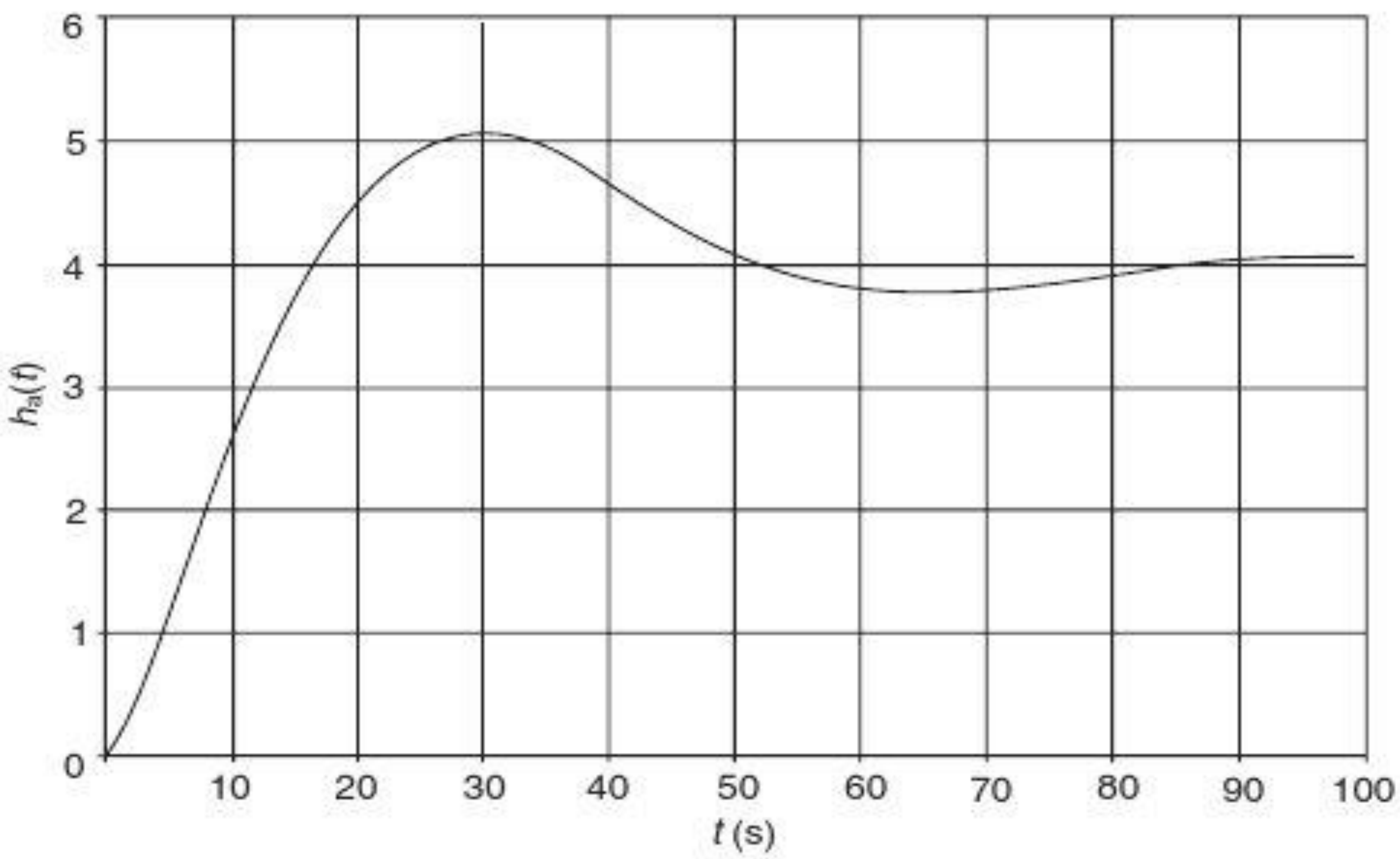

Fig. 4.28 Response of the PI controlled liquid-level system shown in Figure 4.26 to a step change in $h_d(t)$ from 0 to 4 m.

In equation (4.88) the amplitude of the sine term is small, compared with the cosine term, and can be ignored. Hence

$$h_a(t) = 4\left(1 - e^{-0.0417t} \cos 0.0909t\right) \tag{4.89}$$

The time response depicted by equation (4.89) is shown in Figure 4.28.

4.5.4 Proportional plus Integral plus Derivative (PID) control

Most commercial controllers provide full PID (also called three-term) control action. Including a term that is a function of the derivative of the error can, with high-order plants, provide a stable control solution.

Proportional plus Integral plus Derivative control action is expressed as

$$u(t) = K_1 e(t) + K_2 \int e\,dt + K_3 \frac{de}{dt} \tag{4.90}$$

Taking Laplace transforms

$$\begin{aligned} U(s) &= \left(K_1 + \frac{K_2}{s} + K_3 s\right) E(s) \\ &= K_1 \left(1 + \frac{K_2}{K_1 s} + \frac{K_3}{K_1} s\right) E(s) \\ &= K_1 \left(1 + \frac{1}{T_i s} + T_d s\right) E(s) \end{aligned} \tag{4.91}$$

In equation (4.91), T_d is called the derivative action time, and is formally defined as: 'The time interval in which the part of the control signal due to proportional action increases by an amount equal to the part of the control signal due to derivative action when the error is changing at a constant rate' (BS 1523).

Equation (4.91) can also be expressed as

$$U(s) = \frac{K_1(T_i T_d s^2 + T_i s + 1)}{T_i s} E(s) \tag{4.92}$$

4.5.5 The Ziegler–Nichols methods for tuning PID controllers

The selection of the PID controller parameters K_1, T_i and T_d can be obtained using the classical control system design techniques described in Chapters 5 and 6. In the 1940s, when such tools were just being developed, Ziegler and Nichols (1942) devised two empirical methods for obtaining the controller parameters. These methods are still in use.

(a) *The Process Reaction Method:* This is based on the assumption that the open-loop step response of most process control systems has an S-shape, called the process reaction curve, as shown in Figure 4.29. The process reaction curve may be approximated to a time delay D (also called a transportation lag) and a first-order system of maximum tangential slope R as shown in Figure 4.29 (see also Figure 3.13).

The Process Reaction Method assumes that the optimum response for the closed-loop system occurs when the ratio of successive peaks, as defined by equation (3.71), is 4:1. From equation (3.71) it can be seen that this occurs when the closed-loop damping ratio has a value of 0.21. The controller parameters, as a function of R and D, to produce this response, are given in Table 4.2.

