Electrical Circuit-II

6th Lecture

Series and Parallel AC Circuits

(Part 2)

By:

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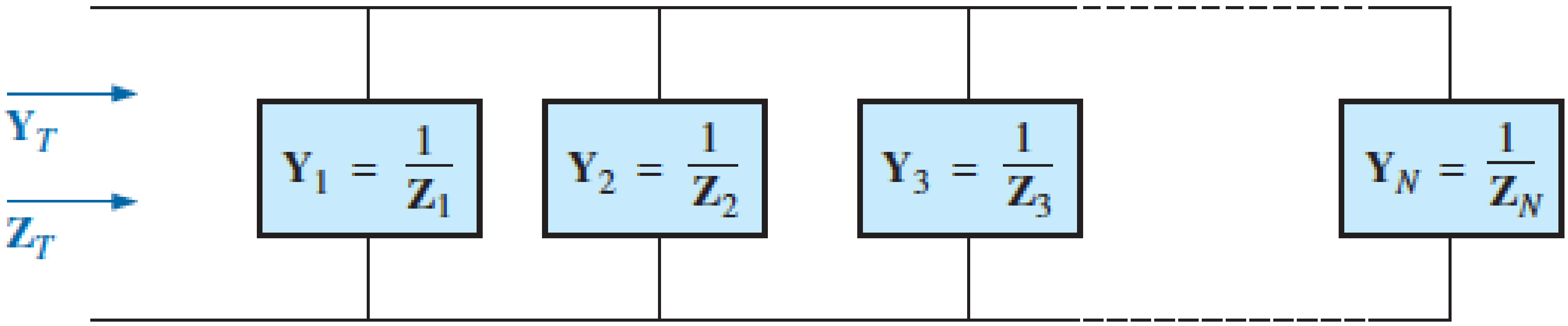
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Parallel Configuration

The discussion for parallel AC circuits is very similar to that for DC circuits. In DC circuits, Conductance (G) was defined as being equal to 1/R. The total conductance of a parallel circuit was then found by adding the conductance of each branch. The total resistance RT is simply 1/GT . In AC circuits, we define Admittance (Y) as being equal to 1/Z. The unit of measure for admittance as defined by the SI system is Siemens.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$$



Parallel Configuration

or, since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}$$

and

$$\mathbf{Z}_T = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}}$$

Parallel Configuration

For two impedances in parallel,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

For *N* parallel equal impedances (**Z**1)

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1}{\mathbf{N}}$$

For three parallel impedances

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3}$$

Parallel Configuration

Conductance of resistance

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ$$

The (1/X) is called Susceptance and is a measure of how susceptible an element is to the passage of current through it. Susceptance is also measured in Siemens and is represented by the capital letter B.

