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State Reduction

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Third stage
2020

State Reduction

- State Reduction is based on:

Two states are equivalent if, for each member of the set of inputs, they give exactly the same output and send the circuit either to the same state or to an equivalent state.

If two states are equivalent, one can be eliminated without effecting the behavior of the FSM.

- Several algorithms exist:
  - Row matching method.
  - Implication table method.

Equivalent States

Equivalent States

- Two states are equivalent if- merging them does not change machines external behaviour.
- *Or*, two states are equivalent if-

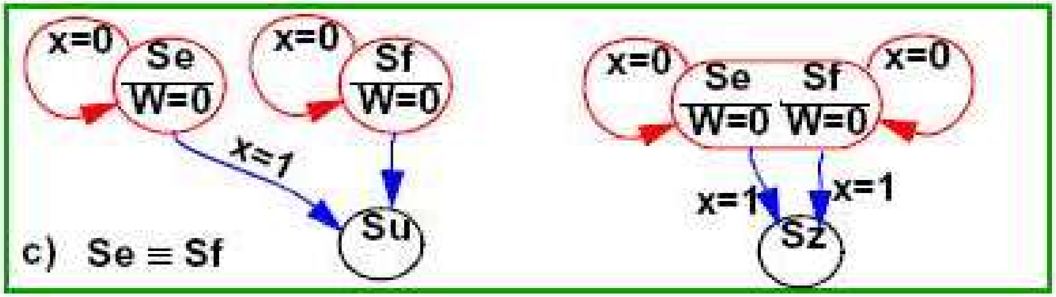
1. There outputs are equivalent.
2. Their next states are the same, or equivalent, or would be equivalent if the states were merged, **for all inputs.**

Examples a) and b)

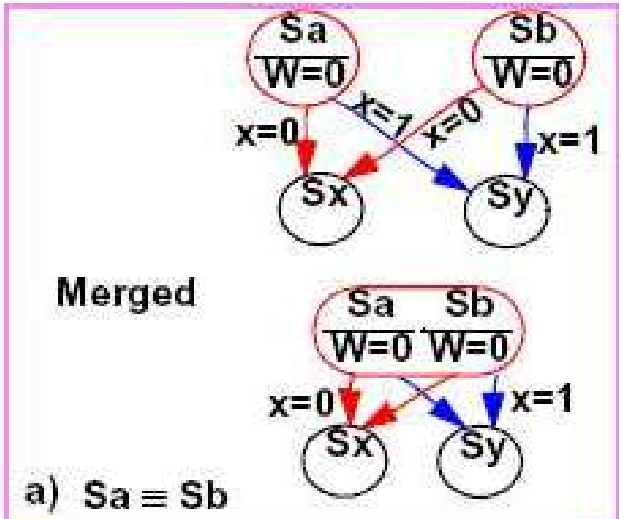
Next states the same
Outputs the same

Example c)

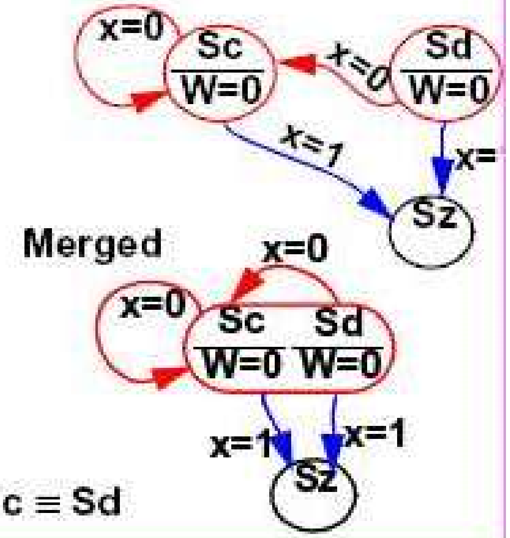
Next states are equivalent if present states merged.
Outputs are the same



c) Se ≡ Sf



a) Sa ≡ Sb



b) Sc ≡ Sd

We now techniques for determining equivalent states in completely specified sequential circuits:

1. Inspection.
2. Partitioning.
3. The implication table.

inspection: two states are equivalent by inspection when the next state row are identical or when the next state rows are identical except for the self loop back entries.

Example 1:

	X	
	0	1
A	B/0	C/1
B	C/0	A/1
C	D/1	B/0
D	C/0	A/1

	X	
	0	1
A	B/0	C/1
B	C/0	A/1
C	B/1	B/0

D=B

HW1:

	X	
	0	1
A	B/0	C/1
B	B/0	A/1
C	D/1	B/0
D	D/0	A/1

	X	
	0	1
A	B/0	C/1
B	C/0	A/1
C	B/1	B/0

HW2:

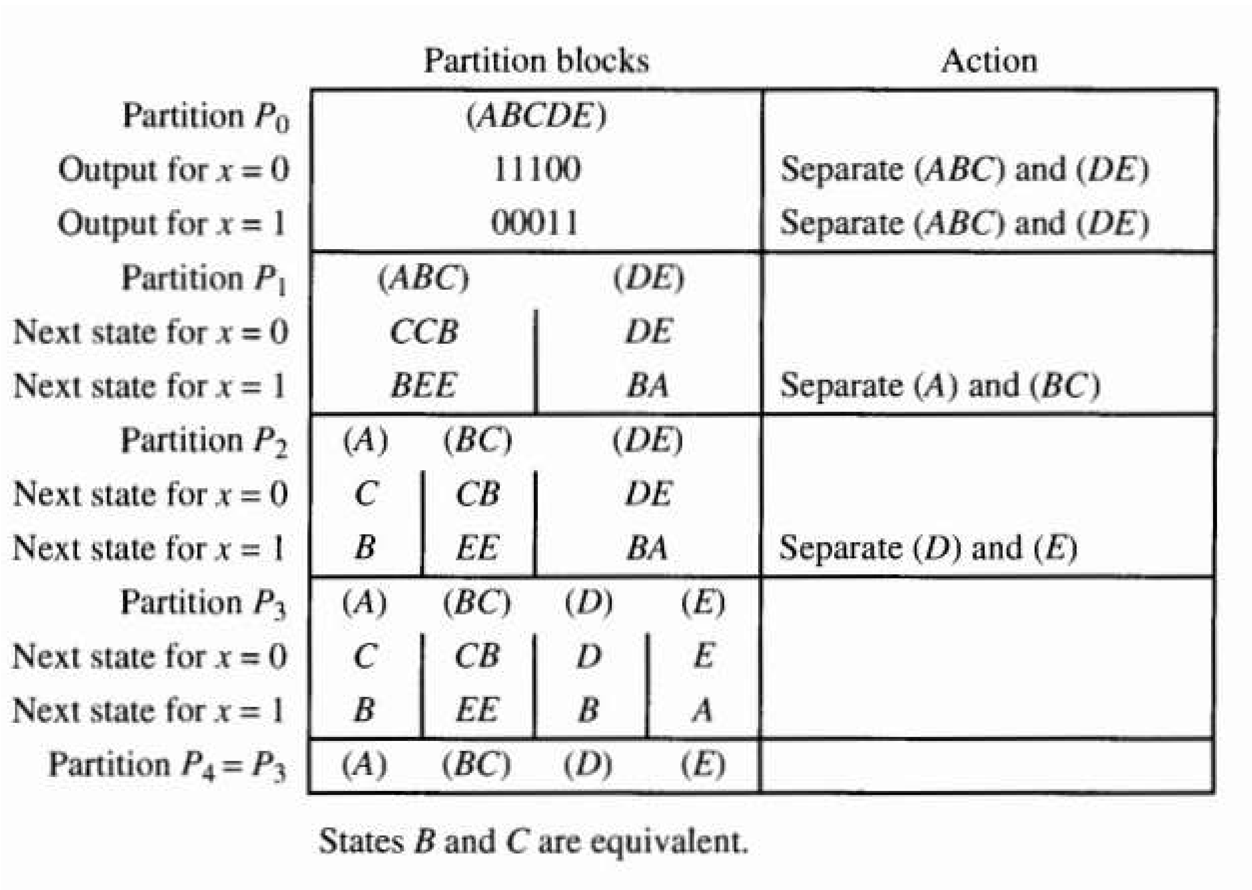
	X	
	0	1
A	B/0	C/1
B	D/0	A/1
C	D/1	B/0
D	B/0	A/1