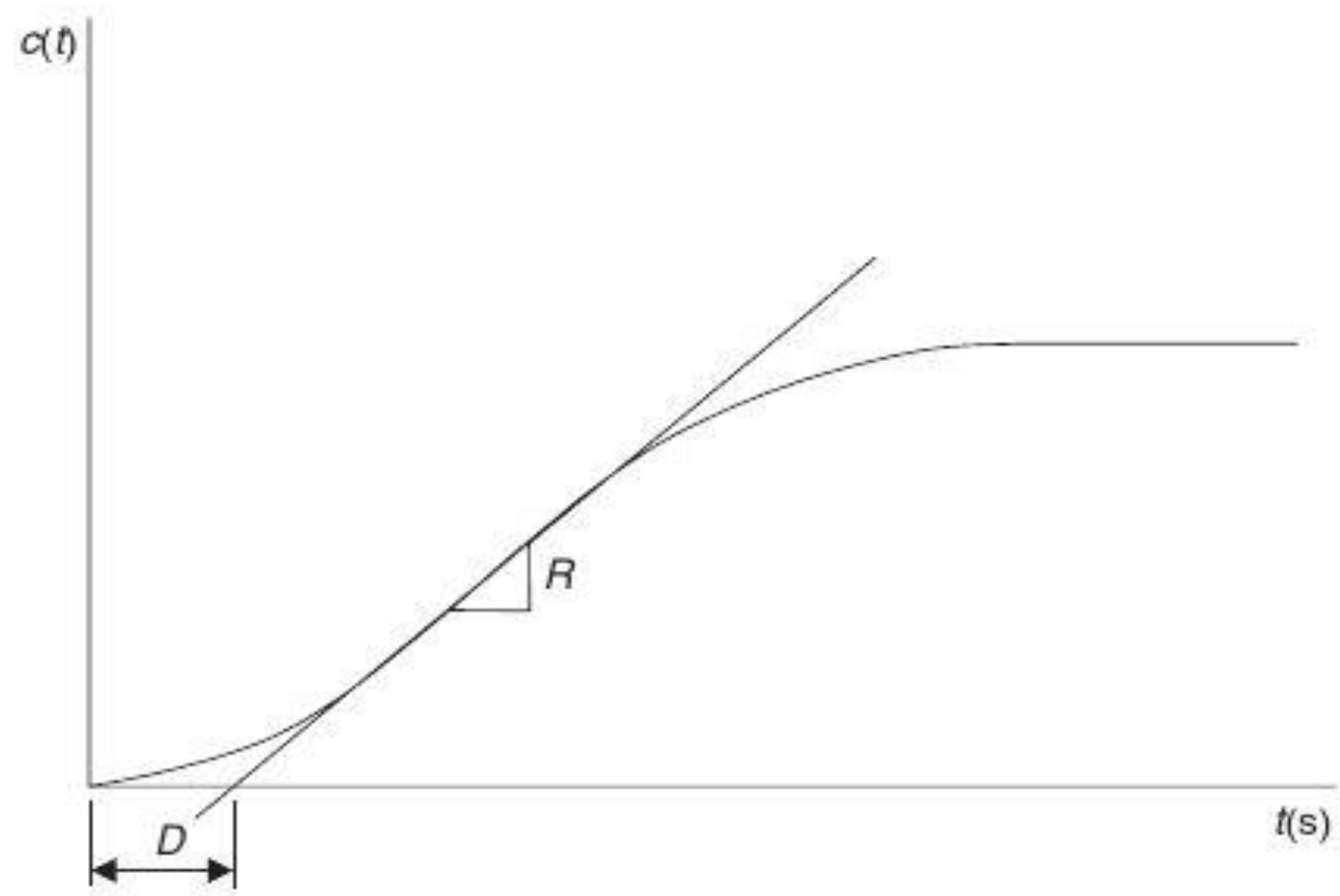

Fig. 4.29 Process reaction curve.

Table 4.2 Ziegler–Nichols PID parameters using the Process Reaction Method

Controller type	K_1	T_i	T_d
P	$1/RD$	–	–
PI	$0.9/RD$	$D/0.3$	–
PID	$1.2/RD$	$2D$	$0.5D$

Table 4.3 Ziegler–Nichols PID parameters using the Continuous Cycling Method

Controller type	K_1	T_i	T_d
P	$K_u/2$	–	–
PI	$K_u/2.2$	$T_u/1.2$	–
PID	$K_u/1.7$	$T_u/2$	$T_u/8$

Note that the Process Reaction Method cannot be used if the open-loop step response has an overshoot, or contains a pure integrator(s).

(b) *The Continuous Cycling Method*: This is a closed-loop technique whereby, using proportional control only, the controller gain K_1 is increased until the system controlled output $c(t)$ oscillates continually at constant amplitude, like a second-order system with no damping. This condition is referred to as marginal stability and is discussed further in Chapters 5 and 6. This value of controller gain is called the ultimate gain K_u, and the time period for one oscillation of $c(t)$ is called the ultimate period T_u. The controller parameters, as a function of K_u and T_u, to provide a similar closed-loop response to the Process Reaction Method, are given in Table 4.3.

The two Ziegler–Nichols PID tuning methods provide a useful 'rule of thumb' empirical approach. The control system design techniques discussed in Chapters 5 and 6 however will generally yield better design solutions.

Of the two techniques, the Process Reaction Method is the easiest and least disruptive to implement. In practice, the measurement of R and D is very subjective, and can lead to errors.

The Continuous Cycling Method, although more disruptive, has the potential to give better results. There is the risk however, particularly with high performance servo-mechanisms, that if K_u is increased by accident to slightly above the marginal stability value, then full instability can occur, resulting in damage to the system.

Actuator saturation and integral wind-up

One of the practical problems of implementing PID control is that of actuator saturation and integral wind-up. Since the range of movement in say, a control valve, has physical limits, once it has saturated, increasing the magnitude of the control signal further has no effect. However, if there is a difference between desired and measured values, the resulting error will cause a continuing increase in the integral term, referred to as integral wind-up. When the error term changes its sign, the integral term starts to 'unwind,' and this can cause long time delays and possible instability. The solution is to limit the maximum value that the integral term can have.

modified PID control schemes have proved their usefulness in providing satisfactory control, although they may not provide optimal control in many given situations.

Outline of the chapter. Section 10–1 has presented introductory material for the chapter. Section 10–2 deals with tuning methods for the basic PID control, commonly known as Ziegler–Nichols tuning rules. Section 10–3 discusses modified PID control schemes, such as PI-D control and I-PD control. Section 10–4 introduces two-degrees-of-freedom PID control schemes. Section 10–5 introduces the concept of robust control using a two-degrees-of-freedom control system as an example.

10–2 TUNING RULES FOR PID CONTROLLERS

PID control of plants. Figure 10–1 shows a PID control of a plant. If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then analytical approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values K_p, T_i, and T_d) based on experimental step responses or based on the value of K_p that results in marginal stability when only the proportional control action is used. Ziegler–Nichols rules, which are presented in the following, are very convenient when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical models.)

Ziegler–Nichols rules for tuning PID controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature. Here, however, we introduce only the Ziegler–Nichols tuning rules.)

There are two methods called Ziegler–Nichols tuning rules. In both methods, they aimed at obtaining 25% maximum overshoot in step response (see Figure 10–2).

