# Interpolation

Interpolation was used for long time to provide an estimate of a tabulated function at values that are not available in the table.

We are given the values of a function $(\boldsymbol{x})$ at different points $\boldsymbol{x_0}, \boldsymbol{x_1}, \ldots \ldots \boldsymbol{x_n}$, we want to find approximate values of the function $\boldsymbol{f(x)}$ for "new" x's that lie between these points for which the function values are given, and this process is called **interpolation**. Continuing our discussion, we write these given values of a function $(f)$ in the form

$$f_0 = f(x_0), f_1 = f(x_1), \ldots\ldots f_n = f(x_n)$$

Or as ordered pairs

$$(x_0, f_0), (x_1, f_1), \ldots \ldots (x_n, f_n)$$

These function values come from a "mathematical" function, such as a logarithm or a Bessel function. More frequently, they may be measured or automatically recorded values of an "empirical" function, such as air resistance of a car or an airplane at different speeds. Other examples of functions that are "empirical" are the yield of a chemical process at different temperatures or the size of the Iraq population as it appears from censuses taken at 10-year intervals.

## Types of interpolation

**A- Linear Interpolation**, The simplest form of interpolation is to connect two data points with a straight line. This technique, called linear interpolation, is depicted graphically in Figure (3.1) using similar triangles

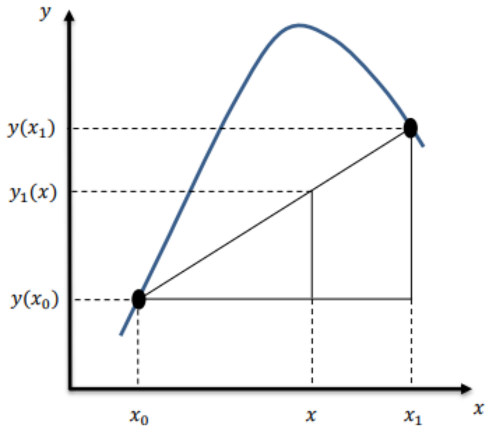



**Figure (3.1) Graphical depiction of linear interpolation**

$$\frac{y_1(x) - y(x_0)}{x - x_0} = \frac{y(x_1) - y(x_0)}{x_1 - x_0}$$

This can be rearranged to yield

$$y_1(x) = y(x_0) + \frac{y(x_1) - y(x_0)}{x_1 - x_0}(x - x_0)$$

Which is a linear-interpolation formula? The notation $y_1(x)$ designates that this is a first-order interpolating polynomial.

**Example:-** Using Linear Interpolation y=sin(x)

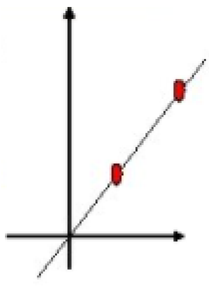
| x | Sin(x) |
|---|---|
| 0 | 0.0000 |
| 0.1 | 0.0998 |
| 0.2 | 0.1987 |
| 0.3 | 0.2955 |
| 0.4 | 0.3894 |

What is sin (0.15)? sin(0.15) ≈ 0.1493

True value (4 decimal digits) sin (0.15) = 0.1494

**Example:-** Given any two points,($x_0$,f($x_0$)),( $x_1$,f($x_1$))

The line that interpolates the two points is:



$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

Find a polynomial that interpolates (1,2) and(2,4).

$$f_1(x) = 2 + \frac{4 - 2}{2 - 1}(x - 1) = 2x$$

**Example: Calculate the unknown value using the interpolation formula from the given set of data. Calculate the value of Y when X value is 60.**

| X | Y |
|---|---|
| 10 | 0 |
| 30 | 40 |
| 50 | 80 |

| | |
|---|---|
| 70 | 120 |
| 90 | 160 |

**S0lution:**

$x=60$ $y=?$

$x_0=50$ $y(x_0) = 80$

$x_1=70$ $y(x_1) = 120$

$y = y(x_0) + (y(x_1)- y(x_0))/( x_1- x_0) * (x - x_0)$
$y= 80 + (120-80)/(70-50) * (60-50)$
$y=80 + 40/20 *10$
$y=80+20$
$y=100$

**example**: Mr. Harry shares details of Sales and profits. He is eager to know the profits of his business when the sales figure reaches $75,00,000. You are required to calculate profits based on the given data:

| Sales | Profits |
|---|---|
| 1,000,000 | 200,000 |
| 2,000,000 | 300,000 |
| 3,000,000 | 400,000 |
| 4,000,000 | 500,000 |
| 5,000,000 | 600,000 |

**Solution:**

$x= 75,00,000$ $y=?$

$x_0= 40,00,000$ $y(x_0) =500,000$

$x_1=5,000,000$ $y(x_1) = 600,000$

$y(x) = y(x_0) + (y(x_1)- y(x_0))/( x_1- x_0) * (x - x_0)$
$y(\$75,00,000)= \$ 5,00,000 + (\$6,00,000 – \$5,00,000)/(\$50,00,000 – \$40,00,000) * (\$75,00,000 – \$40,00,000)$
$y(\$75,00,000)= \$ 5,00,000 + \$1,00,000 / \$10,00,000 * \$ 35,00,000$

y($75,00,000)= $5,00,000 + $ 3,50,000

**H.W1:** Mr. Lark shares details of production and costs. In this era of global recession fears, Mr. Lark is also having a fear of decreasing the demands of his product and eager to know the optimum production level to cover the total cost of his business. You are required to calculate the optimum quantity level of production based on the given data. Lark wants to determine the quantity of production required to cover the estimated cost of $90,00,000.

| Quantity | Cost ($) |
|---|---|
| 10,000 | 4,000,000 |
| 200,000 | 4,500,000 |
| 300,000 | 5,000,000 |
| 400,000 | 5,500,000 |
| 500,000 | 6,000,000 |

**H.W2:** An experiment is used to determine the viscosity of water as a function of temperature. The following table is generated: Problem: Estimate the viscosity when the temperature is 8 degrees.

| Temperature (degree) | Viscosity |
|---|---|
| 0 | 1. 792 |
| 5 | 1. 519 |
| 10 | 1. 308 |
| 15 | 1.140 5 |