	X	
	0	1
A	B/0	C/1
B	C/0	A/1
C	B/1	B/0

Partitioning:
Example:

X

	0	1
A	C/1	B/0
B	C/1	E/0
C	B/1	E/0
D	D/0	B/1
E	E/0	A/1



	Partition blocks	Action
Partition P_0	$(ABCDE)$	
Output for $x = 0$	11100	Separate (ABC) and (DE)
Output for $x = 1$	00011	Separate (ABC) and (DE)
Partition P_1	(ABC) (DE)	
Next state for $x = 0$	CCB DE	
Next state for $x = 1$	BEE BA	Separate (A) and (BC)
Partition P_2	(A) (BC) (DE)	
Next state for $x = 0$	C CB DE	
Next state for $x = 1$	B EE BA	Separate (D) and (E)
Partition P_3	(A) (BC) (D) (E)	
Next state for $x = 0$	C CB D E	
Next state for $x = 1$	B EE B A	
Partition $P_4 = P_3$	(A) (BC) (D) (E)	

States B and C are equivalent.

H.W: Use the three methods to reduce the state table shown in three tables

	x	
	0	1
A	E/0	D/0
B	A/1	F/0
C	C/0	A/1
D	B/0	A/0
E	D/1	C/0
F	C/0	D/1
G	H/1	G/1
H	C/1	B/1

(a)

	x	
	0	1
A	A/0	B/0
B	H/1	C/0
C	E/0	B/0
D	C/1	D/0
E	C/1	E/0
F	F/1	G/1
G	B/0	F/0
H	H/1	C/0

	$x_1 x_2$			
	00	01	11	10
A	D/0	D/0	F/0	A/0
B	C/1	D/0	E/1	F/0
C	C/1	D/0	E/1	A/0
D	D/0	B/0	A/0	F/0
E	C/1	F/0	E/1	A/0
F	D/0	D/0	A/0	F/0
G	G/0	G/0	A/0	A/0
H	B/1	D/0	E/1	A/0

State Reduction:1-Row Matching *Method*

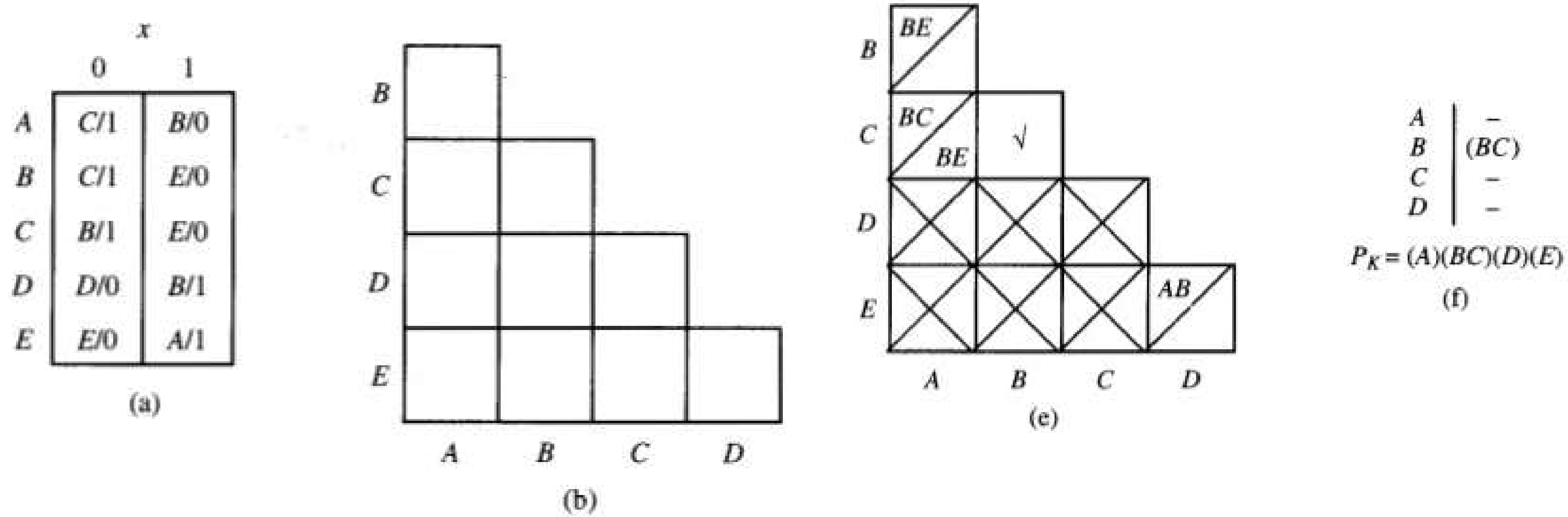
- “Row Matching” is based on the state-transition table:

If two states have the same output, *and* both transition to the same next state, *or* both transition to each other, *or* both self-loop, then **they are equivalent**.

- Combine the equivalent states into a new renamed state.
- Repeat until no more states are combined
- The “row matching” method is not guaranteed to result in the optimal solution in all cases, because it only
- looks at pairs of states.
- For example:

Present State	Next State		Output	
	X=0	X=1	X=0	X=1
S_0	S_1	S_2	0	0
S_1	S_3	S_4	0	0
S_2	S_5	S_6	0	0
S_3	S_7	S_8	0	0
S_4	S_9	S'_{10}	0	0
S_5	S_{11}	S'_{10}	0	0
S_6	S_{13}	S_{14}	0	0
S_7	S_0	S_0	0	0
S_8	S_0	S_0	0	0
S_9	S_0	S_0	0	0
S'_{10}	S_0	S_0	1	0
S_{11}	S_0	S_0	0	0
S_{13}	S_0	S_0	0	0
S_{14}	S_0	S_0	0	0

State Reduction: *Implication Chart (Table) Method*



	x = 0	1
A	C/1	B/0
B	C/1	E/0
C	B/1	E/0
D	D/0	B/1
E	E/0	A/1

(a)

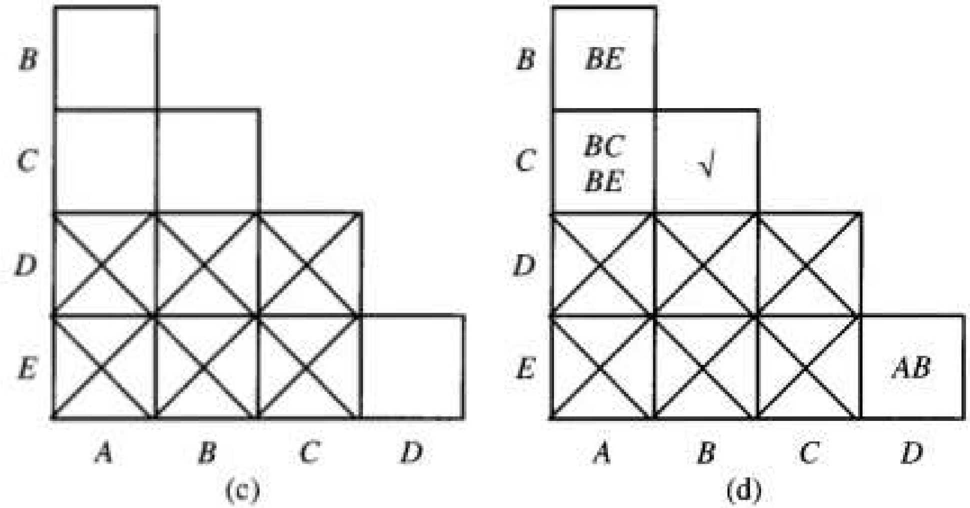
(b)

(e)

A	–
B	(BC)
C	–
D	–

$P_K = (A)(BC)(D)(E)$

(f)



(c)

(d)

The implication table for a five-state

A. State table.
B. Implication table.
C. Output partitioning.
D. Implied pairs.
E. Completed. Table.
F. Equivalence partition.

