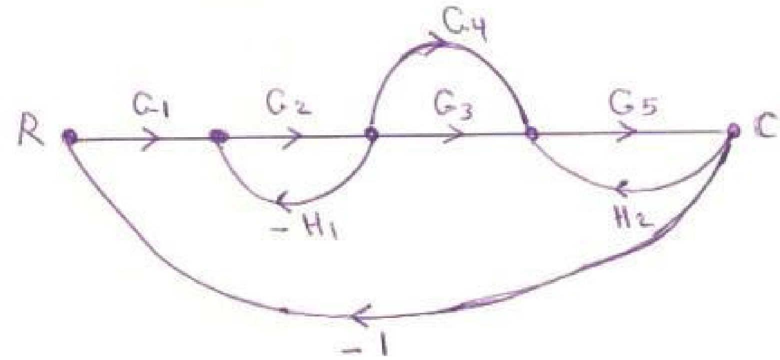
$$w = av + b\vartheta$$
$$x = cw$$
$$y = dw$$

Also, the closed Loop control system represented as follows:



<u>Ex:</u>





<u>Paths</u>

1 - $G_1 G_2 G_3 G_5$

2 - $G_1 G_2 G_4 G_5$

<u>Loops</u>

1 - $G_1 G_2 G_3 G_5 (-1)$

2 - $G_1 G_2 G_4 G_5 (-1)$

3 - $-G_2 H_1$

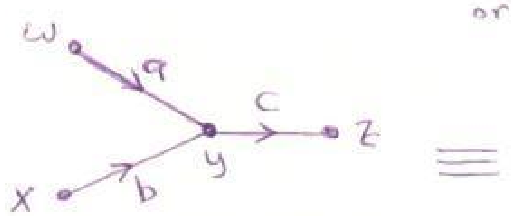
4 - $G_5 H_2$

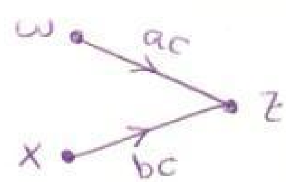
Some of the rules governing flow Graphs

(a) Series paths (These may be Combined)

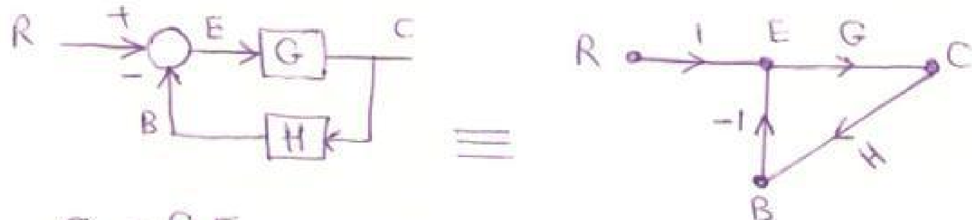
(b) Parallel paths (Addition of transmittance or Gain)

(c) Node absorption : It is a node which represented a Variable which is neither asource or a Sink. This may be elliminated.





(d) Feedback paths

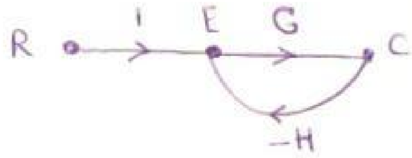


$C = GE$

$B = HC$

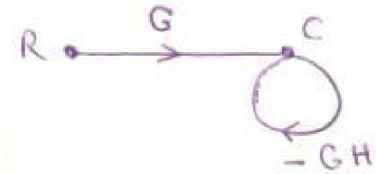
$E = R - B$

Elliminate B we get

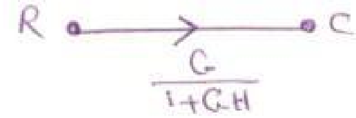


Elliminate E we get

$C = G(R - B)$
$\quad = GR - GHC$



The Final Simplification is



The over all transmittance or T.F can be obtained using Mason's Rule.

