Design of Digital Control system with Dead-Beat Response

The dead-beat response design is characterized by the following design criteria:-

① The system must have zero steady-state error (e_{ss}) at sampling instants for agiven input.

② The response time defined as the time required to reach the steady state and should be minimum.

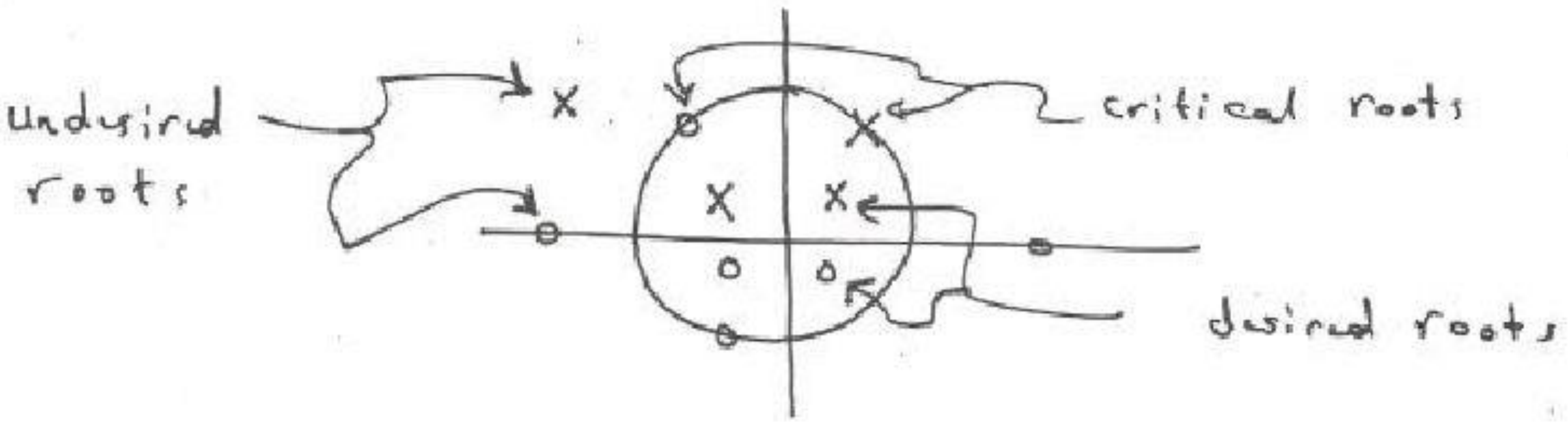
③ The digital Controller $D(z)$, must be physcically realizable.

$$D(z) = \frac{1}{G(z)} * \frac{Q(z)}{1-Q(z)}$$

Prosedure of Design

①. Find $G(z)$ from $G(s)$ with Z.O.H.

②. Determine the undesired roots → undesired Poles, → undesired zeros



3. Determine the $Q(z)$ From: know the order of $G(z)$ and the I/Ps of the system.

* For unit step input:

No. of samples

a. 1st order $\Rightarrow Q(z) = a_0 z^{-1}$

b. 2nd order $\Rightarrow Q(z) = a_0 z^{-1} + a_1 z^{-2}$

c. 3rd order $\Rightarrow Q(z) = a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3}$

* For unit ramp input:

a. 1st order $\Rightarrow Q(z) = a_0 z^{-1} + a_1 z^{-2}$

b. 2nd order $\Rightarrow Q(z) = a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3}$

Note: The start of the $Q(z)$, we know from the found the z^{-1} or z^{-2} in the numinator of $G(z)$ or from the first term of Long division of $G(z)$.