Example :

	x = 0	x = 1
A	E/0	D/0
B	A/1	F/0
C	C/0	A/1
D	B/0	A/0
E	D/1	C/0
F	C/0	D/1
G	H/1	G/1
H	C/1	B/1

(a)

	A	B	C	D	E	F	G
B	✗						
C	✗	✗					
D	BE	✗	✗				
E	✗	AD CF	✗	✗			
F	✗	✗	AD	✗	✗		
G	✗	✗	✗	✗	✗	✗	
H	✗	✗	✗	✗	✗	✗	CH BG

(b)

A	(AD)
B	(BE)
C	(CF)
D	–
E	–
F	–
G	–

$P_K = (AD)(BE)(CF)(G)(H)$

(c)

H.W

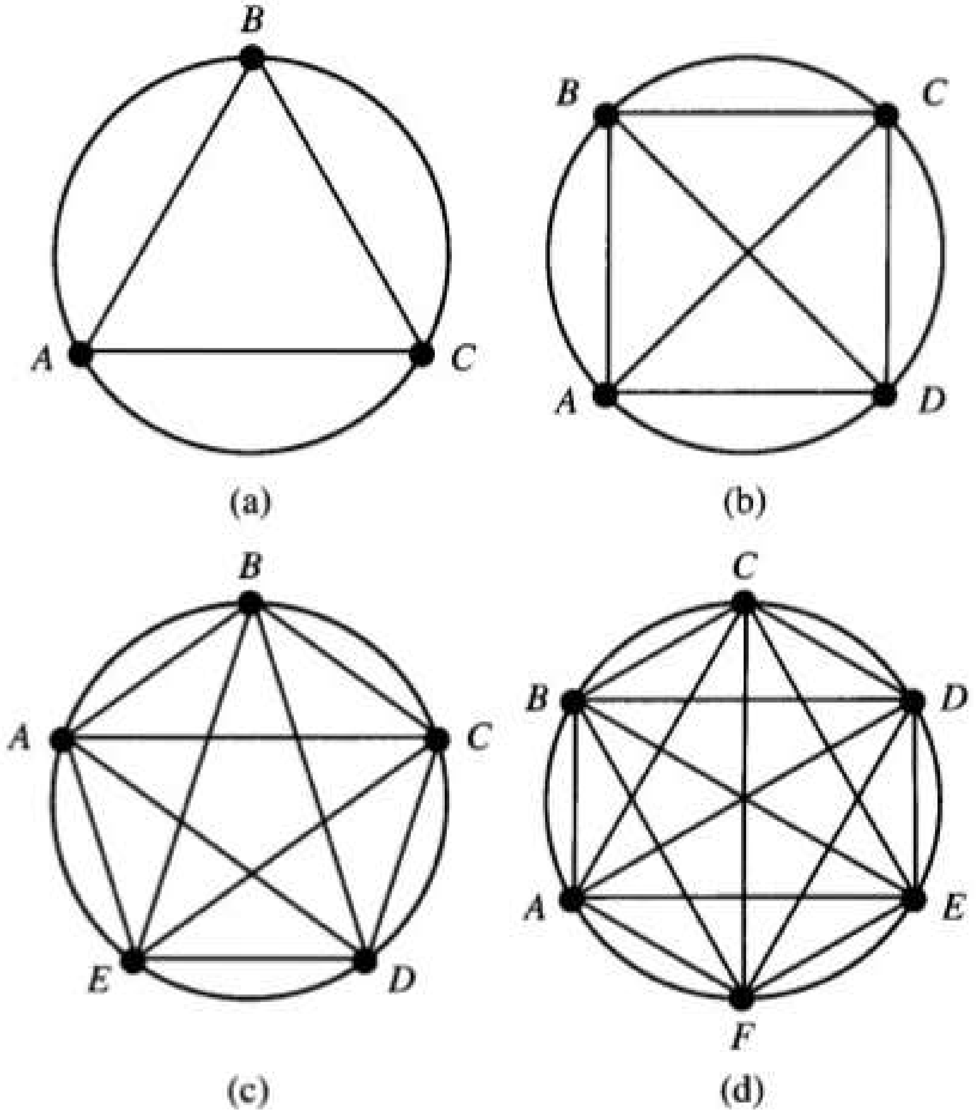
	$x_1 x_2$			
	00	01	11	10
A	*D*/0	*D*/0	*F*/0	*A*/0
B	*C*/1	*D*/0	*E*/1	*F*/0
C	*C*/1	*D*/0	*E*/1	*A*/0
D	*D*/0	*B*/0	*A*/0	*F*/0
E	*C*/1	*F*/0	*E*/1	*A*/0
F	*D*/0	*D*/0	*A*/0	*F*/0
G	*G*/0	*G*/0	*A*/0	*A*/0
H	*B*/1	*D*/0	*E*/1	*A*/0

	x	
	0	1
A	*A*/–	*C*/1
B	*B*/–	*A*/–
C	*G*/–	*E*/0
D	*C*/1	*C*/–
E	*A*/1	*C*/–
F	*D*/–	*A*/–
G	*G*/–	*G*/–
H	*H*/–	*D*/–

Merger Diagrams

The process of finding the maximal compatible sets of states from the compatible pairs derived from the implication table is aided measurably by a graphical technique

- Rule 1: Make each maximal set as large as possible.
- Rule 2:Each state of the maximal set must be interconnected with every other state in the set by a line segment.