First method. In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 10–3. If the plant involves neither inte-

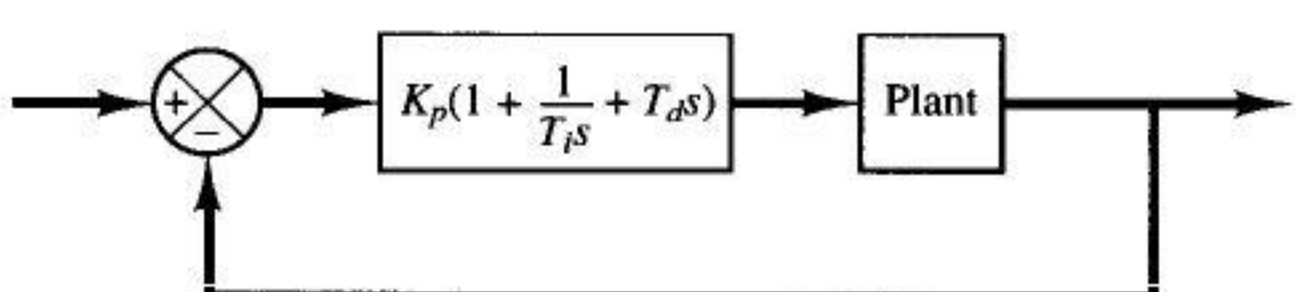

Figure 10–1 PID control of a plant.

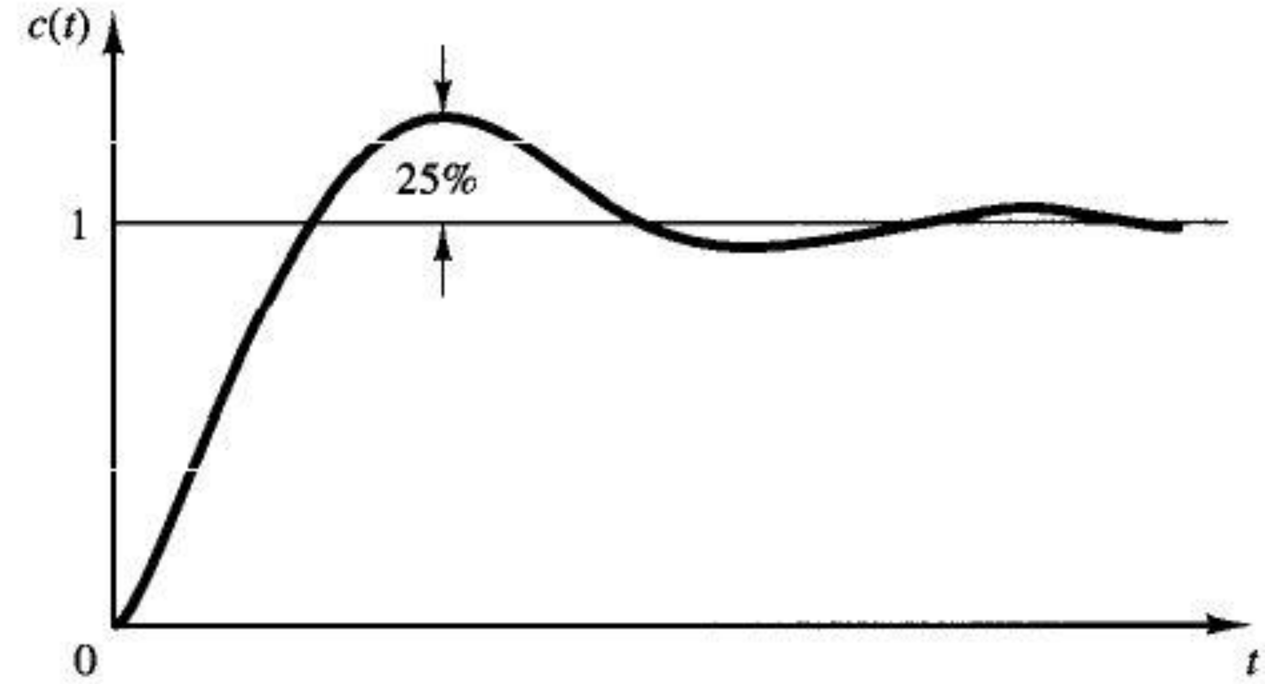

Figure 10–2
Unit-step response curve showing 25% maximum overshoot.

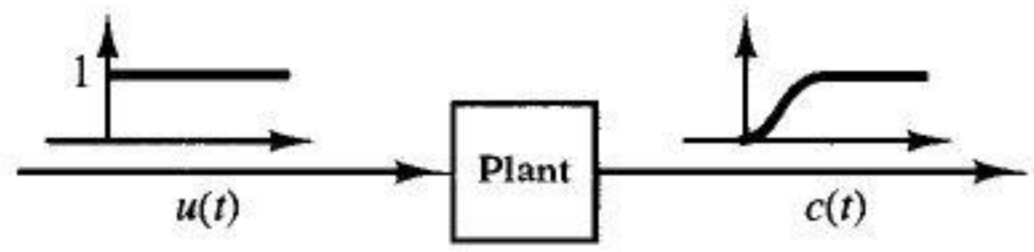

Figure 10–3
Unit-step response of a plant.

grator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look like an S-shaped curve, as shown in Figure 10–4. (If the response does not exhibit an S-shaped curve, this method does not apply.) Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants, delay time L and time constant T. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure 10–4. The transfer function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 10–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