$$\boxed{Q(z) = (\text{undesired zeros}) * F_1(z)}$$

where $F_1(z) = C_0 z^{-1} + C_1 z^{-2} + \text{-----}$

4. Determine the $[1 - Q(z)]$

$$\boxed{1 - Q(z) = (1 - z^{-1})(\text{undesired poles})\, F_2(z)}$$ For unit step input

$$\boxed{1 - Q(z) = (1 - z^{-1})^2(\text{undesired poles})\, F_2(z)}$$ For unit ramp input

where

$$F_2(z) = 1 + b_0 z^{-1} + b_1 z^{-2} + \text{-----}$$

5. By equalization the equations in step 3 and step 4, we get : $a_0, a_1, \ldots$
$b_0, b_1, \ldots$
$c_0, c_1, \ldots$

6. For satisfy the design, we applied the following laws

$$1 - Q(z) \Big|_{z=1} = \text{zero}$$

7. Applied the following formula:

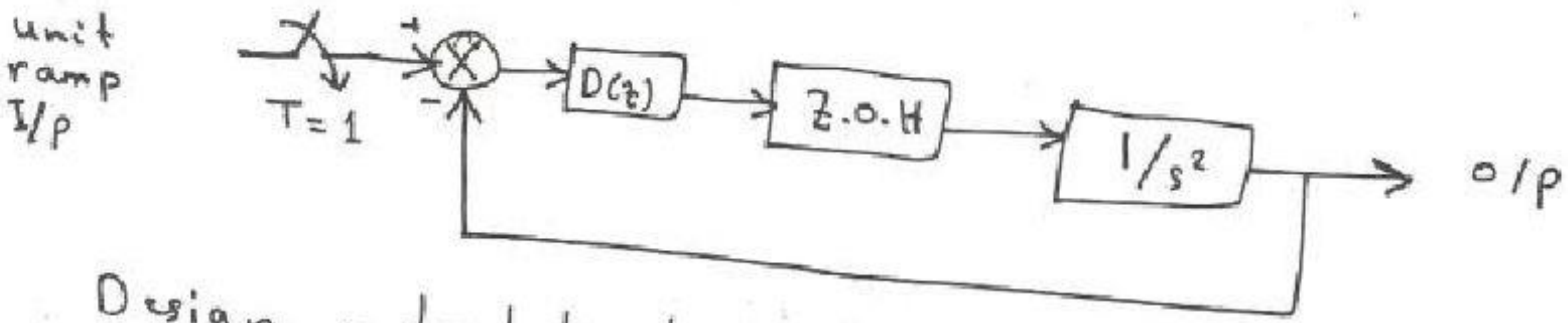
$$D(z) = \frac{1}{G(z)} * \frac{Q(z)}{1 - Q(z)}$$

Note: T.F of unit step = $R(z) = \frac{z}{z-1} = \frac{1}{1 - z^{-1}}$

T.F of unit ramp = $R(z) = \frac{Tz}{(z-1)^2} = \frac{T z^{-1}}{(1 - z^{-1})^2}$

No. of samplers = No. of order system for $G(z)$ + order for I/P

Ex.1: For the system shown in figure



Design a dead-beat digital controller such that it'll not exhibit an inter sampling Ripples. Check o/p then put control algorithm for designed controller.

Solution

$$G(z) = \frac{z-1}{z}\,\mathcal{Z}\,\frac{1}{s^3} = \frac{z-1}{z}\left[\frac{T^2 z(z+1)}{2(z-1)^3}\right], T=1$$

$$\therefore G(z) = \frac{0.5(z+1)}{(z-1)^2} = \frac{0.5\,z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$$

$$Q(z) = a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3}$$

$$\because Q(z) = (\text{undesired zeros})\,F_1(z)$$

$$a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3} = (1+z^{-1})(C_0 z^{-1} + C_1 z^{-2})$$

$$= C_0 z^{-1} + (C_0 + C_1) z^{-2} + C_1 z^{-3}$$

$$\therefore a_0 = C_0 \;;\; a_1 = C_0 + C_1 \;;\; a_2 = C_1$$

$$1 - Q(z) = (1-z^{-1})^2\,(\text{undesired poles})\,F_2(z)$$

$$1 - Q(z) = (1-z^{-1})^2 F_2(z)$$

$$1 - a_0 z^{-1} - a_1 z^{-2} - a_2 z^{-3} = (1 - 2z^{-1} + z^{-2})(1 + b_0 z^{-1})$$