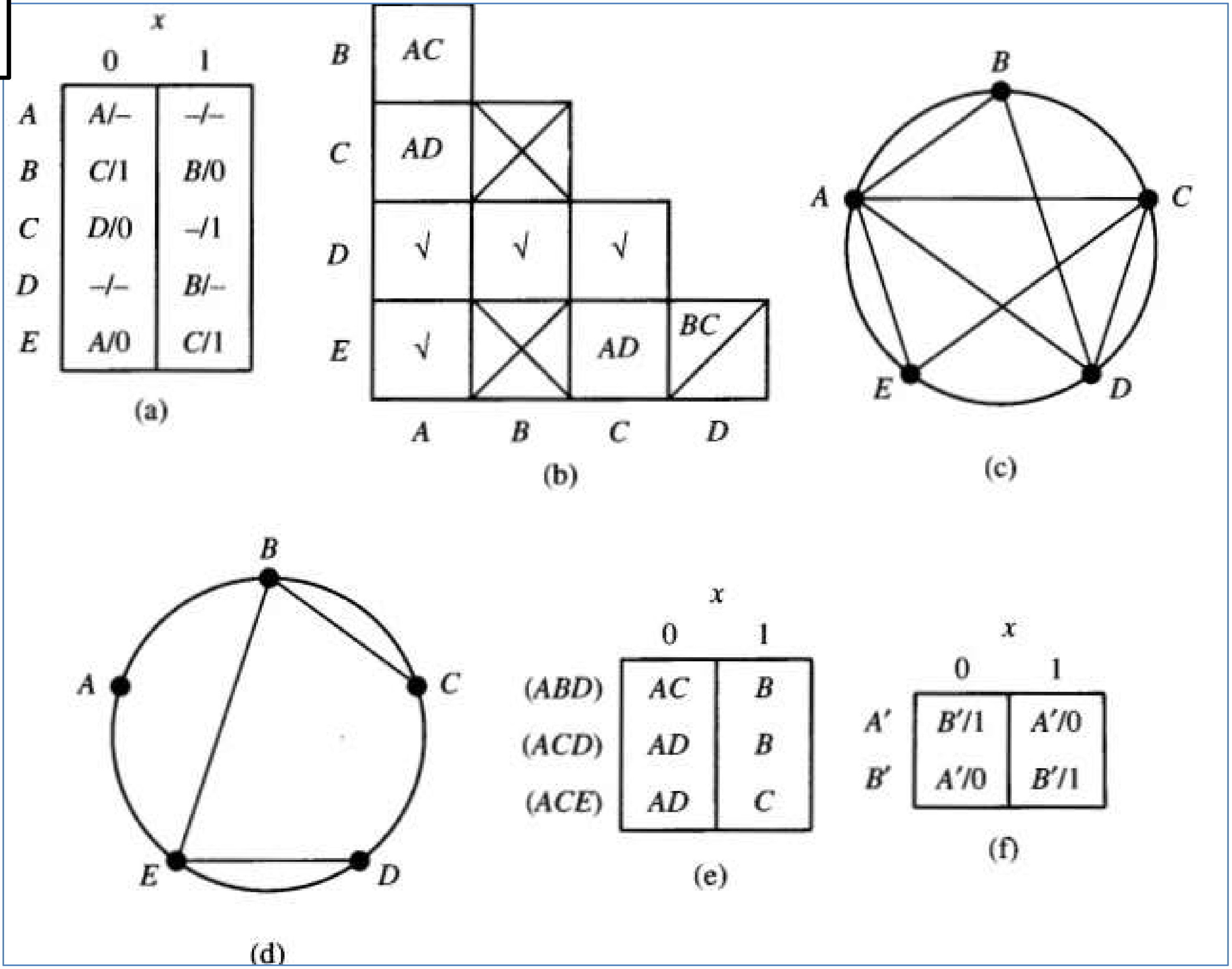


(a) (b)

(c) (d)

The merger diagram: a)three states. b)four states c) five states d) six states

Example:



	x = 0	x = 1
A	A/–	–/–
B	C/1	B/0
C	D/0	–/1
D	–/–	B/–
E	A/0	C/1

(a)

	A	B	C	D
B	AC			
C	AD	✗		
D	√	√	√	
E	√	✗	AD	BC

(b)

(c)

(d)

	x = 0	x = 1
(ABD)	AC	B
(ACD)	AD	B
(ACE)	AD	C

(e)

	x = 0	x = 1
A′	B′/1	A′/0
B′	A′/0	B′/1

(f)

A’=ABD
B’=ACE

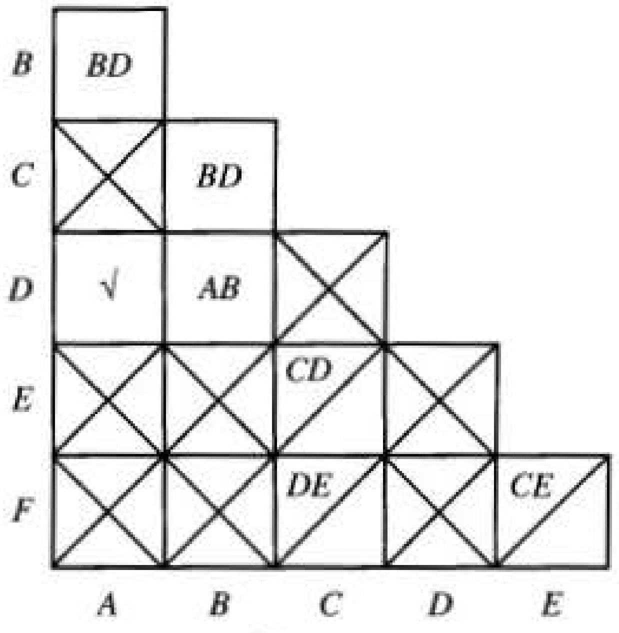
a) State table b)implication table c)maximal compatibles
d) Maximal incompatibles c)closure table f)reduced state table

Example:

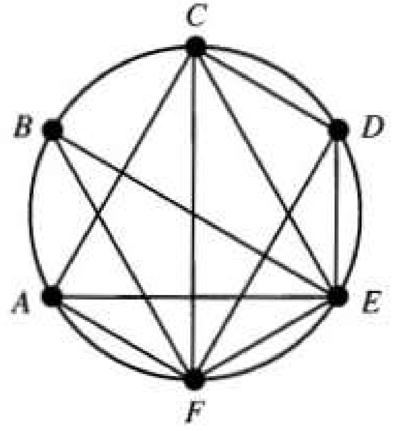
ABD=A’
BC=B’
E=C’
F=D’

	x	
	0	1
A	B/1	D/0
B	–/–	B/0
C	E/0	D/–
D	B/1	A/0
E	–/–	C/1
F	–/0	E/1

(a)



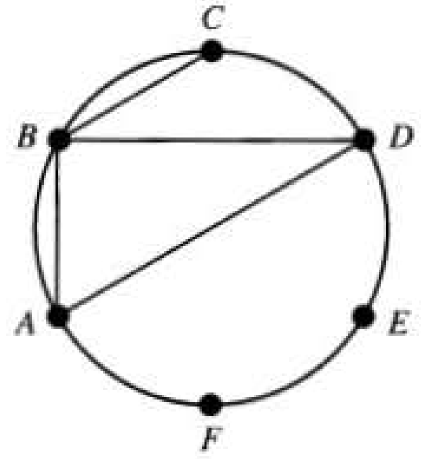
(b)



(e)

	x	
	0	1
A′	A′, B′/1	A′/0
B′	C′/0	A′/0
C′	–/–	B′/1
D′	–/0	C′/1

(f)



(c)

	x	
	0	1
(ABD)	B	ABD
(BC)	E	BD
(E)	–	C
(F)	–	E

(d)

