

**Diyala University
College of Engineering
Computer & Software
Engineering Department**



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Advanced Encryption Standard Algorithm (AES)

Chapter2: Part5

**PRESENTED BY
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Objectives

- ❑ To review a short history of AES**
- ❑ To define the basic structure of AES**
- ❑ To define the transformations used by AES**
- ❑ To define the key expansion process**
- ❑ To discuss different implementations**

2.4.1 Introduction

- **The Advanced Encryption Standard (AES) is a symmetric-key block cipher published by the National Institute of Standards and Technology (NIST) in December 2001.**
- **In February 2001, NIST announced that a draft of the Federal Information Processing Standard (FIPS) was available for public review and comment. Finally, AES was published as FIPS 197 in the Federal Register in December 2001.**
- **The criteria defined by NIST for selecting AES fall into three areas:**
 - 1. Security**
 - 2. Cost**
 - 3. Implementation.**
- **AES is a non-Feistel cipher that encrypts and decrypts a data block of 128 bits. It uses 10, 12, or 14 rounds. The key size, which can be 128, 192, or 256 bits, depends on the number of rounds.**

2.4.1 Introduction

- **RC2 is a block encryption algorithm which may be considered as a proposal for a DES replacement.**
- **The input and output block sizes are 64 bits each.**
- **The key size is variable, from one byte up to 128 bytes, although the current implementation uses eight bytes.**
- **The algorithm is designed to be easy to implement on 16-bit microprocessors. On an IBM AT, the encryption runs about twice as fast as DES (assuming that key expansion has been done)**

2.4.2 Algorithm Description

- We use the term "word" to denote a 16-bit quantity.
- The symbol $+$ will denote twos-complement addition.
- The symbol $\&$ will denote the bitwise "and" operation.
- The term XOR will denote the bitwise "exclusive-or" operation.
- The symbol \sim will denote bitwise complement.
- The symbol \wedge will denote the exponentiation operation.
- The term MOD will denote the modulo operation.

2.4.2 Algorithm Description

There are three separate algorithms involved:

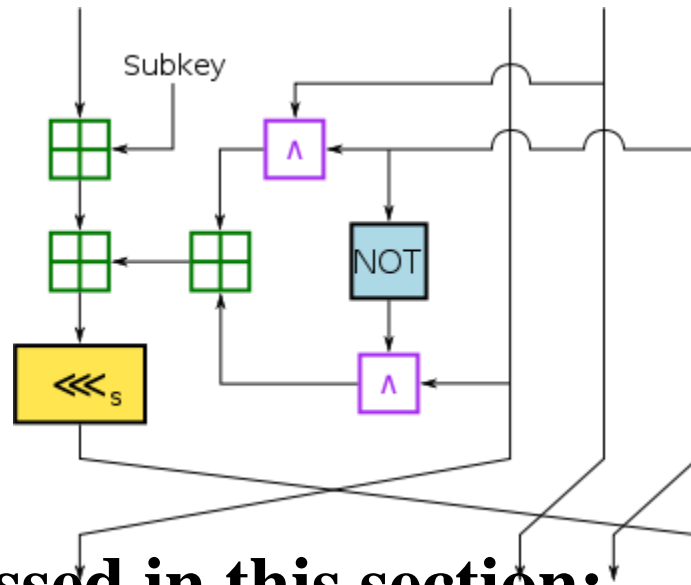
- Key expansion: This takes a (variable-length) input key and produces an expanded key consisting of 64 words $K[0], \dots, K[63]$.**
- Encryption: This takes a 64-bit input quantity stored in words $R[0], \dots, R[3]$ and encrypts it "in place" (the result is left in $R[0], \dots, R[3]$).**
- Decryption: The inverse operation to encryption.**

2.4.3 Key Expansion

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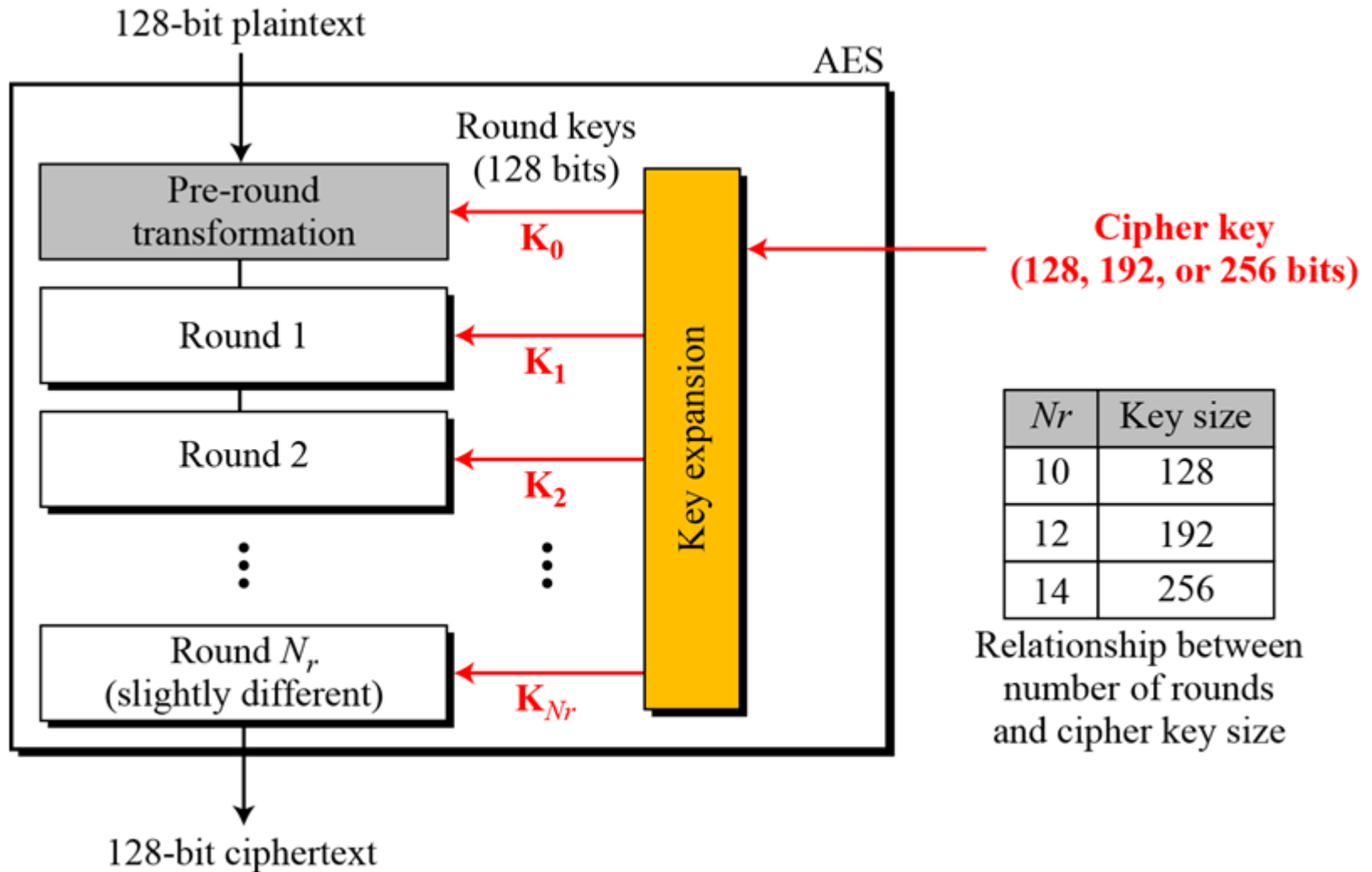
2.4.1 Introduction



