Errors

Occurrence of error is unavoidable in the field of scientific computing. Instead, numerical analysts try to investigate the possible and best ways to minimize the error. The study of the error and how to estimate and minimize it are the fundamental issues in error analysis.

Error Analysis

In numerical analysis we approximate the exact solution of the problem by using numerical method and consequently an error is committed. The numerical error is the difference between the exact solution and the approximate solution.

Definition 2 (Numerical Error). Let x be the exact solution of the underlying problem and x^* its approximate solution, then the error (denoted by e) in solving this problem is

$$e = x - x^*$$

Sources of Error in Numerical Computation

- Blunders (Gross Errors) These errors also called humans errors, and are caused by human's mistakes and oversight and can be minimized by taking care during scientific investigations. These errors will add to the total error of the underlying problem and can significantly affect the accuracy of solution.
- Modelling Errors These errors arise during the modelling process when scientists ignore effecting factors in the model to simplify the problem. Also, these errors known as formulation errors
- Data Uncertainty these errors are due to the uncertainty of the physical problem data and also known as data errors.
- Discretization Errors Computers represent a function of continuous variable by a number of discrete values. Also, scientists approximate and replace complex continuous problems by discrete ones and these results in discretization errors.

Absolute and Relative Errors

Definition3 (Absolute Error). The absolute error $\hat{e}$ of the error e is defined as the absolute value of the error e

$$\hat{e} = |x - x^*|$$

Definition 4 (Relative Error). The relative error e˜ of the error e is defined as the ratio between the absolute error eˆ and the absolute value of the exact solution x

$$\tilde{e} = \frac{\hat{e}}{|x|} = \frac{|x - x^*|}{|x|}, x \neq 0$$

Example 2. Let x = 3.141592653589793 is the value of the constant ratio π correct to 15 decimal places and x* = 3.14159265 be an approximation of x. Compute the following quantities:

a. The error.

b. The absolute error.

c. The relative error.

Solution:

a. The error e = x − x ∗ = 3.141592653589793 − 3.14159265 = 3.589792907376932e − 09

= 3.589792907376932 × 10^{-9} = 0.000000003589792907376932.

b. The absolute error eˆ = |x−x ∗| = |3.141592653589793−3.14159265| = 3.589792907376932e−09.

c. The relative error

$$\tilde{e} = \frac{\hat{e}}{|x|} = \frac{|x - x^*|}{|x|} = \frac{3.141592653589793 - 3.14159265}{3.141592653589793}$$
$$= \frac{3.589792907376932e - 09}{3.141592653589793} = 1.142666571770530e - 09.$$

Round off and Truncation Errors

Computers represent numbers in finite number of digits and hence some quantities cannot be represented exactly. The error caused by replacing a number a by its closest machine number is called the round off error and the process is called correct rounding.

Truncation errors also sometimes called chopping errors are occurred when chopping an infinite number and replaced it by a finite number or by truncated a series after finite number of terms.

Example3. Approximate the following decimal numbers to three digits by using rounding and chopping (truncation) rules:

1. X_1 = 1.34579.
2. X_2 = 1.34679.
3. X_3 = 1.34479.
4. X_4 = 3.34379.
5. X_5 = 2.34579.

Solution:

(i) Rounding:

(a) X_1 = 1.35.

(b) X_2 = 1.35.

(c) X_3 = 1.34.

(d) X_4 = 3.34.

(e) X_5 = 2.35.

(ii) Chopping:

(a) X_1 = 1.34.

(b) X_2 = 1.34.

(c) X_3 = 1.34.

(d) X_4 = 3.34.

(e) X_5 = 2.34