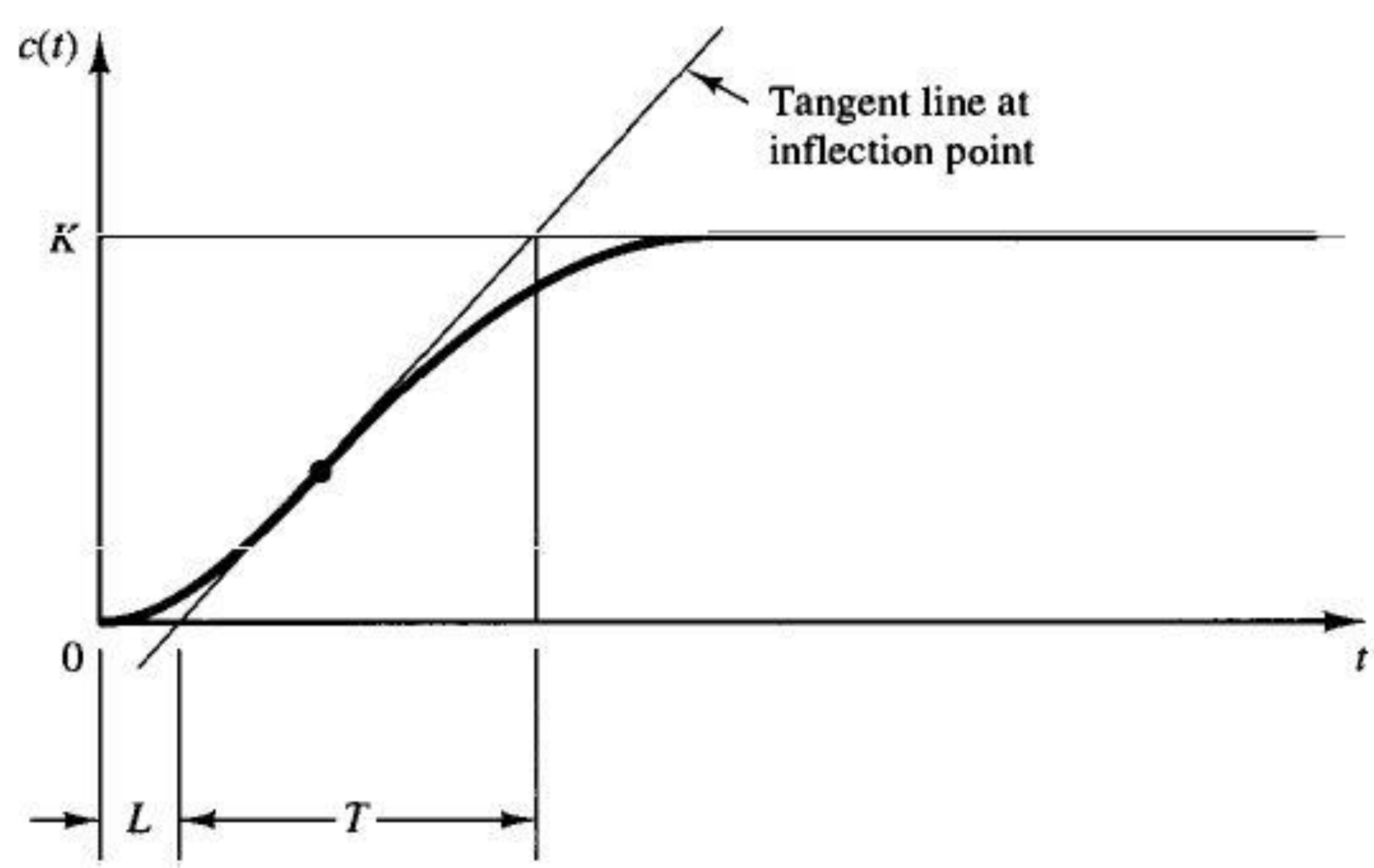

Figure 10–4
S-shaped response curve.

Table 10–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$
$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second method. In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 10–5), increase K_p from 0 to a critical value K_{cr} where the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined (see Figure 10–6). Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 10–2.

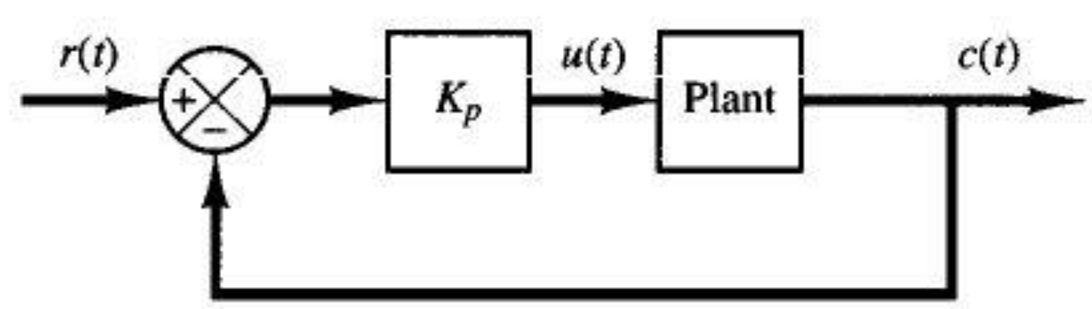

Figure 10–5 Closed-loop system with a proportional controller.

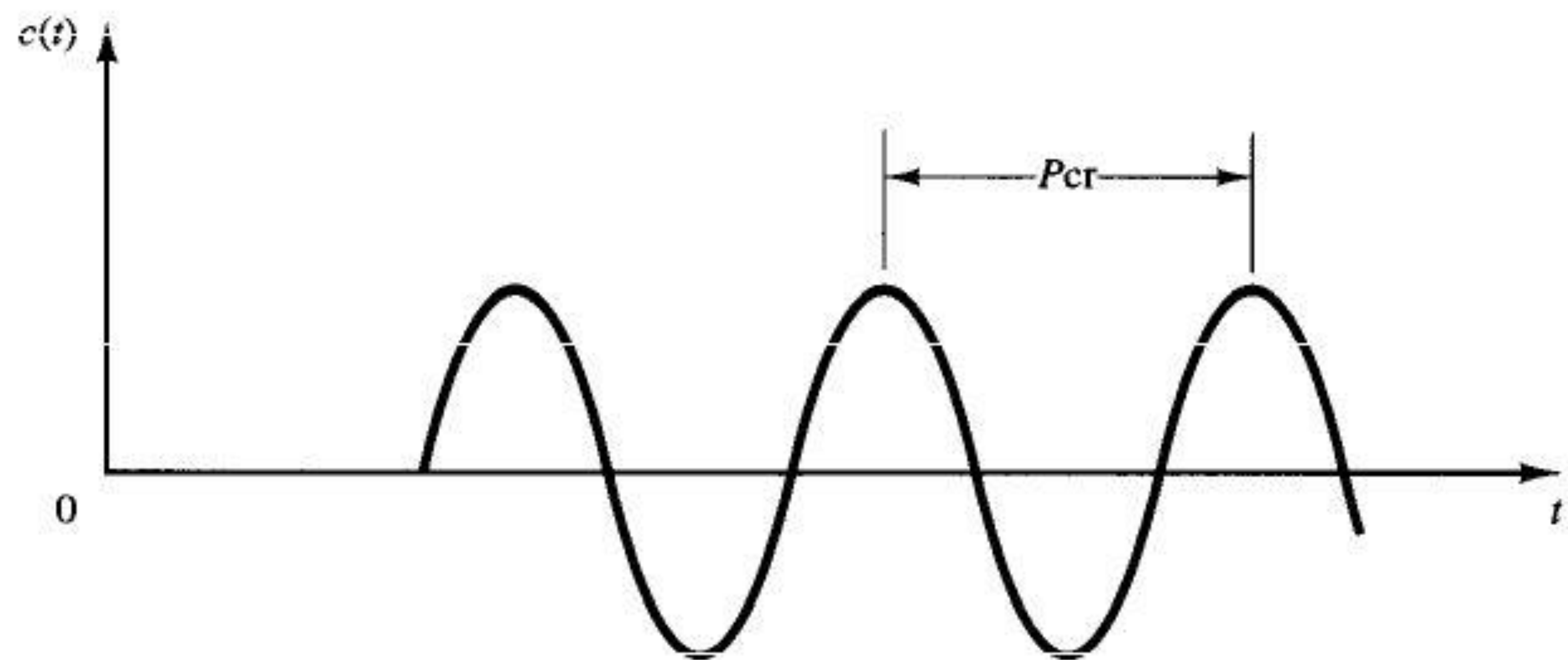

Figure 10–6 Sustained oscillation with period P_{cr}.

Table 10–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125\, P_{cr}s\right)$$

$$= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Comments. Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known. Over many years, such tuning rules proved to be very useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules.)

If the transfer function of the plant is known, a unit-step response may be calculated or the critical gain K_{cr} and critical period P_{cr} may be calculated. Then, using those calculated values, it is possible to determine the parameters K_p, T_i, and T_d from Table 10–1 or 10–2. However, the real usefulness of Ziegler–Nichols tuning rules (and other tuning rules) becomes apparent when the plant dynamics are not known so that no analytical or graphical approaches to the design of controllers are available.