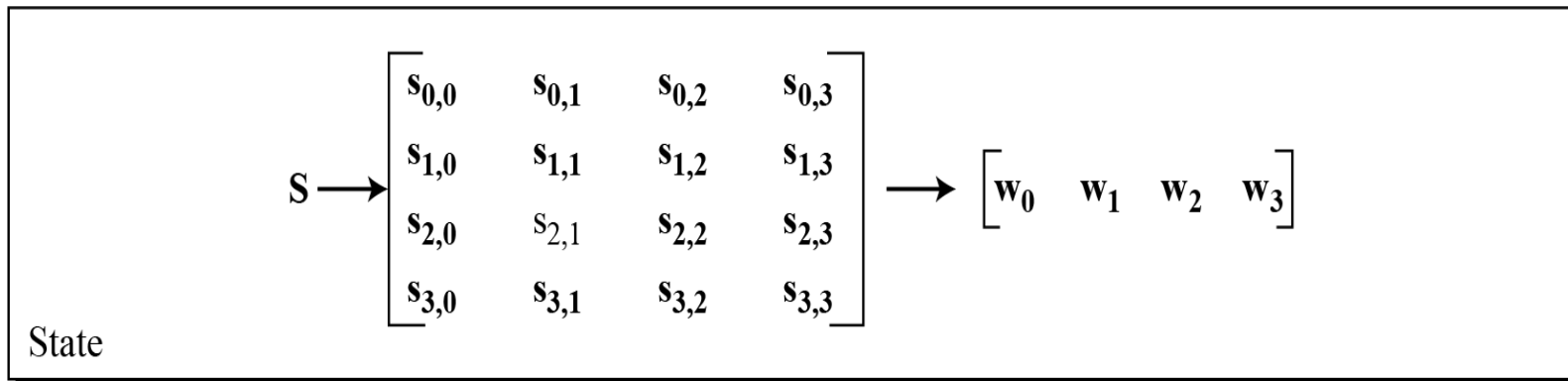
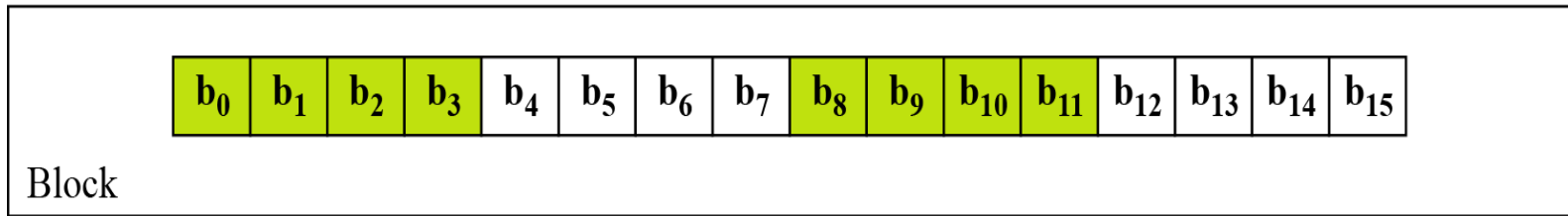
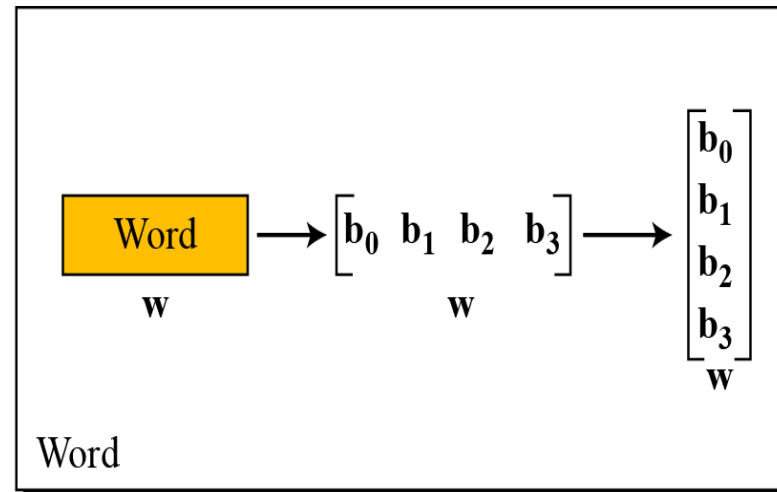
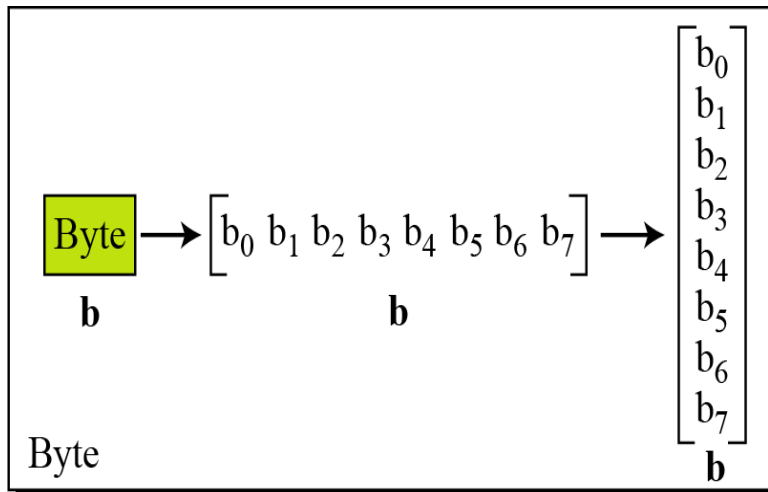
Topics discussed in this section:

1. History
2. Criteria
3. Rounds
4. Data Units
5. Structure of Each Round

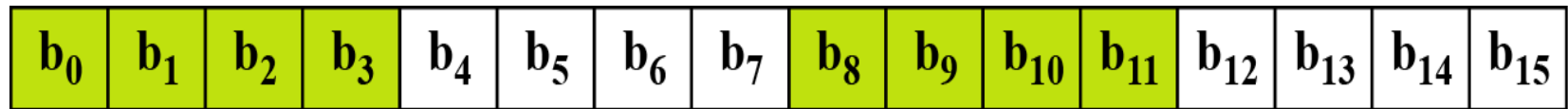
2.4.2 Data Units



2.4.3 Design of AES Encryption Cipher



2.4.4 Block-to-State and State-to-Block Transformation

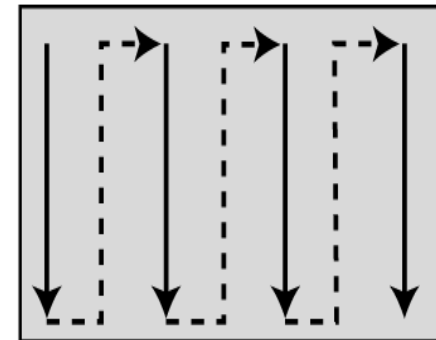


Block

$$s_{i \bmod 4, i/4} \longleftarrow \text{block}_i$$

State

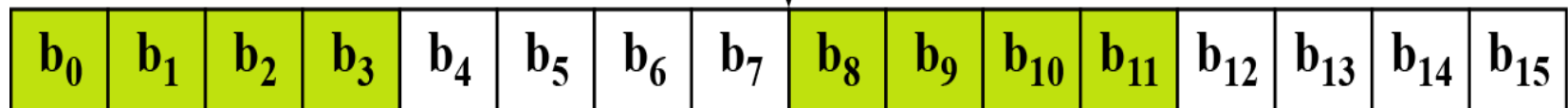
$$\begin{bmatrix} s_{0,0} = b_0 & s_{0,1} = b_4 & s_{0,2} = b_8 & s_{0,3} = b_{12} \\ s_{1,0} = b_1 & s_{1,1} = b_5 & s_{1,2} = b_9 & s_{1,3} = b_{13} \\ s_{2,0} = b_2 & s_{2,1} = b_6 & s_{2,2} = b_{10} & s_{2,3} = b_{14} \\ s_{3,0} = b_3 & s_{3,1} = b_7 & s_{3,2} = b_{11} & s_{3,3} = b_{15} \end{bmatrix}$$



Insertion and extraction flow

$$\text{block}_{i+4j} \longleftarrow s_{i,j}$$

Block



2.4.4 Example: Changing Plaintext to State

Text

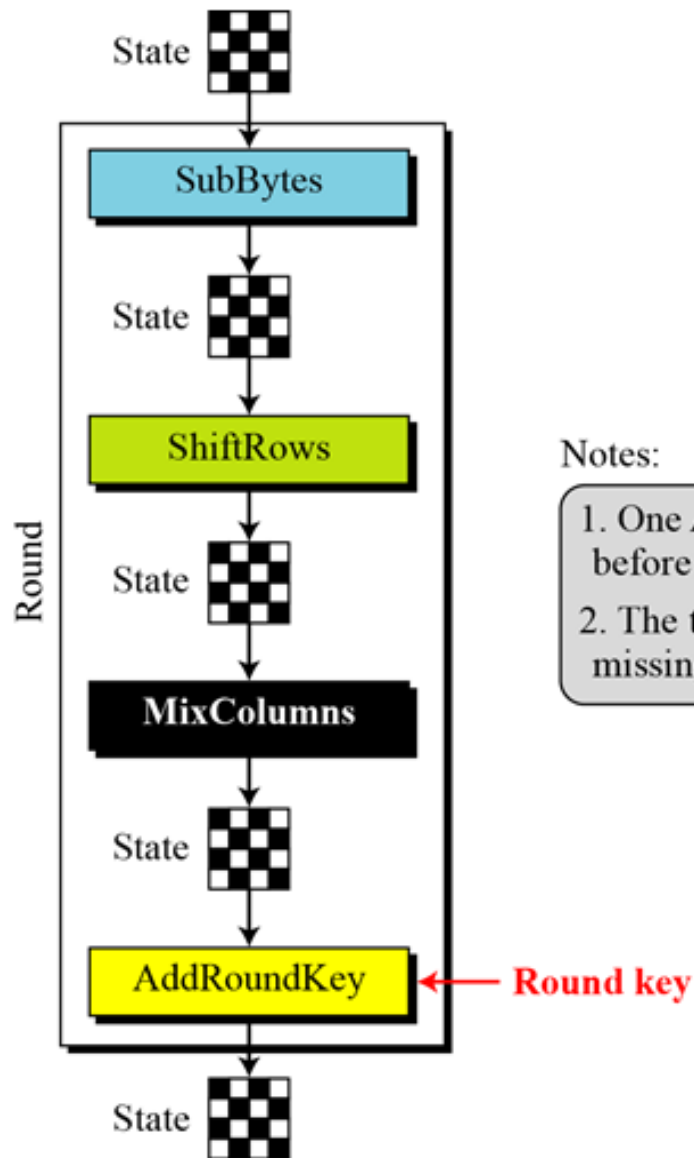
A E S U S E S A M A T R I X **Z Z**

Hexadecimal

00 04 12 14 12 04 12 00 0C 00 13 11 08 23 19 19

$$\begin{bmatrix} 00 & 12 & 0C & 08 \\ 04 & 04 & 00 & 23 \\ 12 & 12 & 13 & 19 \\ 14 & 00 & 11 & 19 \end{bmatrix} \text{State}$$

2.4.5 Structure of each round at the encryption site



Notes:

1. One AddRoundKey is applied before the first round.
2. The third transformation is missing in the last round.

2.4.6 *TRANSFORMATIONS*

To provide security, AES uses four types of transformations:

- 1. Substitution**
- 2. Permutation**
- 3. Mixing**
- 4. Key-Adding**

AES, like DES, uses substitution. AES uses two invertible transformations

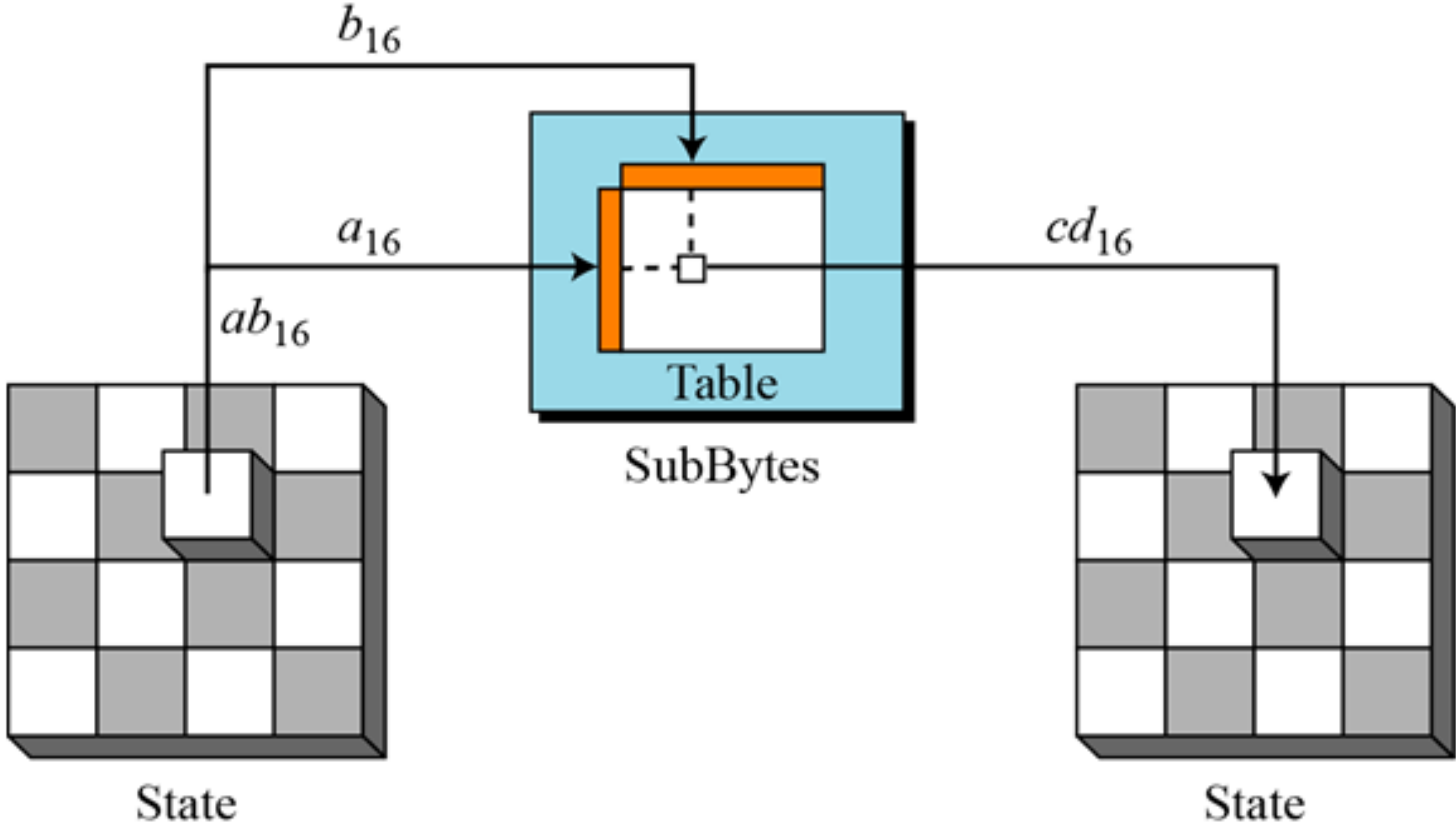
SubBytes

The first transformation, SubBytes, is used at the encryption site. To substitute a byte, we interpret the byte as two hexadecimal digits.

Note

The SubBytes operation involves 16 independent byte-to-byte transformations.

2.4.6.1 Substitution



2.4.6.1 AES S-Box Lookup Table

During encryption each value of the state is replaced with the corresponding SBOX value

AES S-Box Lookup Table

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

For example HEX 19 would get replaced with HEX D4

2.4.6.1 AES Inverse S-Box Lookup Table

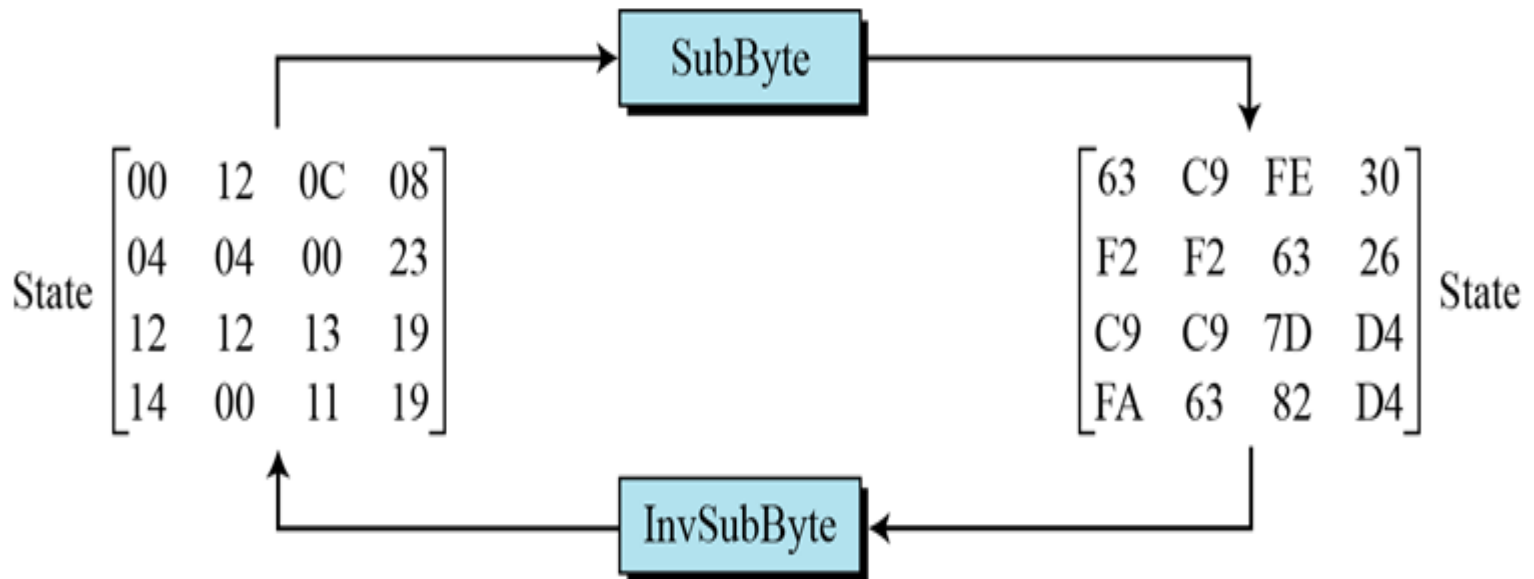
During decryption each value in the state is replaced with the corresponding inverse of the SBOX

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

For example HEX D4 would get replaced with HEX 19

2.4.6.1 Example: Substitution

Figure below shows how a state is transformed using the SubBytes transformation. The figure also shows that the InvSubBytes transformation creates the original one. Note that if the two bytes have the same values, their transformation is also the same.