Susceptance of inductor

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ$$

$$B_L = \frac{1}{X_L} \quad \text{(siemens, S)}$$

$$\mathbf{Y}_L = B_L \angle -90^\circ$$

Parallel Configuration

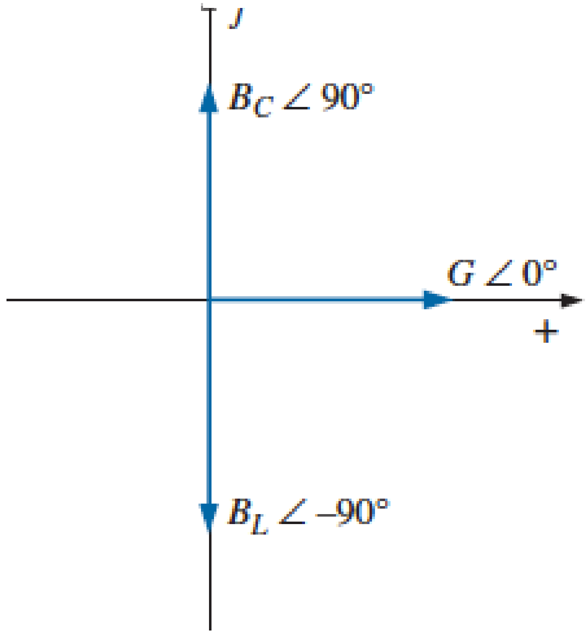
Susceptance of capacitor

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ$$

$$B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

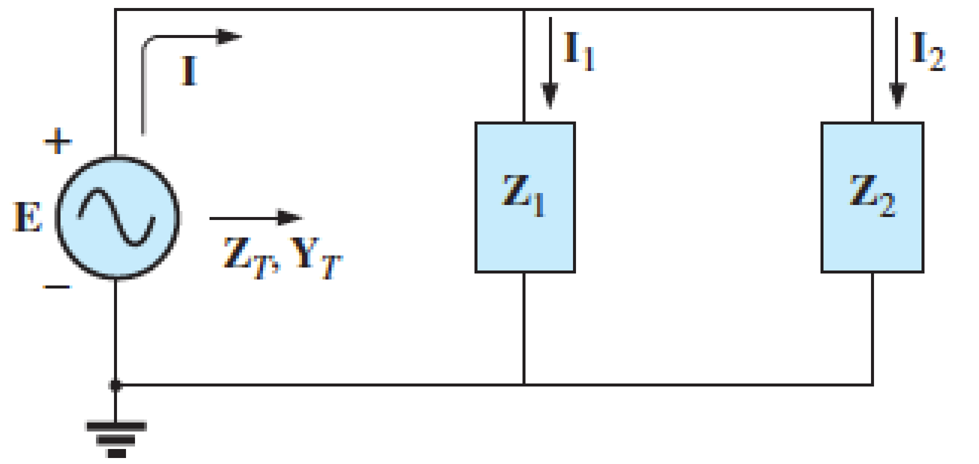
$$\mathbf{Y}_C = B_C \angle 90^\circ$$

Admittance Diagram



Parallel AC Network

$$\mathbf{I} - \mathbf{I}_1 - \mathbf{I}_2 = 0$$



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T$$

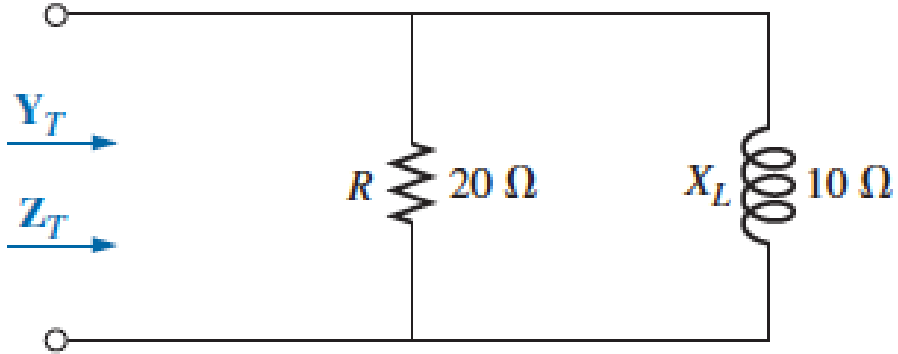
$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1} = \mathbf{E}\mathbf{Y}_1$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2} = \mathbf{E}\mathbf{Y}_2$$

The power to the network can be determined by

$$P = EI\cos\theta_T$$

Example



For the network shown

a. Calculate the input impedance.

b. Draw the impedance diagram.

c. Find the admittance of each parallel branch.

d. Determine the input admittance and draw the admittance diagram.