Generally, for plants with complicated dynamics but no integrators, Ziegler–Nichols tuning rules can be applied. However, if the plant has an integrator, these rules may not be applied in some cases. To illustrate such a case where Ziegler–Nichols rules do not apply, consider the following case: Suppose that a unity-feedback control system has a plant whose transfer function is

$$G(s) = \frac{(s + 2)(s + 3)}{s(s + 1)(s + 5)}$$

Because of the presence of an integrator, the first method does not apply. Referring to

Figure 10–3, the step response of this plant will not have an S-shaped response curve; rather, the response increases with time. Also, if the second method is attempted (see Figure 10–5), the closed-loop system with a proportional controller will not exhibit sustained oscillations whatever value the gain K_p may take. This can be seen from the following analysis. Since the characteristic equation is

$$s(s + 1)(s + 5) + K_p(s + 2)(s + 3) = 0$$

or

$$s^3 + (6 + K_p)s^2 + (5 + 5K_p)s + 6K_p = 0$$

the Routh array becomes

$$\begin{array}{lcc} s^3 & 1 & 5 + 5K_p \\ s^2 & 6 + K_p & 6K_p \\ s^1 & \dfrac{30 + 29K_p + 5K_p^2}{6 + K_p} & 0 \\ s^0 & 6K_p & \end{array}$$

The coefficients in the first column are positive for all values of positive K_p. Thus, in the present case the closed-loop system will not exhibit sustained oscillations and, therefore, the critical gain value K_{cr} does not exist. Hence, the second method does not apply.

If the plant is such that Ziegler–Nichols rules can be applied, then the plant with a PID controller tuned by Ziegler–Nichols rules will exhibit approximately 10% ~ 60% maximum overshoot in step response. On the average (experimented on many different plants), the maximum overshoot is approximately 25%. (This is quite understandable because the values suggested in Tables 10–1 and 10–2 are based on the average.) In a given case, if the maximum overshoot is excessive, it is always possible (experimentally or otherwise) to make fine tuning so that the closed-loop system will exhibit satisfactory transient responses. In fact, Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning.

EXAMPLE 10–1 Consider the control system shown in Figure 10–7 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25%.

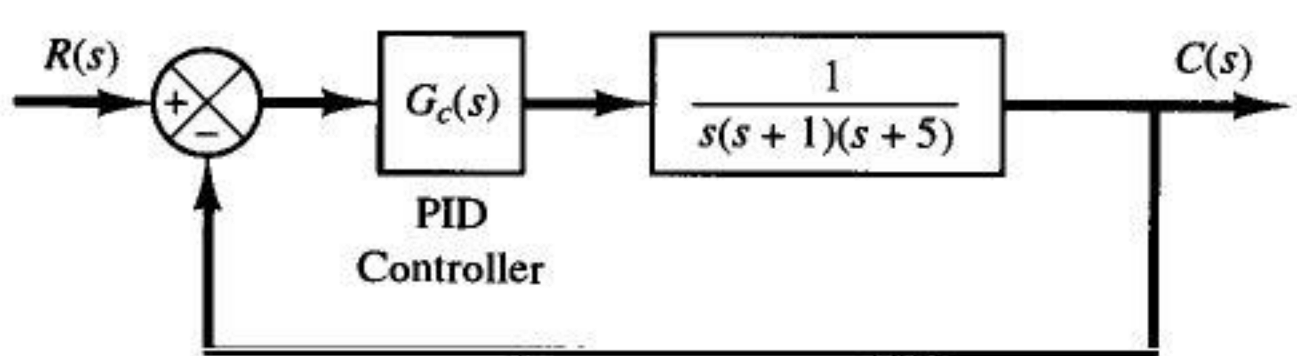

Figure 10–7 PID-controlled system.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} (= 30), the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 10–2, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 18$$

$$T_i = 0.5P_{cr} = 1.405$$

$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$

$$= \frac{6.3223(s + 1.4235)^2}{s}$$

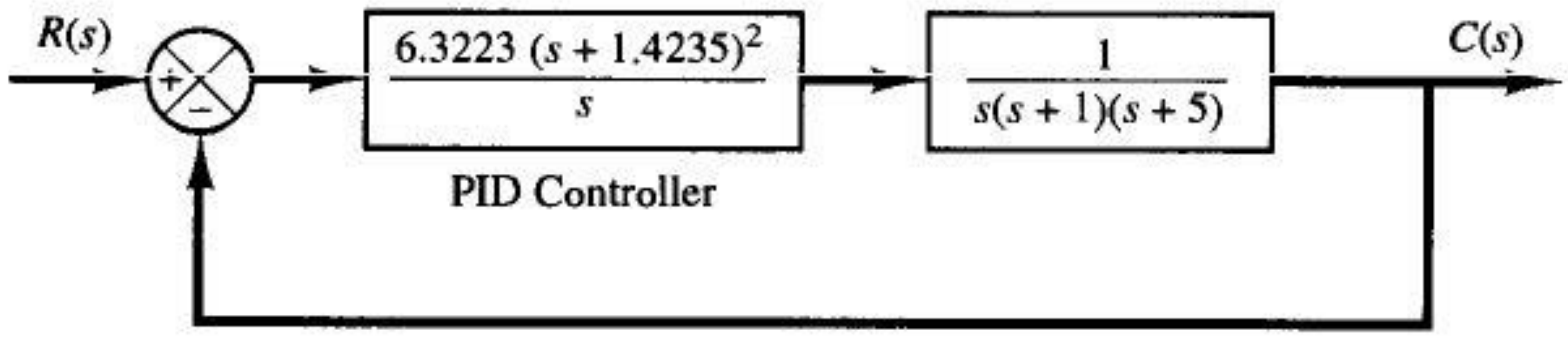

Figure 10–8 Block diagram of the system with PID controller designed by use of Ziegler–Nichols tuning rule (second method).

The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 10–8.

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. See MATLAB Program 10–1. The resulting unit-step response curve is shown in Figure 10–9. The maximum overshoot in the unit-step response is approximately 62%. The amount of maximum overshoot is excessive. It can be reduced by fine tuning the controller parameters. Such fine tuning can be made on the computer. We find that by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$, that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\frac{(s + 0.65)^2}{s} \qquad (10\text{–}1)$$

the maximum overshoot in the unit-step response can be reduced to approximately 18% (see Figure 10–10). If the proportional gain K_p is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, using the PID controller

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\frac{(s + 0.65)^2}{s} \qquad (10\text{–}2)$$

then the speed of response is increased, but the maximum overshoot is also increased to approximately 28%, as shown in Figure 10–11. Since the maximum overshoot in this case is fairly close to 25% and the response is faster than the system with $G_c(s)$ given by Equation (10–1), we may consider $G_c(s)$ as given by Equation (10–2) as acceptable. Then the tuned values of K_p, T_i, and T_d become

$$K_p = 39.42, \qquad T_i = 3.077, \qquad T_d = 0.7692$$

It is interesting to observe that these values respectively are approximately twice the values suggested by the second method of the Ziegler–Nichols tuning rule. The important thing to note here is that the Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

It is instructive to note that, for the case where the double zero is located at $s = -1.4235$, increasing the value of K_p increases the speed of response, but as far as the percentage maximum

MATLAB Program 10–1

```
% ---------- Unit-step response ----------

num = [0   0   6.3223   18   12.811];
den = [1   6   11.3223   18   12.811];
step(num,den)
grid
title('Unit-Step Response')
```