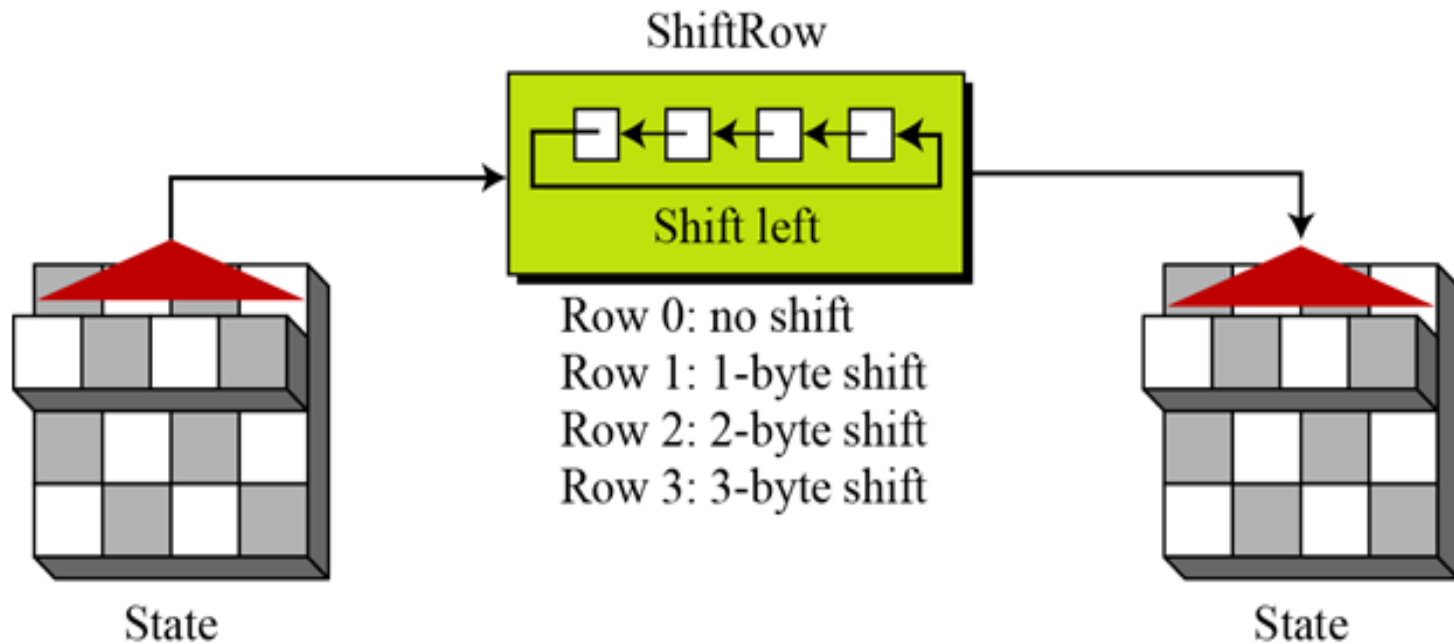


2.4.6.2 Permutation

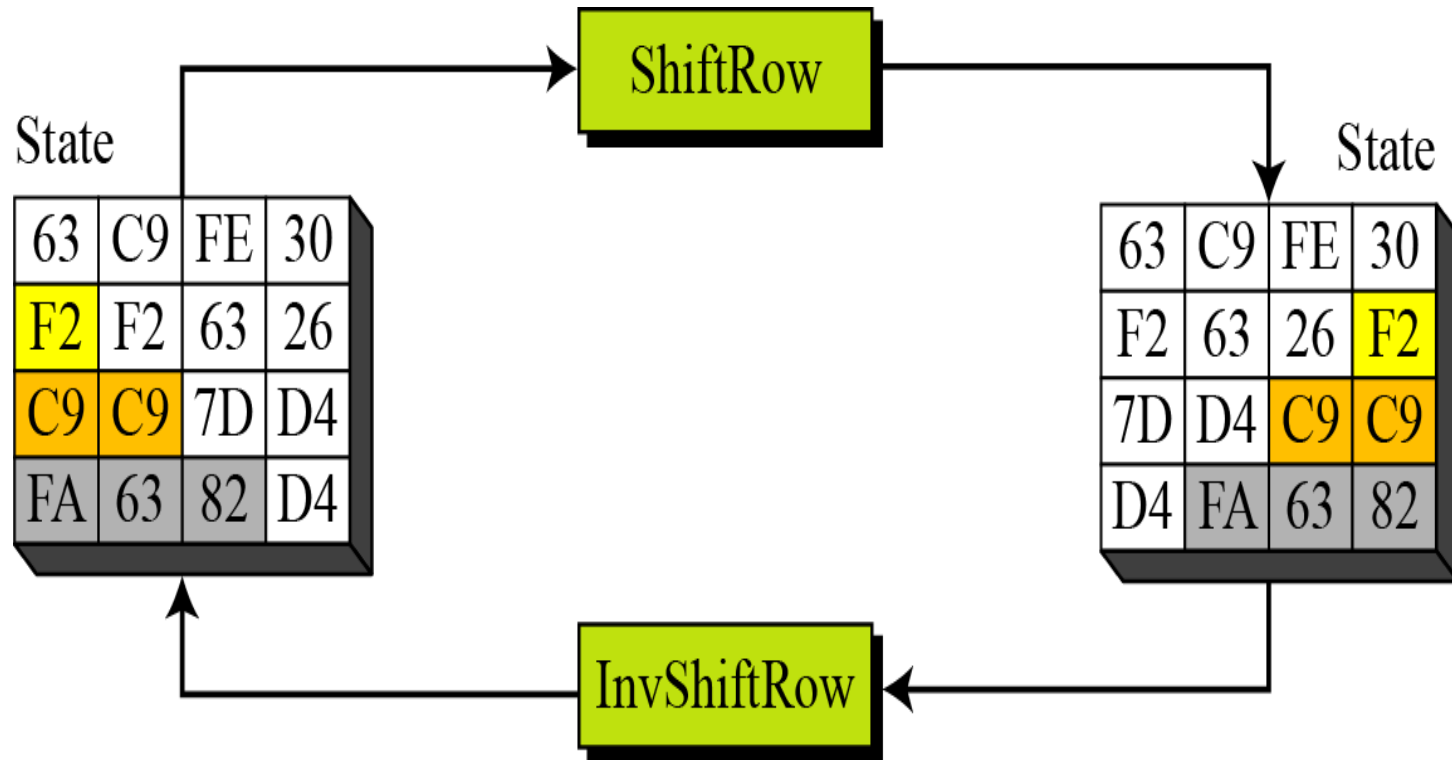
Another transformation found in a round is shifting, which permutes the bytes.

ShiftRows

In the encryption, the transformation is called ShiftRows.



2.4.6.2 *Permutation Shift Rows Example 1*



2.4.6.2 *Permutation Shift Rows Example2*

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

1 5 9 13
2 6 10 14
3 7 11 15
4 8 12 16

Row1 0
Row2 1
Row3 2
Row4 3

From	To
1 5 9 13	1 5 9 13
2 6 10 14	6 10 14 2
3 7 11 15	11 15 3 7
4 8 12 16	16 4 8 12

From	To
1 5 9 13	1 5 9 13
2 6 10 14	14 2 6 10
3 7 11 15	11 15 3 7
4 8 12 16	8 12 16 4

2.4.6.3 Mixing

We need an interbyte transformation that changes the bits inside a byte, based on the bits inside the neighboring bytes. We need to mix bytes to provide diffusion at the bit level.

$$\begin{array}{l} ax + by + cz + dt \\ ex + fy + gz + ht \\ ix + jy + kz + lt \\ mx + ny + oz + pt \end{array} \begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \left[\begin{array}{c} \text{Green Box} \\ \text{Green Box} \\ \text{Green Box} \\ \text{Green Box} \end{array} \right] = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

New matrix **Constant matrix** Old matrix

2.4.6.3 Mixing / Constant Matrices

Constant matrices used by MixColumns and InvMixColumns

$$\begin{array}{c} \left[\begin{array}{cccc} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{array} \right] \\ C \end{array} \xleftrightarrow{\text{Inverse}} \begin{array}{c} \left[\begin{array}{cccc} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{array} \right] \\ C^{-1} \end{array}$$

MixColumns

The MixColumns transformation operates at the column level; it transforms each column of the state to a new column.