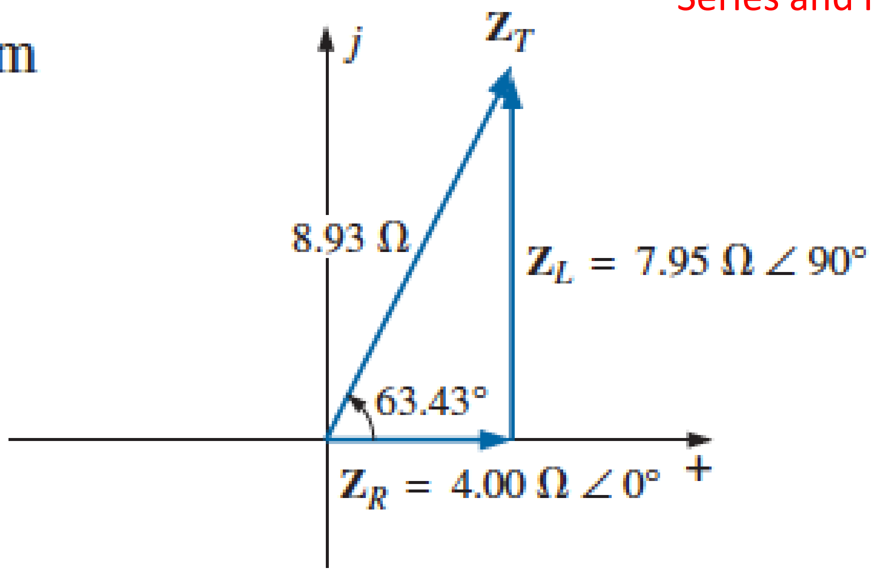
Solutions:

a. $$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega \angle 0^\circ)(10\ \Omega \angle 90^\circ)}{20\ \Omega + j\,10\ \Omega}$$

$$= \frac{200\ \Omega \angle 90^\circ}{22.361 \angle 26.57^\circ} = \mathbf{8.93\ \Omega \angle 63.43^\circ}$$

$$= \mathbf{4.00\ \Omega + j\,7.95\ \Omega} = R_T + j\,X_{L_T}$$

b. The impedance diagram



c. $$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{20\ \Omega}\angle 0^\circ = \mathbf{0.05\ S\ \angle 0^\circ}$$

$$= \mathbf{0.05\ S + \mathit{j}\ 0}$$

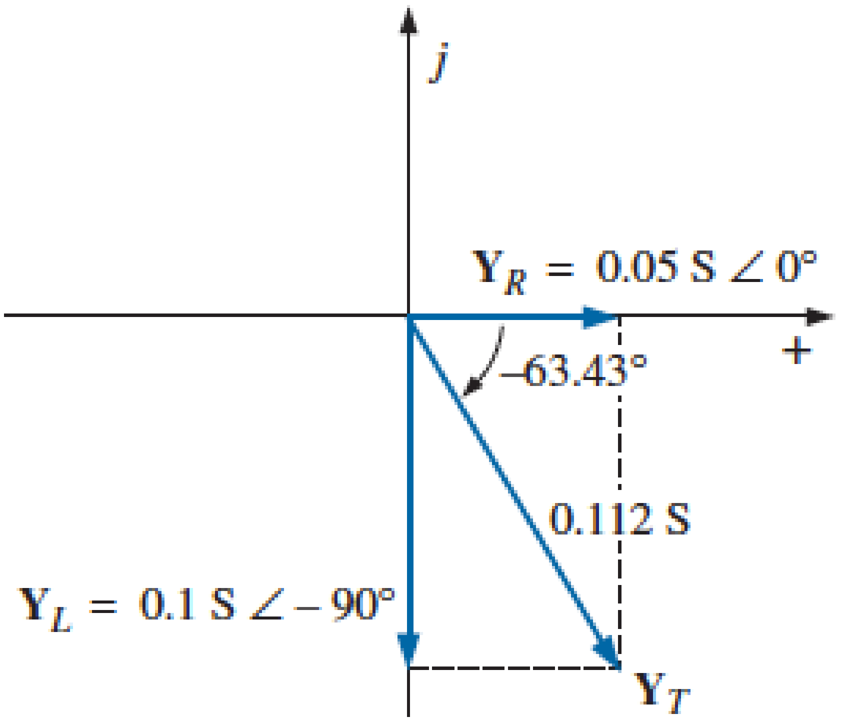
$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10\ \Omega}\angle -90^\circ$$

$$= \mathbf{0.1\ S\ \angle -90^\circ = 0 - \mathit{j}\ 0.1\ S}$$

d. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \text{S} + j\,0) + (0 - j\,0.1\ \text{S})$$