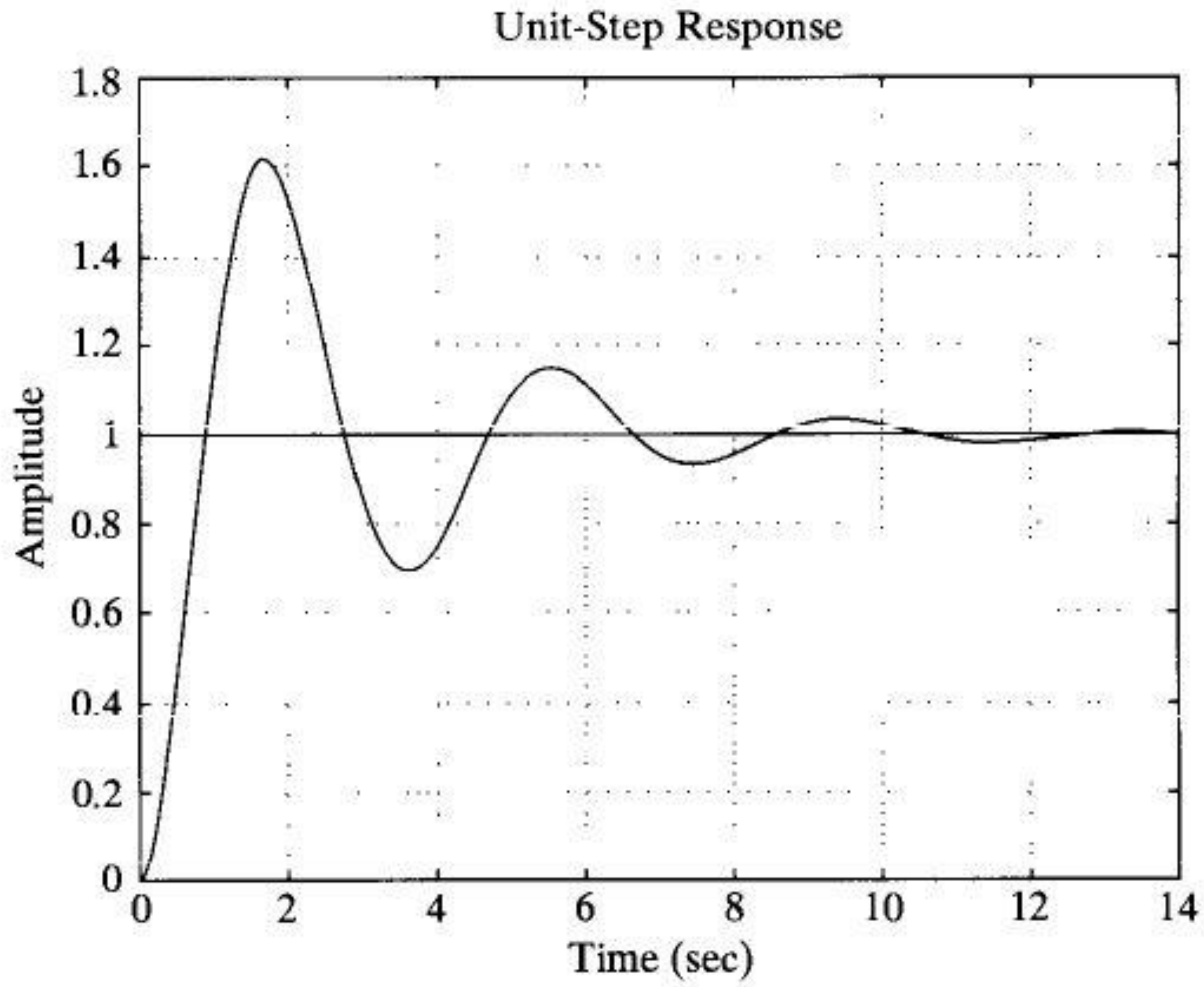

Figure 10–9 Unit-step response curve of PID-controlled system designed by use of Ziegler–Nichols tuning rule (second method).

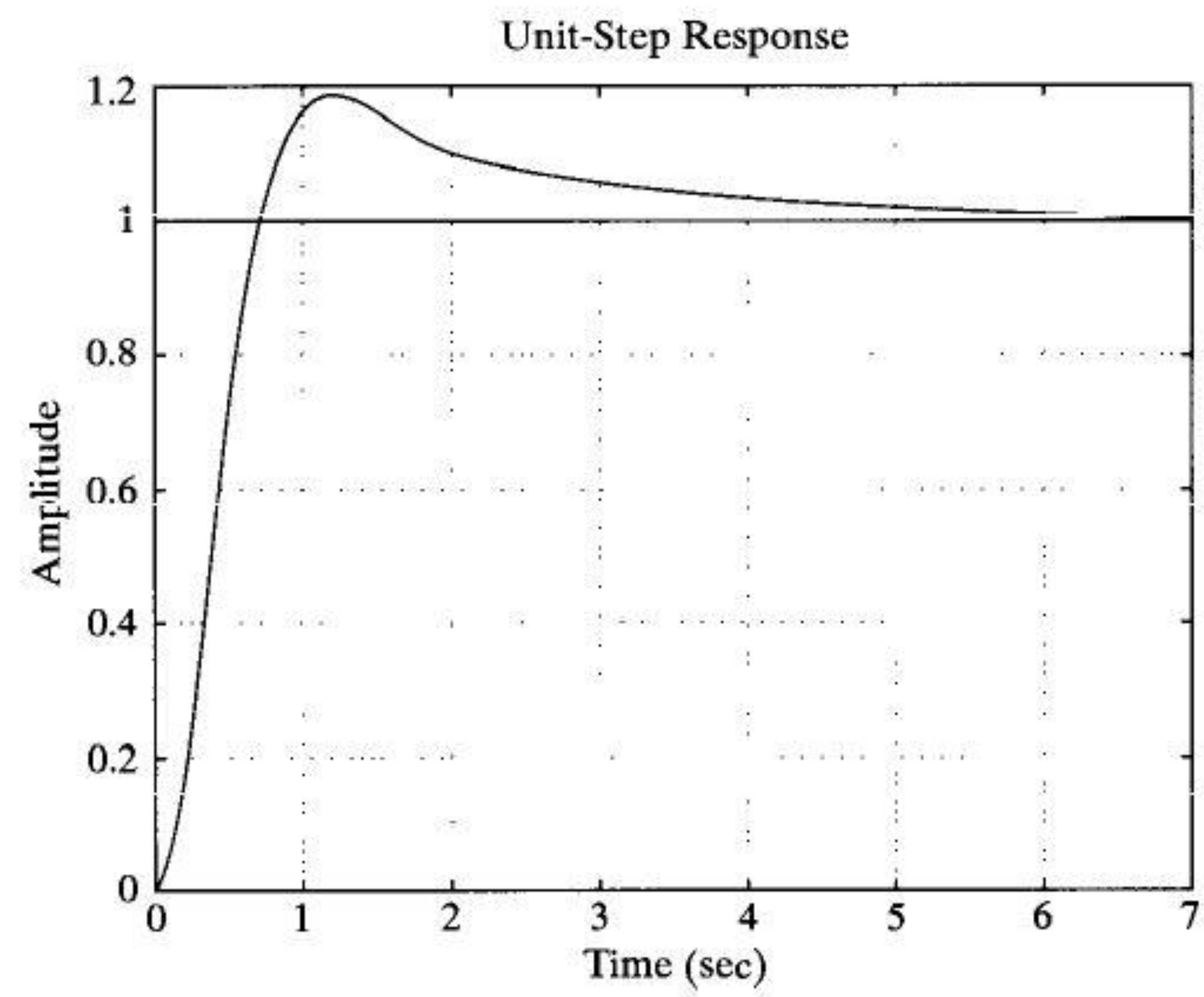

Figure 10–10 Unit-step response of the system shown in Figure 10–7 with PID controller having parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$.