2.4.6.3 Mixing / Example

Multiplication Matrix

2	3	1	1
1	2	3	1
1	1	2	3
3	1	1	2

16 byte State

b1	b5	b9	b13
b2	b6	b10	b14
b3	b7	b11	b15
b4	b8	b12	b16

$$b1 = (b1 * 2) \text{ XOR } (b2 * 3) \text{ XOR } (b3 * 1) \text{ XOR } (b4 * 1)$$

$$b2 = (b1 * 1) \text{ XOR } (b2 * 2) \text{ XOR } (b3 * 3) \text{ XOR } (b4 * 1)$$

$$b3 = (b1 * 1) \text{ XOR } (b2 * 1) \text{ XOR } (b3 * 2) \text{ XOR } (b4 * 3)$$

$$b4 = (b1 * 3) \text{ XOR } (b2 * 1) \text{ XOR } (b3 * 1) \text{ XOR } (b4 * 2)$$

$$b5 = (b5 * 2) \text{ XOR } (b6 * 3) \text{ XOR } (b7 * 1) \text{ XOR } (b8 * 1)$$

$$b6 = (b5 * 1) \text{ XOR } (b6 * 2) \text{ XOR } (b7 * 3) \text{ XOR } (b8 * 1)$$

$$b7 = (b5 * 1) \text{ XOR } (b6 * 1) \text{ XOR } (b7 * 2) \text{ XOR } (b8 * 3)$$

$$b8 = (b5 * 3) \text{ XOR } (b6 * 1) \text{ XOR } (b7 * 1) \text{ XOR } (b8 * 2)$$

2.4.6.3 Mixing / Galois Field Multiplication

The result of the multiplication is simply the result of a lookup of the L table, followed by the addition of the results, followed by a lookup to the E table. The addition is a regular mathematical addition represented by +, not a bitwise AND.

L Table

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	00	19	01	32	02	1A	C6	4B	C7	1B	68	33	EE	DF	03	
1	64	04	E0	0E	34	8D	81	EF	4C	71	08	C8	F8	69	1C	C1
2	7D	C2	1D	B5	F9	B9	27	6A	4D	E4	A6	72	9A	C9	09	78
3	65	2F	8A	05	21	0F	E1	24	12	F0	82	45	35	93	DA	8E
4	96	8F	DB	BD	36	D0	CE	94	13	5C	D2	F1	40	46	83	38
5	66	DD	FD	30	BF	06	8B	62	B3	25	E2	98	22	88	91	10
6	7E	6E	48	C3	A3	B6	1E	42	3A	6B	28	54	FA	85	3D	BA
7	2B	79	0A	15	9B	9F	5E	CA	4E	D4	AC	E5	F3	73	A7	57
8	AF	58	A8	50	F4	EA	D6	74	4F	AE	E9	D5	E7	E6	AD	E8
9	2C	D7	75	7A	EB	16	0B	F5	59	CB	5F	B0	9C	A9	51	A0
A	7F	0C	F6	6F	17	C4	49	EC	D8	43	1F	2D	A4	76	7B	B7
B	CC	BB	3E	5A	FB	60	B1	86	3B	52	A1	6C	AA	55	29	9D
C	97	B2	87	90	61	BE	DC	FC	BC	95	CF	CD	37	3F	5B	D1
D	53	39	84	3C	41	A2	6D	47	14	2A	9E	5D	56	F2	D3	AB
E	44	11	92	D9	23	20	2E	89	B4	7C	B8	26	77	99	E3	A5
F	67	4A	ED	DE	C5	31	FE	18	0D	63	8C	80	C0	F7	70	07

2.4.6.3 Mixing / Galois Field Multiplication

E Table

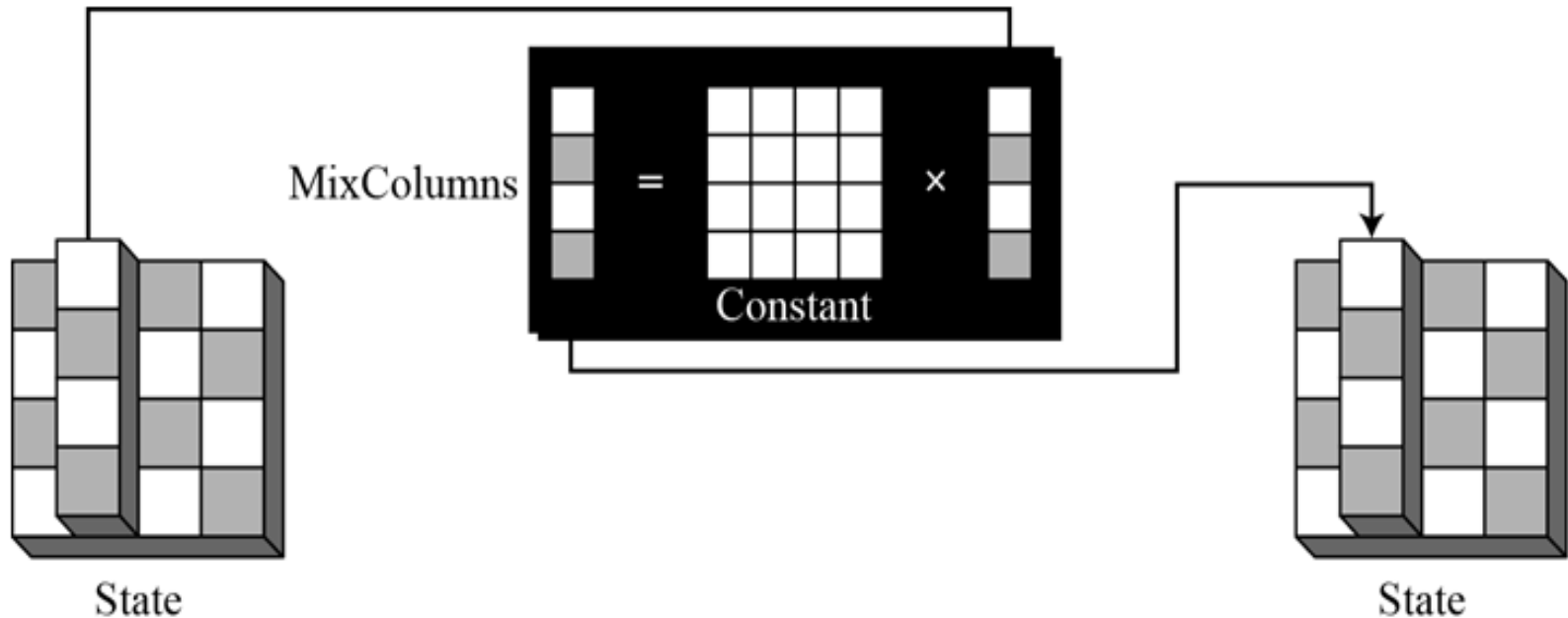
	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	01	03	05	0F	11	33	55	FF	1A	2E	72	96	A1	F8	13	35
1	5F	E1	38	48	D8	73	95	A4	F7	02	06	0A	1E	22	66	AA
2	E5	34	5C	E4	37	59	EB	26	6A	BE	D9	70	90	AB	E6	31
3	53	F5	04	0C	14	3C	44	CC	4F	D1	68	B8	D3	6E	B2	CD
4	4C	D4	67	A9	E0	3B	4D	D7	62	A6	F1	08	18	28	78	88
5	83	9E	B9	D0	6B	BD	DC	7F	81	98	B3	CE	49	DB	76	9A
6	B5	C4	57	F9	10	30	50	F0	0B	1D	27	69	BB	D6	61	A3
7	FE	19	2B	7D	87	92	AD	EC	2F	71	93	AE	E9	20	60	A0
8	FB	16	3A	4E	D2	6D	B7	C2	5D	E7	32	56	FA	15	3F	41
9	C3	5E	E2	3D	47	C9	40	C0	5B	ED	2C	74	9C	BF	DA	75
A	9F	BA	D5	64	AC	EF	2A	7E	82	9D	BC	DF	7A	8E	89	80
B	9B	B6	C1	58	E8	23	65	AF	EA	25	6F	B1	C8	43	C5	54
C	FC	1F	21	63	A5	F4	07	09	1B	2D	77	99	B0	CB	46	CA
D	45	CF	4A	DE	79	8B	86	91	A8	E3	3E	42	C6	51	F3	0E
E	12	36	5A	EE	29	7B	8D	8C	8F	8A	85	94	A7	F2	0D	17
F	39	4B	DD	7C	84	97	A2	FD	1C	24	6C	B4	C7	52	F6	01