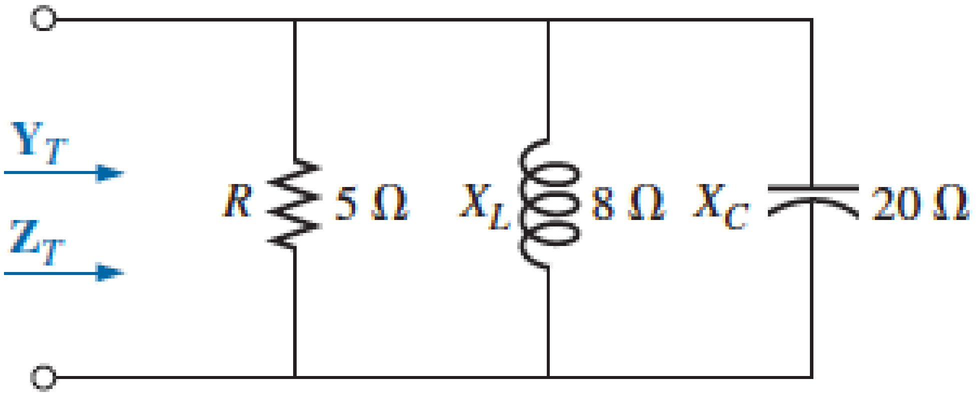
$$= \mathbf{0.05\ S - \mathit{j}\ 0.1\ S} = G - jB_L$$

The admittance diagram



Example

Repeat the previous example for the network shown



Solutions:

a.
$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_R} + \dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_C}}$$

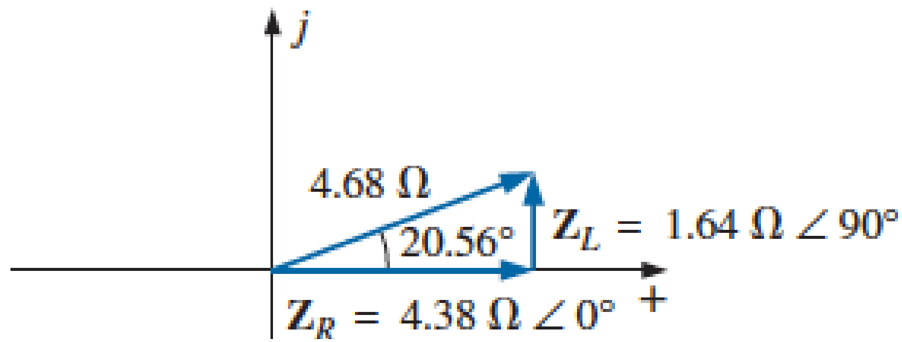
$$= \frac{1}{\dfrac{1}{5\ \Omega \angle 0^\circ} + \dfrac{1}{8\ \Omega \angle 90^\circ} + \dfrac{1}{20\ \Omega \angle -90^\circ}}$$

$$= \frac{1}{0.2\ \text{S} \angle 0^\circ + 0.125\ \text{S} \angle -90^\circ + 0.05\ \text{S} \angle 90^\circ}$$

$$= \frac{1}{0.2\ \text{S} - j\,0.075\ \text{S}} = \frac{1}{0.2136\ \text{S} \angle -20.56^\circ}$$

