7.3 Ideal sampling

An ideal sample $f^*(t)$ of a continuous signal $f(t)$ is a series of zero width impulses spaced at sampling time T seconds apart as shown in Figure 7.4.

The sampled signal is represented by equation (7.1).

$$f^*(t) = \sum_{k=-\infty}^{\infty} f(kT)\delta(t - kT) \tag{7.1}$$

where $\delta(t - kT)$ is the unit impulse function occurring at $t = kT$.

A sampler (i.e. an A/D converter) is represented by a switch symbol as shown in Figure 7.5. It is possible to reconstruct $f(t)$ approximately from $f^*(t)$ by the use of a hold device, the most common of which is the zero-order hold (D/A converter) as shown in Figure 7.6. From Figure 7.6 it can be seen that a zero-order hold converts a series of impulses into a series of pulses of width T. Hence a unit impulse at time t is converted into a pulse of width T, which may be created by a positive unit step at time t, followed by a negative unit step at time $(t - T)$, i.e. delayed by T.

The transfer function for a zero-order hold is

$$\begin{aligned} \mathscr{L}[f(t)] &= \frac{1}{s} - \frac{1}{s}e^{-Ts} \\ G_h(s) &= \frac{1 - e^{-Ts}}{s} \end{aligned} \tag{7.2}$$

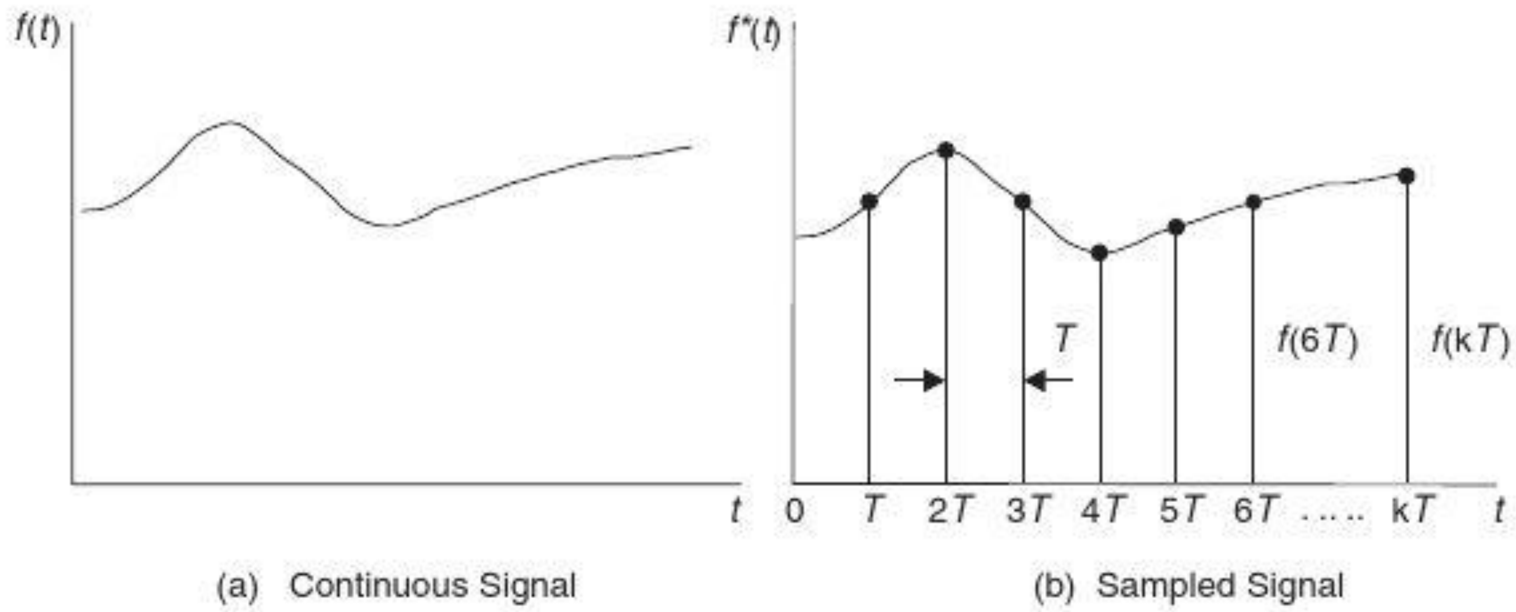

Fig. 7.4 The sampling process.

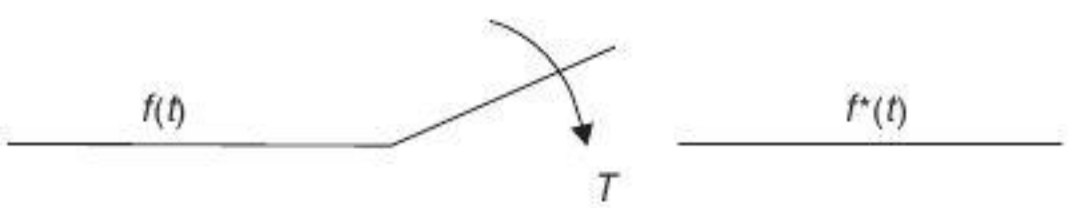

Fig. 7.5 A sampler.