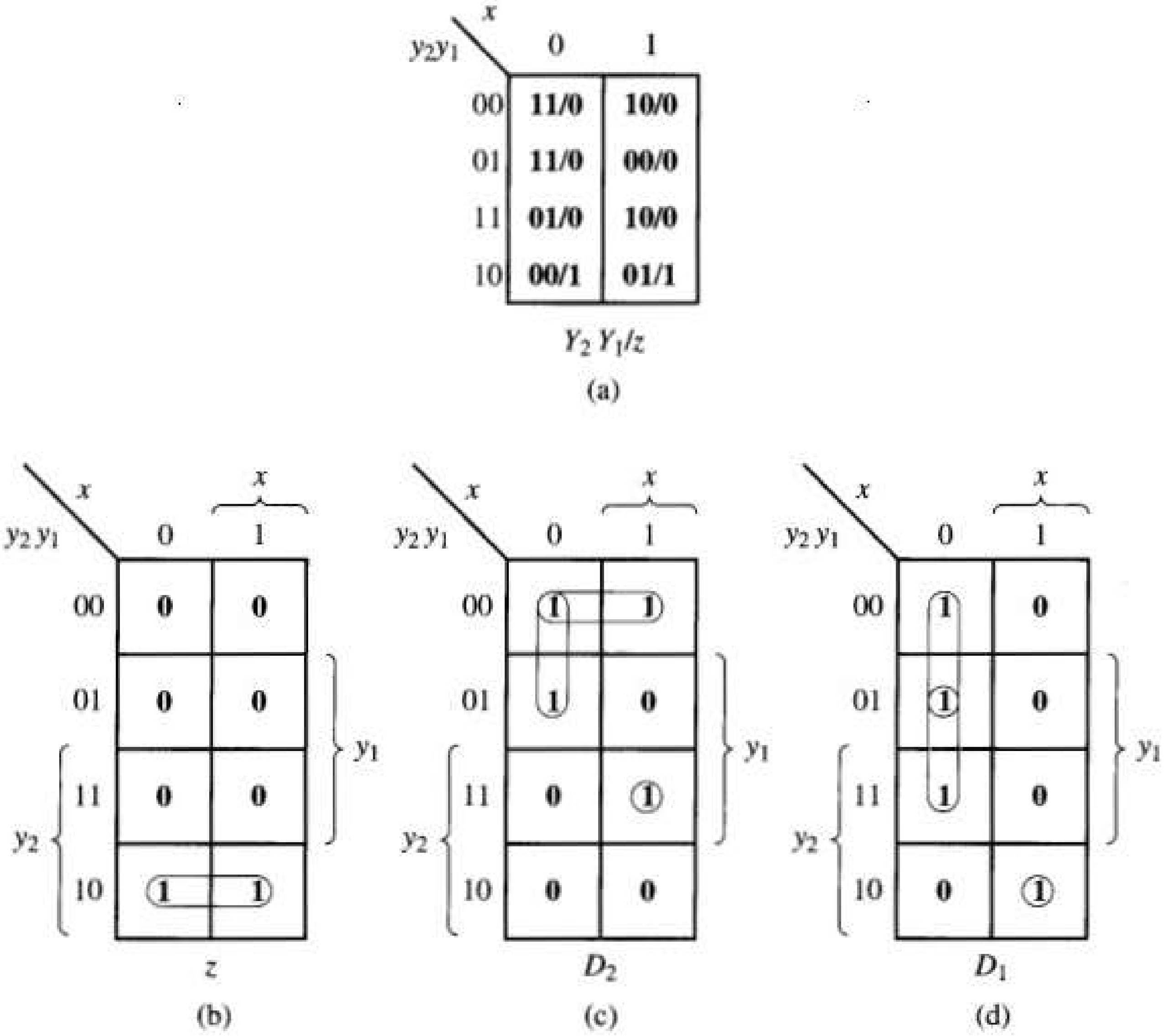
a) State table b)implication table c)maximal compatibles
e) Maximal incompatibles d)closure table f)reduced state table

State Assignment

	Assignments		
	1	2	3
States	y_1y_2	y_1y_2	y_1y_2
A	00	00	00
B	01	11	10
C	11	01	01
D	10	10	11

Example:

	x = 0	x = 1
A	C/0	D/0
B	C/0	A/0
C	B/0	D/0
D	A/1	B/1



(a)

(b)

(c)

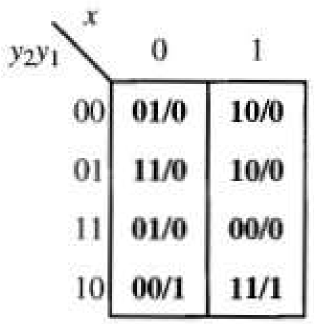
(d)

Assignment 1.

$$D_2 = \bar{y}_1\bar{y}_2 + \bar{x}\bar{y}_2 + xy_1y_2$$
$$D_1 = \bar{x}\bar{y}_2 + \bar{x}y_1 + x\bar{y}_1y_2$$
$$z = \bar{y}_1y_2$$

Example:

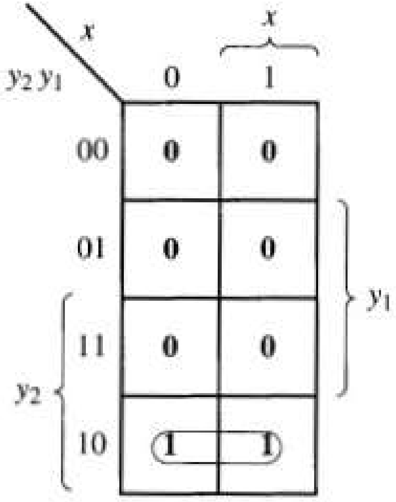
	x = 0	1
A	*C*/0	*D*/0
B	*C*/0	*A*/0
C	*B*/0	*D*/0
D	*A*/1	*B*/1



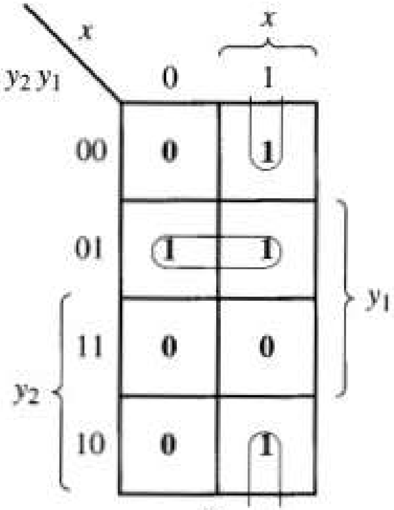
y_2y_1 \ x	0	1
00	01/0	10/0
01	11/0	10/0
11	01/0	00/0
10	00/1	11/1

$Y_2\,Y_1/z$

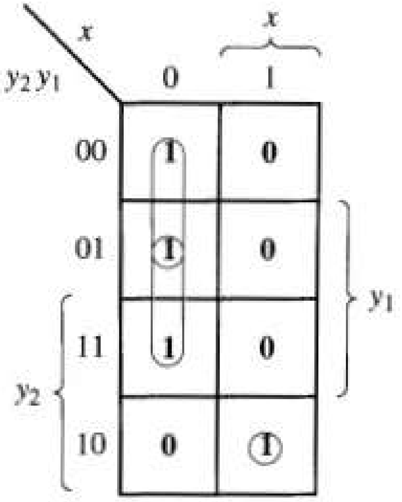
(a)



(b)



(c)



(d)

Assignment 2.

$$D_2 = x\bar{y}_1 + y_1\bar{y}_2$$
$$D_1 = \bar{x}\bar{y}_2 + \bar{x}y_1 + x\bar{y}_1y_2$$
$$z = \bar{y}_1y_2$$