For example if the two Hex values being multiplied are $AF * 8$ we first lookup L (AF) index which returns B7 and then lookup L (08) which returns 4B.

Once the L table lookup is complete we can then simply add the numbers together. The only trick being that if the addition result is greater than FF we subtract FF from the addition result.

For example $AF+B7= 166$. Because $166 > FF$, we perform: $166-FF$ which gives us 67.

2.4.6.3 Mixing



Note

The MixColumns and InvMixColumns transformations are inverses of each other.

2.4.6.3 Mix Column Example

Mix Column Example During Encryption

Input = D4 BF 5D 30

Output (0) = (D4 * 2) XOR (BF*3) XOR (5D*1) XOR (30*1)
= E(L(D4) + L(02)) XOR E(L(BF) + L(03)) XOR 5D XOR 30
= E(41 + 19) XOR E(9D + 01) XOR 5D XOR 30
= E(5A) XOR E(9E) XOR 5D XOR 30

= B3 XOR DA XOR 5D XOR 30
= **04**

Output (1) = (D4 * 1) XOR (BF*2) XOR (5D*3) XOR (30*1)
= D4 XOR E(L(BF)+L(02)) XOR E(L(5D)+L(03)) XOR 30
= D4 XOR E(9D+19) XOR E(88+01) XOR 30
= D4 XOR E(B6) XOR E(89) XOR 30
= D4 XOR 65 XOR E7 XOR 30
= **66**

Output (2) = (D4 * 1) XOR (BF*1) XOR (5D*2) XOR (30*3)
= D4 XOR BF XOR E(L(5D)+L(02)) XOR E(L(30)+L(03))
= D4 XOR BF XOR E(88+19) XOR E(65+01)
= D4 XOR BF XOR E(A1) XOR E(66)
= D4 XOR BF XOR BA XOR 50
= **81**

Output (3) = (D4 * 3) XOR (BF*1) XOR (5D*1) XOR (30*2)
= E(L(D4)+L(3)) XOR BF XOR 5D XOR E(L(30)+L(02))
= E(41+01) XOR BF XOR 5D XOR E(65+19)
= E(42) XOR BF XOR 5D XOR E(7E)
= 67 XOR BF XOR 5D XOR 60
= **E5**

2.4.6.3 Mix Column Example

Mix Column During Decryption

Input 04 66 81 E5

Output (0) - (04 * 0E) XOR (66*0B) XOR (81*0D) XOR (E5*09)
- E(L(04)+L(0E)) XOR E(L(66)+L(0B)) XOR E(L(81)+L(0D)) XOR E(L(E5)+L(09))
- E(32+DF) XOR E(1E+68) XOR E(58+EE) XOR E(20+C7)
- E(111-FF) XOR E(86) XOR E(146-FF) XOR E(E7)
- E(12) XOR E(86) XOR E(47) XOR E(E7)
- 38 XOR B7 XOR D7 XOR 8C
- D4

Output (1) - (04 * 09) XOR (66*0E) XOR (81*0B) XOR (E5*0D)
- E(L(04)+L(09)) XOR E(L(66)+L(0E)) XOR E(L(81)+L(0B)) XOR E(L(E5)+L(0D))
- E(32+C7) XOR E(1E+DF) XOR E(58+68) XOR E(20+ EE)
- E(F9) XOR E(FD) XOR E(C0) XOR E(10E-FF)
- E(F9) XOR E(FD) XOR E(C0) XOR E(0F)
- 24 XOR 52 XOR FC XOR 35
- BF