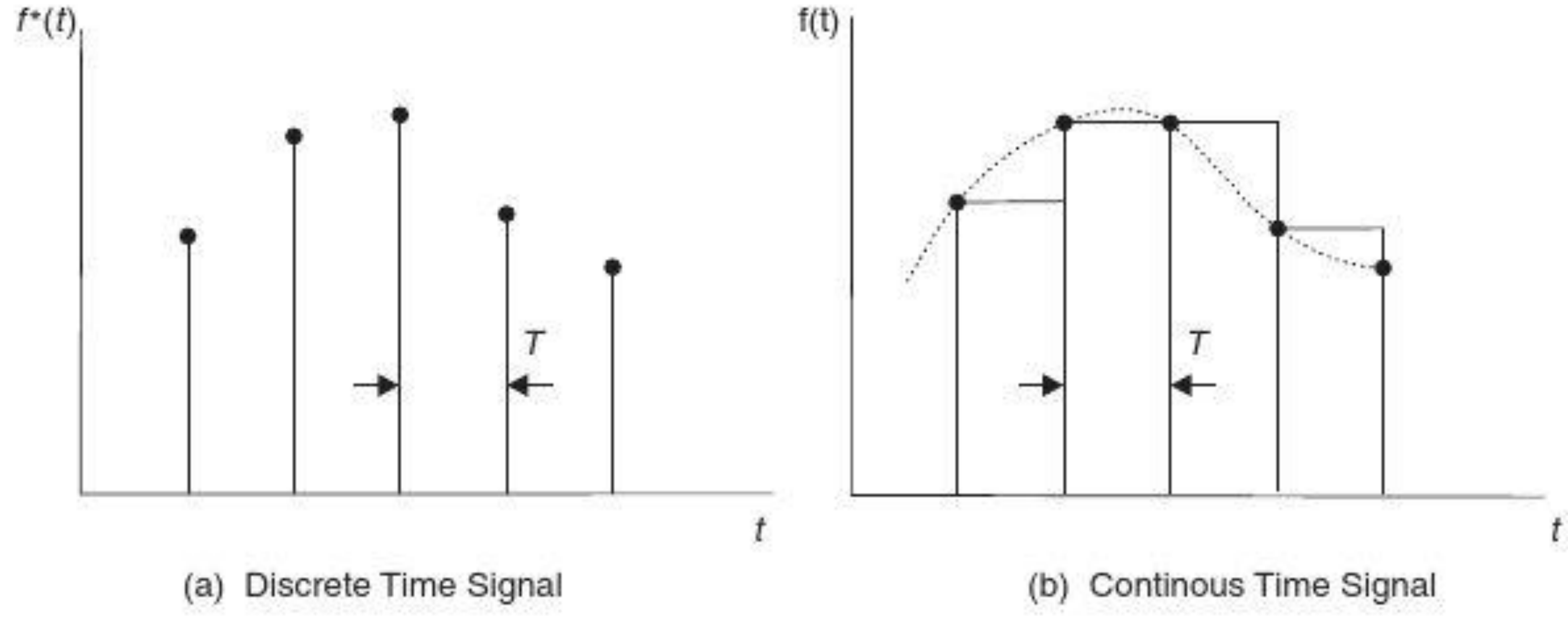

Fig. 7.6 Construction of a continuous signal using a zero-order hold.

7.4 The *z*-transform

The z-transform is the principal analytical tool for single-input–single-output discrete-time systems, and is analogous to the Laplace transform for continuous systems.

Conceptually, the symbol z can be associated with discrete time shifting in a difference equation in the same way that s can be associated with differentiation in a differential equation.

Taking Laplace transforms of equation (7.1), which is the ideal sampled signal, gives

$$F^*(s) = \mathscr{L}[f^*(t)] = \sum_{k=0}^{\infty} f(kT)e^{-kTs} \tag{7.3}$$

or

$$F^*(s) = \sum_{k=0}^{\infty} f(kT)\left(e^{sT}\right)^{-k} \tag{7.4}$$

Define z as

$$z = e^{sT} \tag{7.5}$$

then

$$F(z) = \sum_{k=0}^{\infty} f(kT)z^{-k} = Z[f(t)] \tag{7.6}$$

In 'long-hand' form equation (7.6) is written as

$$F(z) = f(0) + f(T)z^{-1} + f(2T)z^{-2} + \cdots + f(kT)z^{-k} \tag{7.7}$$

Example 7.1

Find the z-transform of the unit step function $f(t) = 1$.