overshoot is concerned, varying gain K_p has very little effect. The reason for this may be seen from the root-locus analysis. Figure 10–12 shows the root-locus diagram for the system designed by use of the second method of Ziegler–Nichols tuning rules. Since the dominant branches of root loci are along the $\zeta = 0.3$ lines for a considerable range of K, varying the value of K (from 6 to 30) will not change the damping ratio of the dominant closed-loop poles very much. However, varying the location of the double zero has a significant effect on the maximum overshoot, because the damping ratio of the dominant closed-loop poles can be changed significantly. This can also be seen from the root-locus analysis. Figure 10–13 shows the root-locus diagram for the system where the PID controller has the double zero at $s = -0.65$. Notice the change of the root-locus configuration. This change in the configuration makes it possible to change the damping ratio of the dominant closed-loop poles.

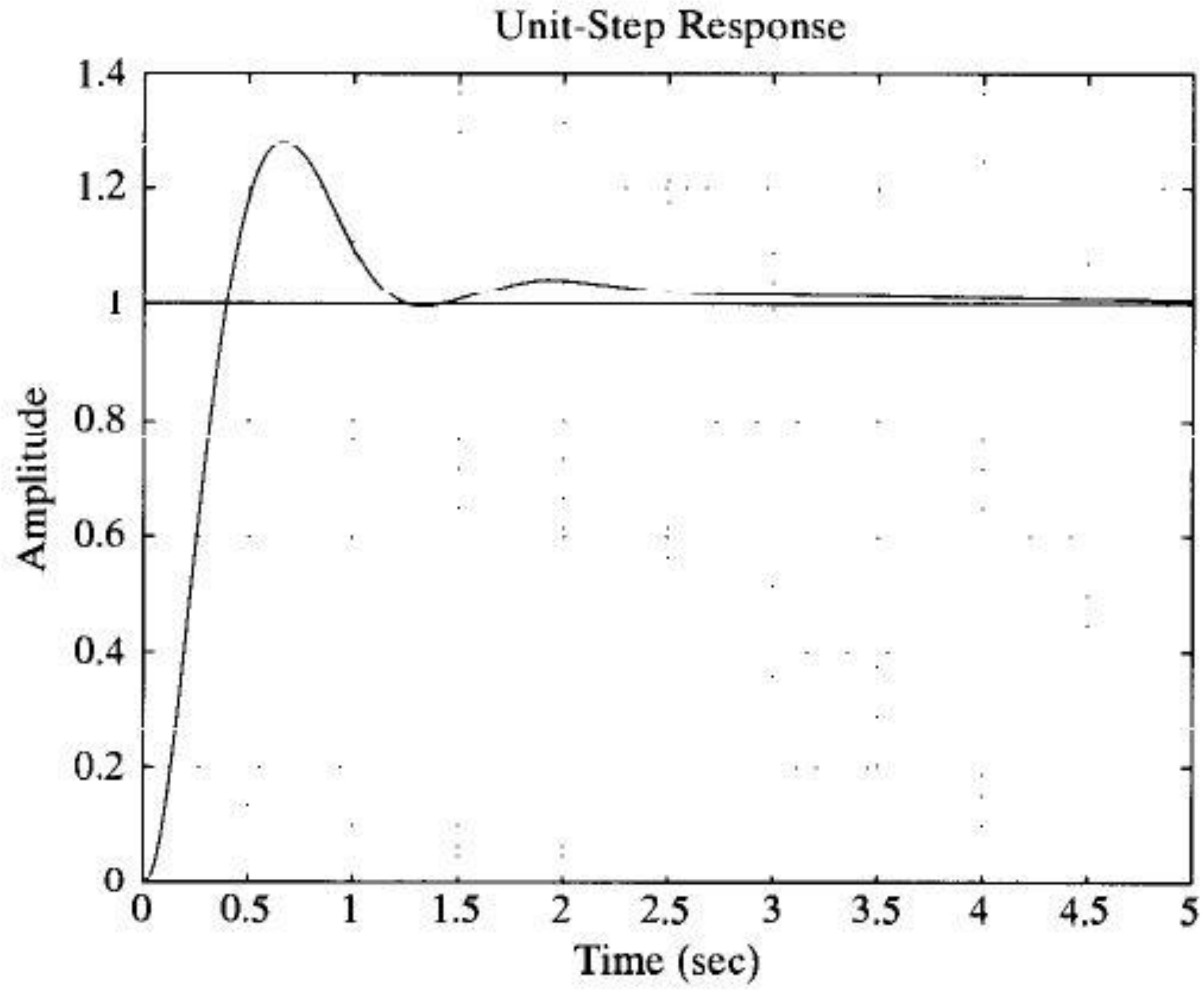

Figure 10–11 Unit-step response of the system shown in Figure 10–7 with PID controller having parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$.