$$= \mathbf{4.68\ \Omega \angle 20.56^\circ}$$

b. The impedance diagram



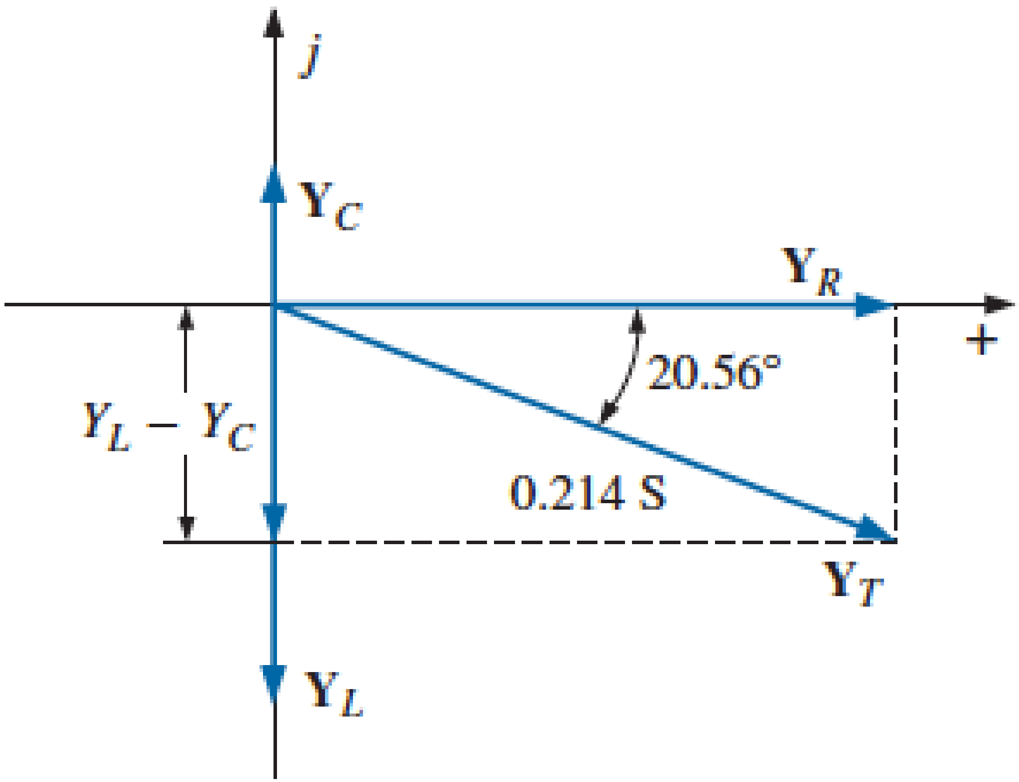
c. $$\mathbf{Y}_R = G\angle 0° = \frac{1}{R}\angle 0° = \frac{1}{5\ \Omega}\angle 0°$$
$$= \mathbf{0.2\ S\angle 0° = 0.2\ S} + \boldsymbol{j}\,\mathbf{0}$$

$$\mathbf{Y}_L = B_L\angle -90° = \frac{1}{X_L}\angle -90° = \frac{1}{8\ \Omega}\angle -90°$$
$$= \mathbf{0.125\ S\angle -90° = 0} - \boldsymbol{j}\,\mathbf{0.125\ S}$$