Mason's Rule

It is an algorithim use to Calculate the total transfer function $T(s)$ for a control system represented by a signal flow graph according to the equation:

$$T(s) = \frac{1}{\Delta} \sum_{i=1}^{n} P_i \Delta_i$$

Where

T -- is the total T.F of a system.

P_i -- Path gain or transmittance of ith forward path.

Δ -- determinant of graph.

$$\Delta = 1 - \sum_{i=1}^{n} L_i + \sum_{i,j=1}^{n} L_i L_j - \sum_{i,j,k=1}^{n} L_i L_j L_k + \sum \cdots$$

where

$\sum_{i=1}^{n} L_i$ .... Sum of all individual Loop gains.

$\sum_{i,j=1}^{n} L_i L_j$ ... Sum of gain products of all possible combinations of two non touching Loops.

$\sum_{i,j,k=1}^{n} L_i L_j L_k$ .... Sum of gain products of all possible combinations of three nontouching Loops.

Δi ... Cofactor of the ith forward path determinant of the graph with the Loops touching the ith forward path removed, that is the cofactor Δi is obtained from Δ by removing the Loops that touch path P_i.

<u>Note</u>

* The summations are taken over all possible paths from input to output.
* In Δ the multiplication of touch loop equal zero.

<u>Ex:</u> Find Δ we have L_1, L_2, L_3 that not touch each other.

$$\sum_{i=1}^{3} L_i = L_1 + L_2 + L_3$$

$$\sum_{\substack{i=1\\ j=1}}^{3} L_i L_j = L_1 L_2 + L_1 L_3 + L_2 L_3$$

$$\sum_{\substack{i=1\\ j=1\\ k=1}}^{3} L_i L_j L_k = L_1 L_2 L_3$$

$$\therefore \Delta = 1 - (L_1 + L_2 + L_3) + L_1L_2 + L_1L_3 + L_2L_3 - (L_1L_2L_3)$$

Ex: Find Δ we have L_1, L_2, L_3, L_4 that non touch-

$$\sum_{i=1}^{4} L_i = L_1 + L_2 + L_3 + L_4$$

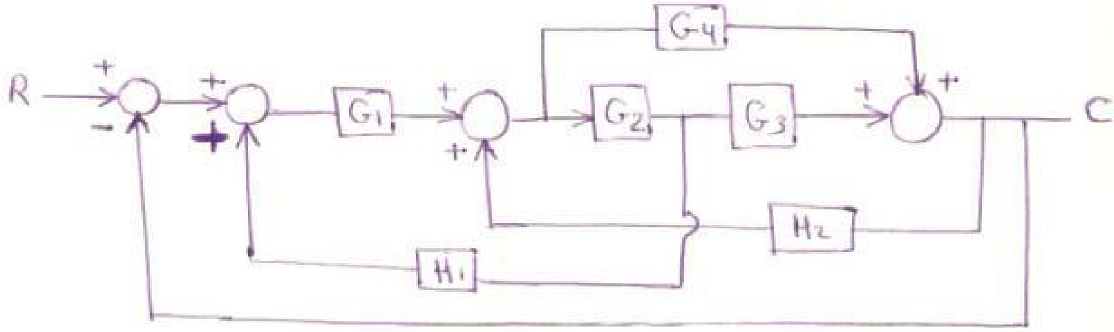
$$\sum_{\substack{i=1 \\ j=1}}^{4} L_iL_j = L_1L_2 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4 + L_3L_4$$