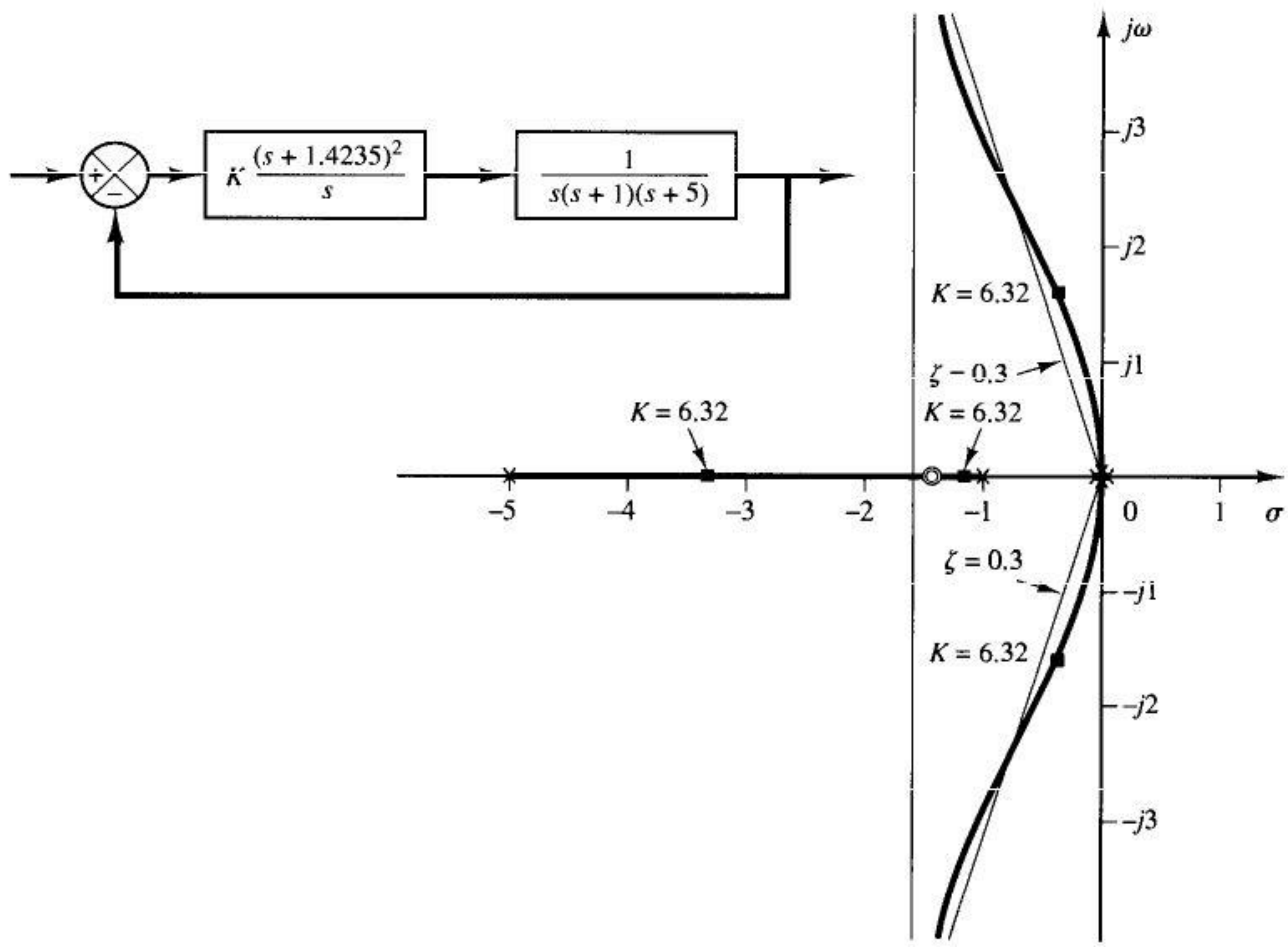

Figure 10–12 Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

In Figure 10–13, notice that, in the case where the system has gain $K = 30.322$, the closed-loop poles at $s = -2.35 \pm j4.82$ act as dominant poles. Two additional closed-loop poles are very near the double zero at $s = -0.65$, with the result that these closed-loop poles and the double zero almost cancel each other. The dominant pair of closed-loop poles indeed determines the nature of the response. On the other hand, when the system has $K = 13.846$, the closed-loop poles at $s = -2.35 \pm j2.62$ are not quite dominant because the two other closed-loop poles near the dou-

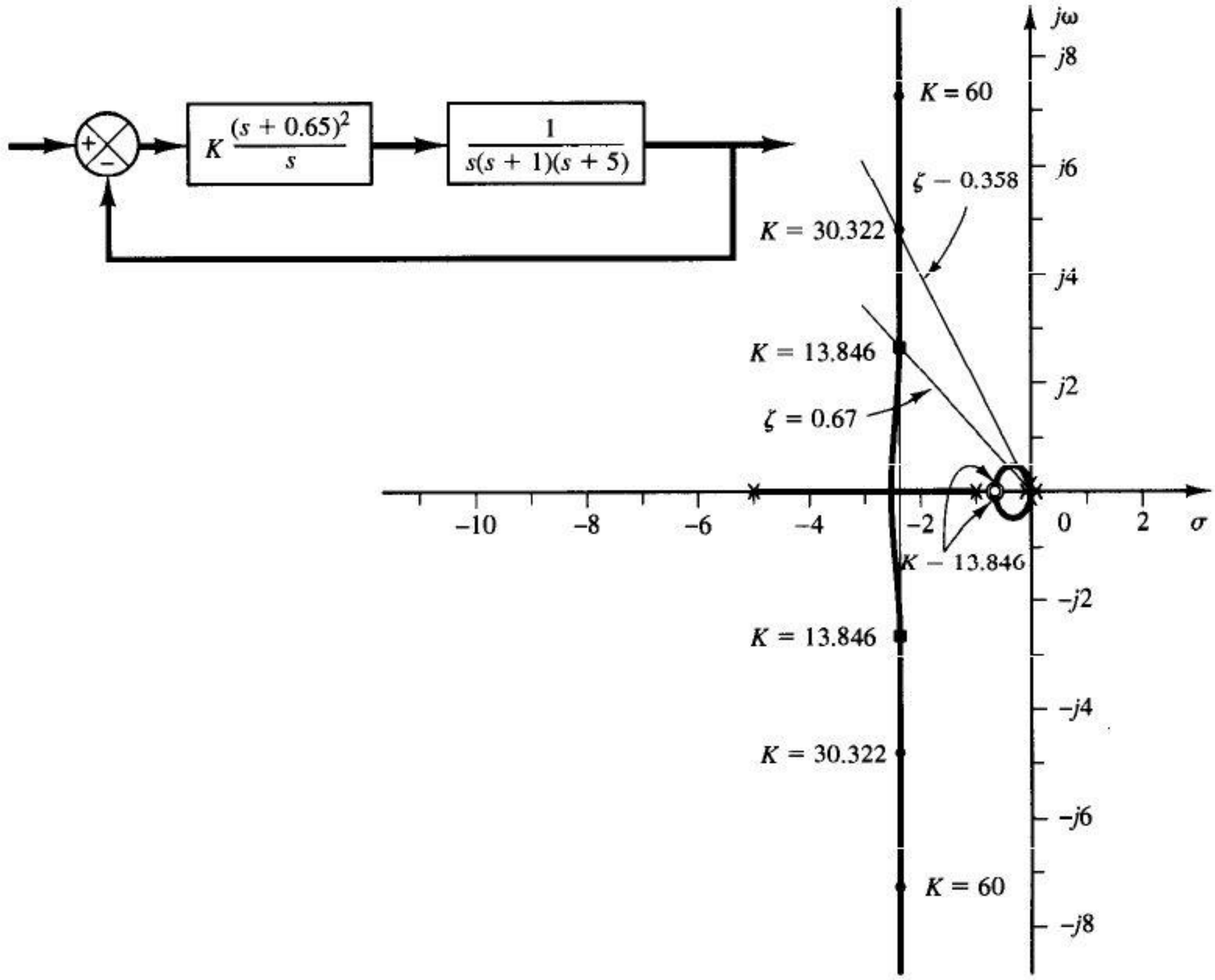

Figure 10–13
Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (10–1) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (10–2).

ble zero at $s = -0.65$ have considerable effect on the response. The maximum overshoot in the step response in this case (18%) is much larger than the case where the system is of second-order having only dominant closed-loop poles. (In the latter case the maximum overshoot in the step response would be approximately 6%.)

10–3 MODIFICATIONS OF PID CONTROL SCHEMES

Consider the basic PID control system shown in Figure 10–14(a), where the system is subjected to disturbances and noises. Figure 10–14(b) is a modified block diagram of the same system. In the basic PID control system such as the one shown in Figure 10–14(b), if the reference input is a step function, then, because of the presence of the derivative term in the control action, the manipulated variable $u(t)$ will involve an impulse function (delta function). In an actual PID controller, instead of the pure derivative term $T_d s$ we employ

$$\frac{T_d s}{1 + \gamma T_d s}$$

where the value of γ is somewhere around 0.1. Therefore, when the reference input is a step function, the manipulated variable $u(t)$ will not involve an impulse function, but will involve a sharp pulse function. Such a phenomenon is called *set-point kick*.