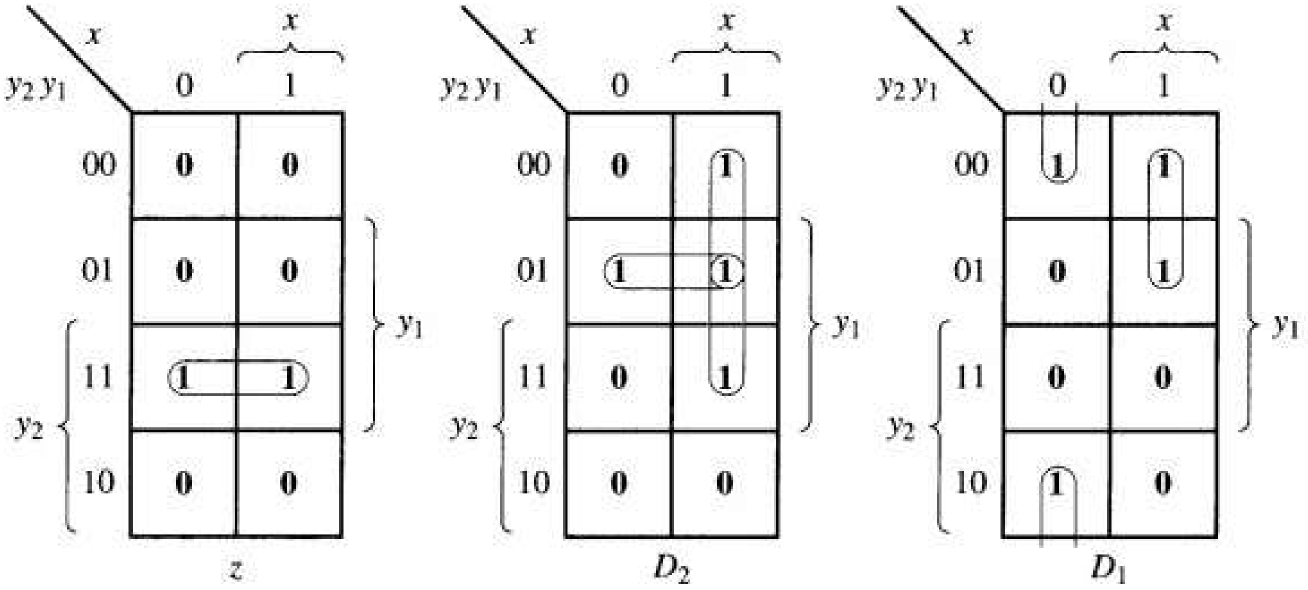
Example:

	$x = 0$	$x = 1$
A	C/0	D/0
B	C/0	A/0
C	B/0	D/0
D	A/1	B/1

$y_2y_1 \backslash x$	0	1
00	01/0	11/0
01	10/0	11/0
11	00/1	10/1
10	01/0	00/0

$Y_2 Y_1/z$

(a)

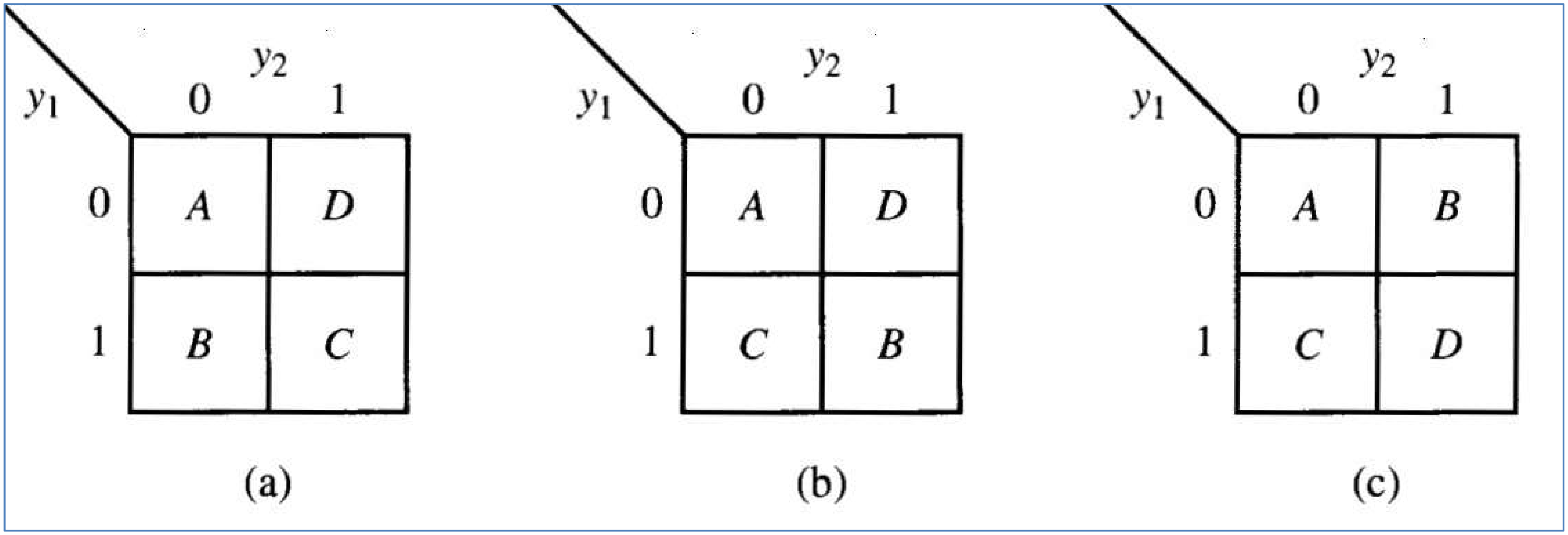


(b) (c) (d)

Assignment 3.

$$D_2 = y_1\bar{y}_2 + x\bar{y}_2 + xy_1$$
$$D_1 = x\bar{y}_2 + \bar{x}\bar{y}_1$$
$$z = y_1 y_2$$

Discussion



(a) (b) (c)

State adjacencies for four state assignments a) Assignment 1 b) Assignment 2 c) Assignment 3

Count the number of inputs to gates, assignment 1 =20, assignment 2 =18, assignment 3 =15 , assignment 3 gives the best results because gives better grouping of ones and zeros on K-maps for D1,D2 and z.

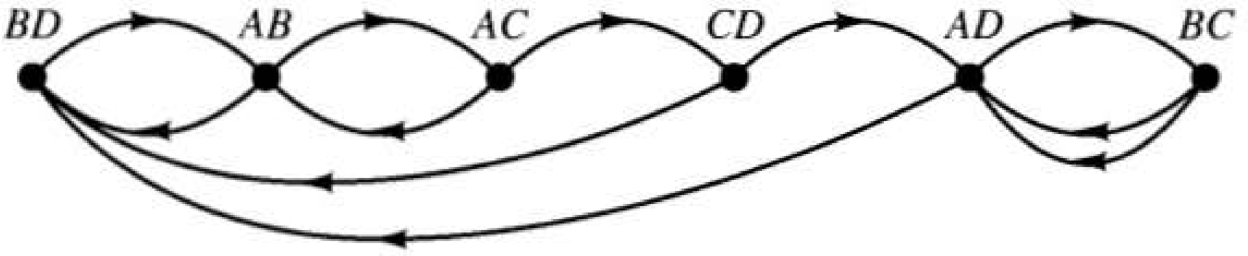
Implication Graph

Another tool that to selecting good state assignments for a sequential circuit is flow graph whose nodes represent pairs of states

Example:

	x = 0	x = 1
A	B/0	C/0
B	D/0	A/1
C	A/1	D/0
D	D/1	B/1

(a)

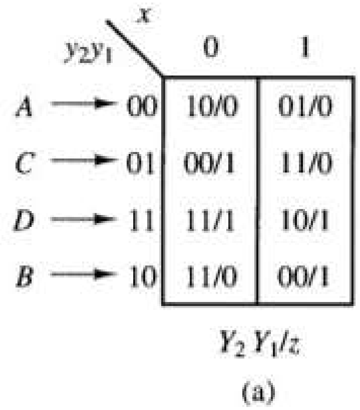


(b)

a)State table
b) Implication graph

Rule 1. *B* adj *D*.