$$= 1 + (b_0 - 2) z^{-1} + (1 - 2b_0) z^{-2} + b_0 z^{-3}$$

$$a_0 = 2 - b_0 \;;\; a_1 = 2b_0 - 1 \;;\; a_2 = -b_0$$

$$\therefore C_0 = 2 - b_0 \;;\; C_0 + C_1 = 2b_0 - 1 \;;\; C_1 = -b_0$$

$$2 - b_0 - b_0 = 2b_0 - 1$$

$$C_0 = 2 - 0.75 = 1.25$$

$$C_1 = -0.75$$

$$Q(z) = (1+z^{-1})(1.25\,z^{-1} - 0.75\,z^{-2})$$

$$= 1.25\,z^{-1}(1+z^{-1})(1-0.6\,z^{-1})$$

$$1 - Q(z) = (1-z^{-1})^2(1+0.75\,z^{-1})$$

For cheacking : $1 - Q(z)\big|_{z=1} = 0$

$$1 - 1 = 0$$

$$\therefore D(z) = \frac{1}{G(z)} * \frac{Q(z)}{1-Q(z)}$$

$$= \frac{\cancel{(1-z^{-1})^2}}{0.5\,\cancel{z^{-1}}\,\cancel{(1+z^{-1})}} * \frac{1.25\,\cancel{z^{-1}}\,\cancel{(1+z^{-1})}(1-0.6\,z^{-1})}{\cancel{(1-z^{-1})^2}(1+0.75\,z^{-1})}$$

$$= \frac{2.5(1-0.6\,z^{-1})}{(1+0.75\,z^{-1})}$$

$$\frac{M(z)}{E(z)} = \frac{2.5 - 1.5\,z^{-1}}{1+0.75\,z^{-1}}$$

$$\boxed{m(kT) = -0.75\,m((k-1)T) + 2.5\,e(kT) - 1.5\,e((k-1)T)}$$

Control algorithm or Control Law or Control action.

* Cheack for Ripples :

$$M(z) = \frac{Q(z) * R(z)}{G(z)}$$

$$M(z) = \frac{1.25 z^{-1} \cancel{(1+z^{-1})}(1-0.6z^{-1}) * \frac{\cancel{z^{-1}}}{\cancel{(1-z^{-1})^2}}}{\frac{0.5 \cancel{z^{-1}} \cancel{(1+z^{-1})}}{\cancel{(1-z^{-1})^2}}}$$

$$M(z) = \frac{2.5 z^{-1}(1-0.6 z^{-1})}{1}$$

$$M(z) = 2.5 z^{-1} - 1.5 z^{-2} + 0 + 0$$

$\therefore$ The system has no ripples

ملاحظة :- عندما يوجد ripple فإننا نضيف sampler

الى $Q(z)$ أي تصبح

$$Q(z) = a_0 z^{-1} + a_1 z^{-2} + a_2 z^{-3} + a_3 z^{-4}$$

وكذلك يجب أن يكون الـ e_{ss} = zero

وإذا لم يكن كذلك أيضاً نضيف sampler

Ex.2: For a unity feedback control system with O/L T.F

$$G(s) = \frac{e^{-5s}}{(10s+1)}$$ with unit step change in input.

Design a dead-beat digital controller such that it'll not exhibit an inter sampling Ripples. Take sampling rate to be 0.2 Hz. Check o/p then put control algorithm for designed controller. [redacted]