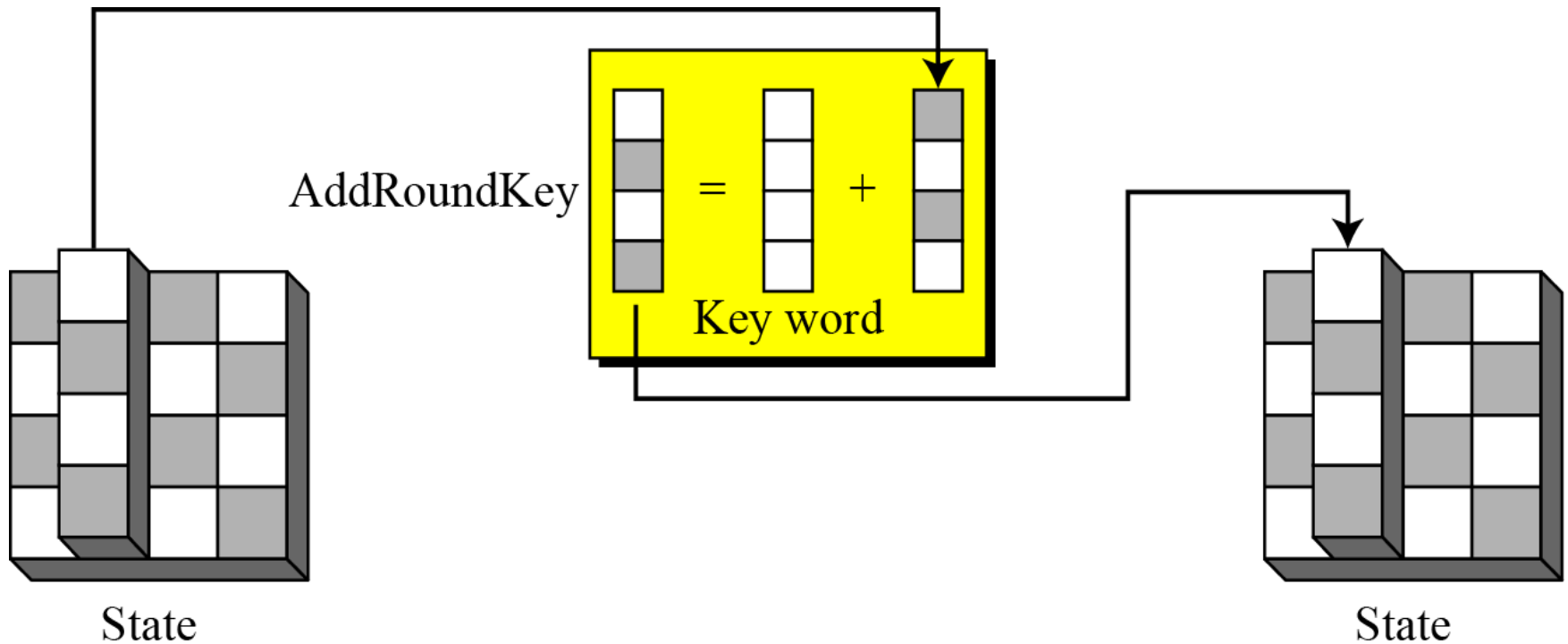
Output (2) - (04 * 0D) XOR (66*09) XOR (81*0E) XOR (E5*0B)
- E(L(04)+L(0D)) XOR E(L(66)+L(09)) XOR E(L(81)+L(0E)) XOR E(L(E5)+L(0B))
- E(32+EE) XOR E(1E+C7) XOR E(58+DF) XOR E(20+68)
- E(120-FF) XOR E(E5) XOR E(137-FF) XOR E(88)
- E(21) XOR E(E5) XOR E(38) XOR E(88)
- 34 XOR 7B XOR 4F XOR 5D
- 5D

Output (3) - (04 * 0B) XOR (66*0D) XOR (81*09) XOR (E5*0E)
- E(L(04)+L(0B)) XOR E(L(66)+L(0D)) XOR E(L(81)+L(09)) XOR E(L(E5)+L(0E))
- E(32+68) XOR E(1E+EE) XOR E(58+C7) XOR E(20+DF)
- E(9A) XOR E(10C-FF) XOR E(11F-FF) XOR E(FF)
- E(9A) XOR E(0D) XOR E(20) XOR E(FF)
- 2C XOR F8 XOR E5 XOR 01
- 30

2.4.6.4 Add Round Key

AddRoundKey

AddRoundKey proceeds one column at a time. AddRoundKey adds a round key word with each state column matrix; the operation in AddRoundKey is matrix addition.



2.4.6.4 Add Round Key

Each of the 16 bytes of the state is XORed against each of the 16 bytes of a portion of the expanded key for the current round. The Expanded Key bytes are never reused. So once the first 16 bytes are XORed against the first 16 bytes of the expanded key then the expanded key bytes 1-16 are never used again. The next time the Add Round Key function is called bytes 17-32 are XORed against the state.

The first time Add Round Key gets executed

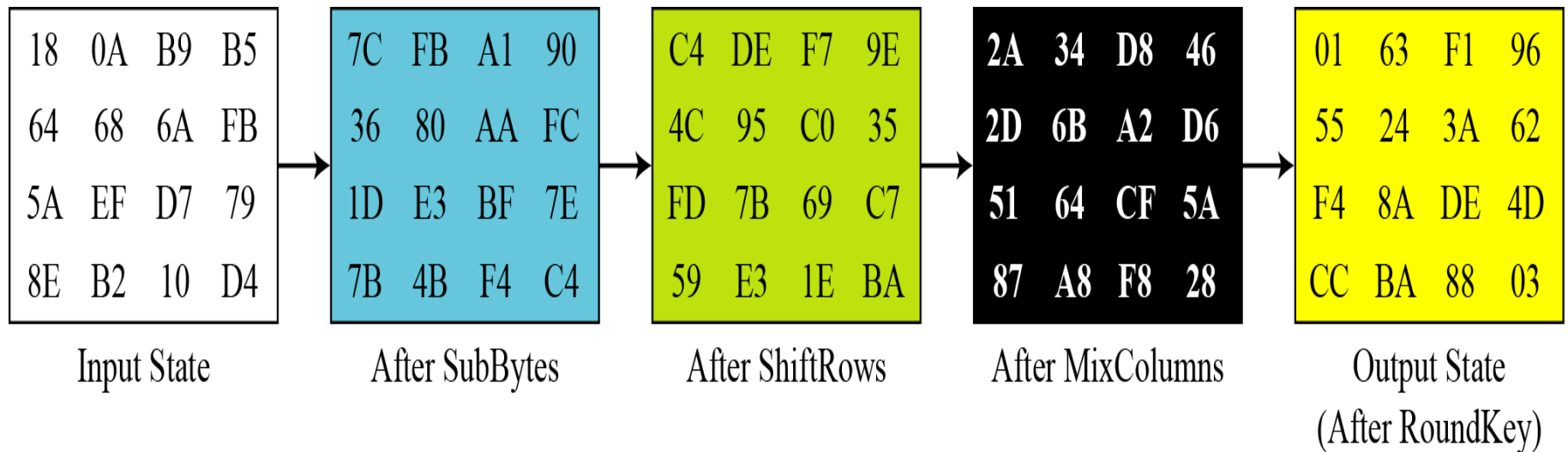
State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR
Exp Key	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

The second time Add Round Key is executed

State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR	XOR
Exp Key	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32

And so on for each round of execution.

2.4.6.5 Whole AES Example



2.4.6.6 ANALYSIS OF AES

AES was designed after DES. Most of the known attacks on DES were already tested on AES.

Brute-Force Attack

AES is definitely more secure than DES due to the larger-size key.

Statistical Attacks

Numerous tests have failed to do statistical analysis of the ciphertext.

Differential and Linear Attacks

There are no differential and linear attacks on AES as yet.

2.4.6.6 ANALYSIS OF AES

Statistical Attacks

Numerous tests have failed to do statistical analysis of the ciphertext.

Differential and Linear Attacks

There are no differential and linear attacks on AES as yet.

AES can be implemented in software, hardware, and firmware. The implementation can use table lookup process or routines that use a well-defined algebraic structure.

The algorithms used in AES are so simple that they can be easily implemented using cheap processors and a minimum amount of memory.

End of Chapter2/ Part5