Rule 2. *B* adj *C*, *D* adj *A*, *A* adj *D*, *D* adj *B*.



	y_2y_1 \ x	0	1
$A \longrightarrow$	00	10/0	01/0
$C \longrightarrow$	01	00/1	11/0
$D \longrightarrow$	11	11/1	10/1
$B \longrightarrow$	10	11/0	00/1

$Y_2 Y_1/z$

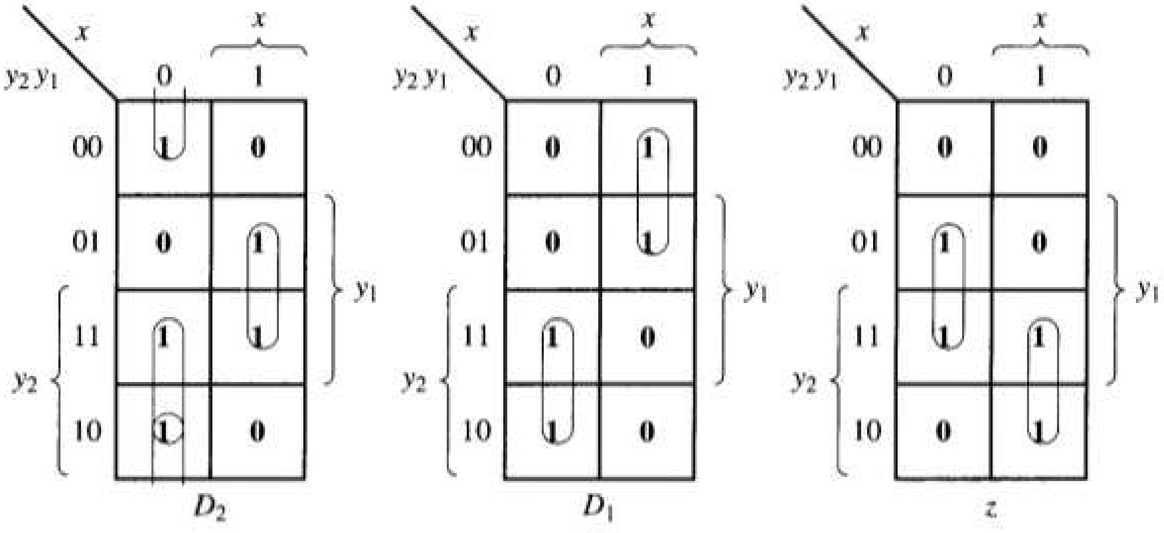
(a)

By using table c

$$D_2 = \bar{x}y_2 + \bar{x}\bar{y}_1 + xy_1$$
$$D_1 = \bar{x}y_2 + x\bar{y}_2$$
$$z = xy_2 + \bar{x}y_1$$

By using table b is not optimal

$$D_2 = xy_1\bar{y}_2 + \bar{x}\bar{y}_1 + \bar{x}y_2 + \bar{y}_1y_2$$
$$D_1 = \bar{y}_1\bar{y}_2 + x\bar{y}_1$$
$$z = xy_2 + \bar{y}_1y_2 + \bar{x}y_1\bar{y}_2$$

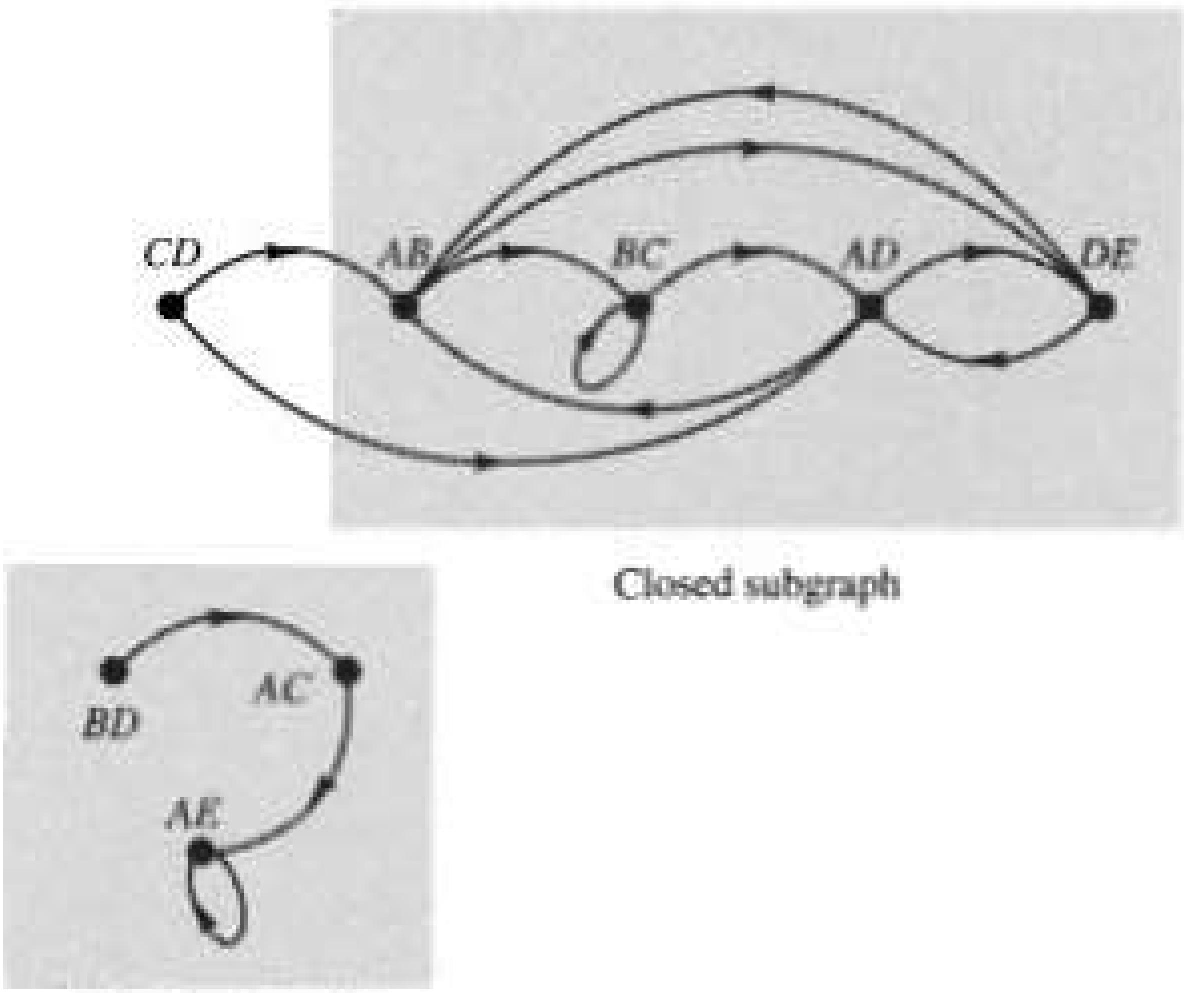


(b)

Example:

	x	
	0	1
A	B/0	E/0
B	C/1	D/1
C	B/0	A/0
D	A/0	D/0
E	B/1	A/1

(a)



Closed subgraph

Closed subgraph

(b)

Example:

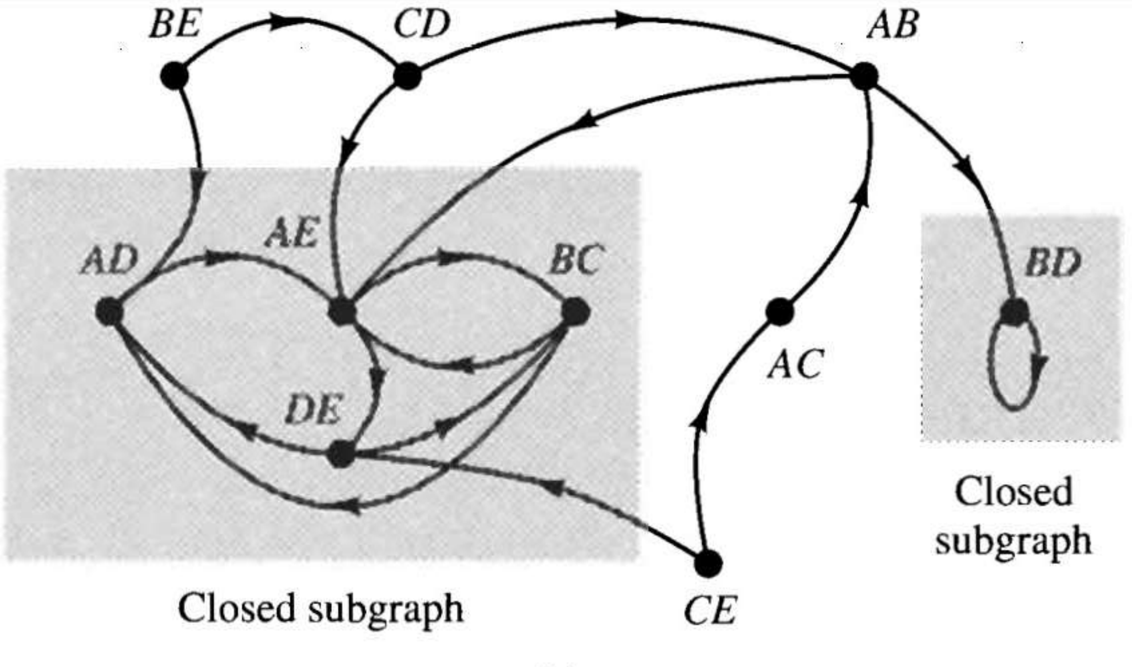
Rule 1. A adj C, B adj D, A adj D.

Rule 2. A adj D, B adj E, A adj B, C adj D.

Rule 3. From rules 1 and 2 we plot the implication graph

	x = 0	1
A	$E/0$	$B/0$
B	$A/1$	$D/1$
C	$E/0$	$A/0$
D	$A/0$	$B/1$
E	$D/0$	$C/0$

(a)



(b)

y_1 \ $y_3 y_2$	00	01	11	10
0	0 A	2 D	6 B	4 C
1	1 E	3	7	5

y_3 (columns 11, 10); y_2 (columns 01, 11); y_1 (row 1)

(c)

a) State table b) closed subgraphs c)state assignment

$$D_3 = x\bar{y}_3$$

$$D_2 = x\bar{y}_3\bar{y}_1 + xy_2 + \bar{x}y_1$$

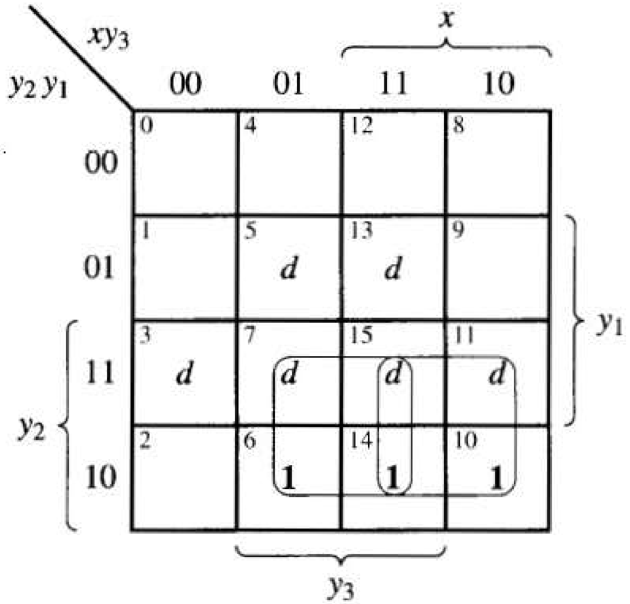
$$D_1 = \bar{x}\bar{y}_2\bar{y}_1$$

$$z = xy_2 + y_3y_2$$

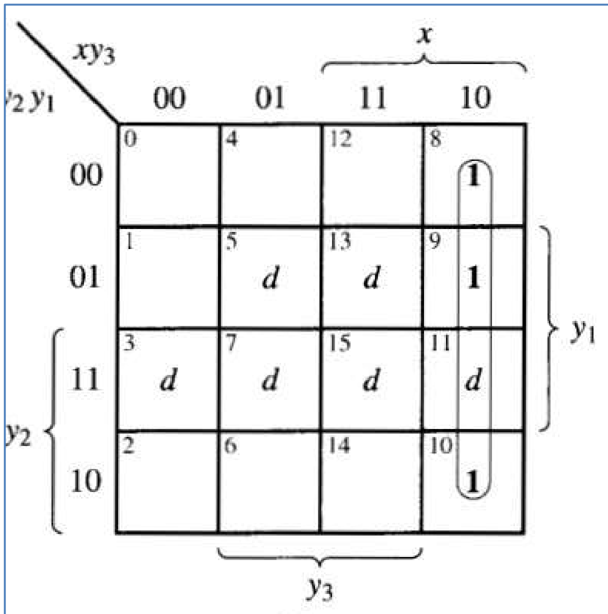
	$y_3y_2y_1$ \ x	0	1
A	000	001/0	110/0
B	110	000/1	010/1
C	100	001/0	000/0
D	010	000/0	110/1
E	001	010/0	100/0

$Y_3\,Y_2\,Y_1/z$

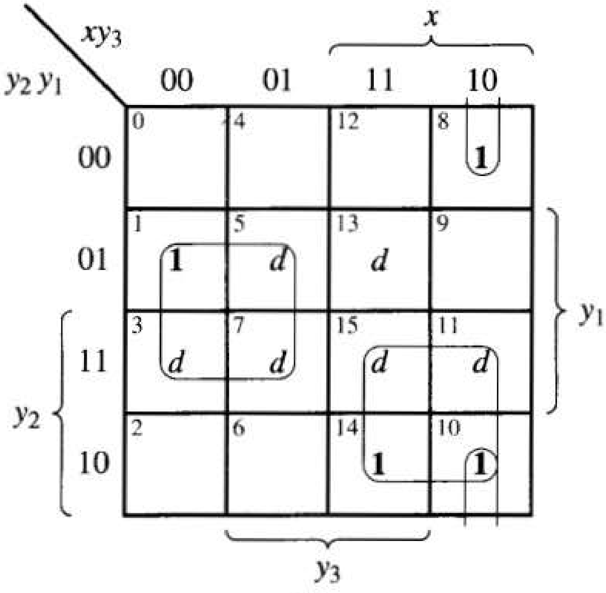
(a)



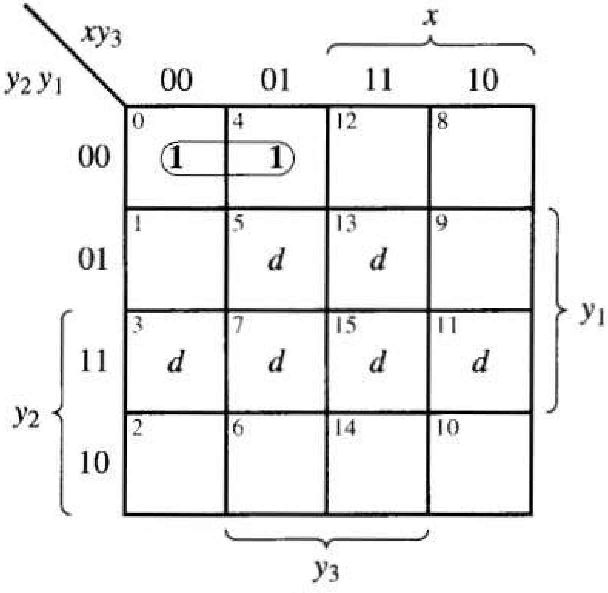
(b)



(c)



(d)



(e)