Solution

$$G(z) = \frac{z-1}{z} \mathcal{Z} \frac{e^{-5s}}{s(10s+1)}$$

$$= \frac{z-1}{z} \mathcal{Z} \frac{0.1\, e^{-5s}}{s(s+0.1)}$$

$$T\ \text{sampler} = \frac{1}{0.2} = 5\ \text{sec}.$$

$$G(z) = \frac{z-1}{z^2} \left[\mathcal{Z} \frac{A}{s} + \mathcal{Z} \frac{B}{s+0.1} \right]$$

$$A = 1 \quad ; \quad B = -1$$

$$G(z) = \frac{z-1}{z^2} \left[\frac{z}{z-1} - \frac{z}{z - e^{-0.5}} \right]$$

$$= \frac{z-1}{z^2} \left[\frac{z}{z-1} - \frac{z}{z-0.606} \right]$$

$$= \frac{1}{z} - \frac{z-1}{z(z-0.606)}$$

$$= \frac{\cancel{z} - 0.606 - \cancel{z} + 1}{z(z-0.606)}.$$

$$= \frac{0.393}{z(z-0.606)}$$

$$G(z) = \frac{0.393\, z^{-2}}{1 - 0.606\, z^{-1}}$$

$$\because\ Q(z) = +a_0\, z^{-2}$$

$$Q(z) = (\text{undesired zeros})\, F_1(z)$$

$$a_0\, z^{-2} = 1 * (C_0 z^{-1} + C_1 z^{-2})$$

$\therefore \; C_0 = 0 \;;\; C_1 = a_0$

$1 - Q(z) = (1 - z^{-1})\,(\text{undesired poles})\; F_2(z)$

$1 - a_0 z^{-2} = (1 - z^{-1})(1 + b_0 z^{-1})$

$1 - a_0 z^{-2} = 1 + (b_0 - 1) z^{-1} - b_0 z^{-2}$

$b_0 - 1 = 0 \Rightarrow b_0 = 1$

$a_0 = b_0 = 1 = C_1$

$\therefore Q(z) = z^{-2}$

$1 - Q(z) = (1 - z^{-1})(1 + z^{-1})$

$1 - Q(z) \Big|_{z=1} = \text{zero}$

$1 - 1 = 0$

$$\therefore D(z) = \frac{1 - 0.606 z^{-1}}{0.393 \cancel{z^{-2}}} * \frac{\cancel{z^{-2}}}{(1 - z^{-1})(1 + z^{-1})}$$

$$D(z) = \frac{2.56 (1 - 0.606 z^{-1})}{1 - z^{-2}}$$

$$D(z) = \frac{2.56 - 1.56 z^{-1}}{1 - z^{-2}}$$

$$\frac{M(z)}{E(z)} = \frac{2.56 - 1.56 z^{-1}}{1 - z^{-2}}$$

$$m(kT) = m((k-2)T) + 2.56\, e(kT) - 1.56\, e((k-1)T)$$

Control algorithm

* Cheack Ripples

$$M(z) = \frac{R(z) * Q(z)}{G(z)}$$

$$M(z) = \frac{\frac{1}{1-z^{-1}} * \cancel{z^{-2}}}{\frac{0.393\cancel{z^{-2}}}{1-0.606z^{-1}}}$$

$$M(z) = \frac{2.56(1-0.606z^{-1})}{(1-z^{-1})}$$

$$M(z) = \frac{2.56-1.56z^{-1}}{1-z^{-1}}$$

by Long division

$$
\begin{array}{r|l}
 & 2.56 + z^{-1} + z^{-2} \\
\hline
1-z^{-1} & \cancel{2.56} - 1.56z^{-1} \\
 & \mp\cancel{2.56} \pm 2.56z^{-1} \\
\hline
 & \cancel{z^{-1}} \\
 & \mp\cancel{z^{-1}} \pm z^{-2} \\
\hline
 & \cancel{z^{-2}} \\
 & \mp\cancel{z^{-2}} \pm z^{-3} \\
\hline
 & z^{-3}
\end{array}
$$

∴ The system has no ripples

H.w

For the system shown in figure below, the transfer function of the controlled is given by:

$$G_p(z) = \frac{0.00039\, z^{-1}(1 + 2.78 z^{-1})(1 + 0.2 z^{-1})}{(1 - z^{-1})^2 (1 - 0.286 z^{-1})}$$

Design a controller $G_c(z)$ which produces an output response with zero steady-state error and minimum settling time to a unit ramp I/P.