$$\sum_{i,j,k=1}^{4} L_iL_jL_k = L_1L_2L_3 + L_1L_2L_4 + L_2L_3L_4 + L_1L_3L_4$$

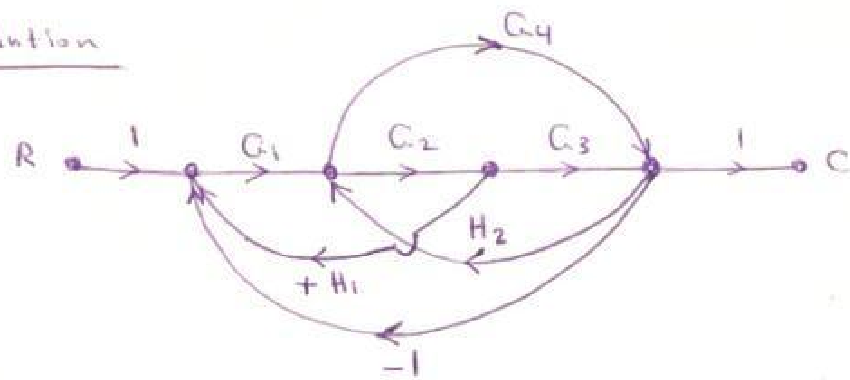
$$\sum_{i,j,k,m=1}^{4} L_iL_jL_kL_m = L_1L_2L_3L_4$$

$$\therefore \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2 + L_1L_3 + L_1L_4 +$$
$$+ L_2L_3 + L_2L_4 + L_3L_4 - (L_1L_2L_3 + L_1L_2L_4 +$$
$$+ L_2L_3L_4 + L_1L_3L_4) + L_1L_2L_3L_4$$

Ex₁: Find the T.F $\left(\frac{C}{R}\right)$ for the control system given by:



Solution



Paths

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4$

Loops

$L_1 = + G_1 G_2 H_1$

$L_2 = - G_1 G_2 G_3$

$L_3 = G_2 G_3 H_2$

$L_4 = - G_1 G_4$

$L_5 = G_4 H_2$

$$\sum_{i=1}^{5} L_i = L_1 + L_2 + L_3 + L_4 + L_5$$
$$= G_1G_2H_1 - G_1G_2G_3 + G_2G_3H_2 - G_1G_4 + G_4H_2$$

$$\sum_{\substack{i=1\\j=1}}^{5} L_iL_j = L_1L_2 + L_1L_3 + L_1L_4 + L_1L_5 + L_2L_3 + L_2L_4 + L_2L_5 + L_3L_4 + L_3L_5 + L_4L_5$$
$$= 0$$

$$\sum_{i,j,k=1}^{5} L_iL_jL_k = 0$$

$$\sum_{i,j,k,m=1}^{5} L_iL_jL_kL_m = 0$$

$$\sum_{i,j,k,m,n=1}^{5} L_iL_jL_kL_mL_n = 0$$

① - Forward path P_1

$\Delta_i = 1 - \Sigma + \Sigma - \Sigma$

$\Delta_1 = 1$

$P_1\Delta_1 = G_1G_2G_3$

② - Forward path P_2

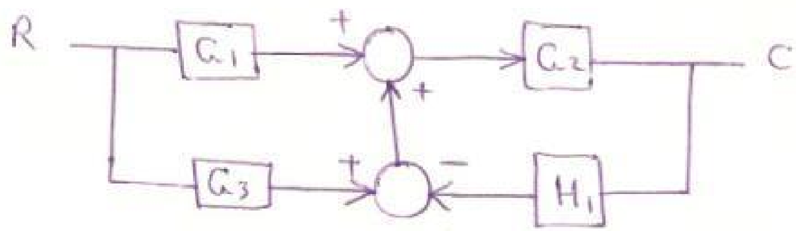
$\Delta_2 = 1$

$P_2\Delta_2 = G_1G_4$

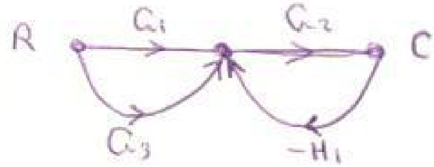
$$\therefore \frac{C}{R} = \frac{\sum P_i\Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_1G_4}{1 - G_1G_2H_1 + G_1G_2G_3 - G_2G_3H_2 + G_1G_4 - G_4H_2}$$

Ex2: Determine $\left(\frac{C}{R}\right)$ for the system given by:



Solution



Paths:

$P_1 = G_1 G_2$

$P_2 = G_3 G_2$

Loops:

$L_1 = -G_2 H_1$

$\Delta = 1 - (-G_2 H_1)$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$\frac{C}{R} = \frac{\sum P_i \Delta_i}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 + G_3 G_2}{1 + G_2 H_1}$$