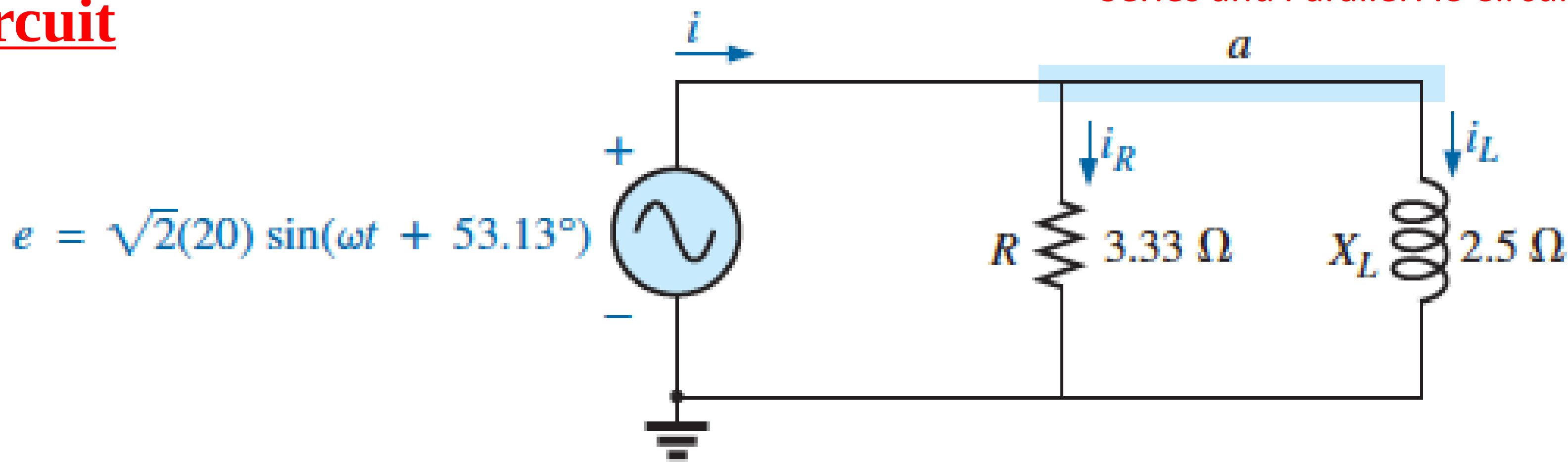
$$\mathbf{Y}_C = B_C\angle 90° = \frac{1}{X_C}\angle 90° = \frac{1}{20\ \Omega}\angle 90°$$
$$= \mathbf{0.050\ S\angle +90° = 0} + \boldsymbol{j}\,\mathbf{0.050\ S}$$

d. $$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$$
$$= (0.2\ \text{S} + j\,0) + (0 - j\,0.125\ \text{S}) + (0 + j\,0.050\ \text{S})$$
$$= 0.2\ \text{S} - j\,0.075\ \text{S} = \mathbf{0.214\ S\angle -20.56°}$$

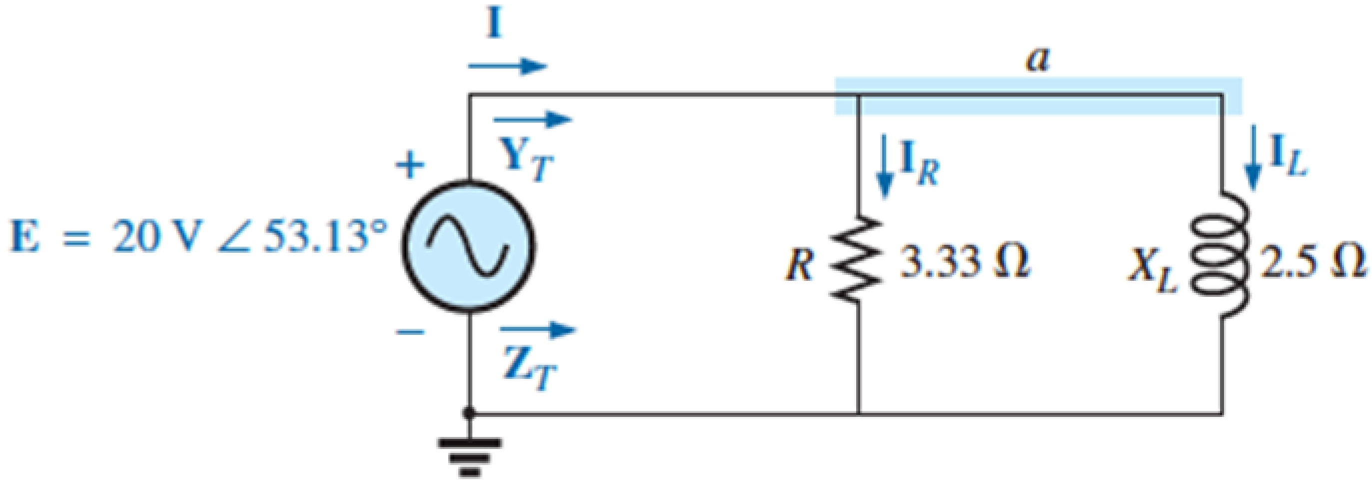
The admittance diagram



R – L Circuit



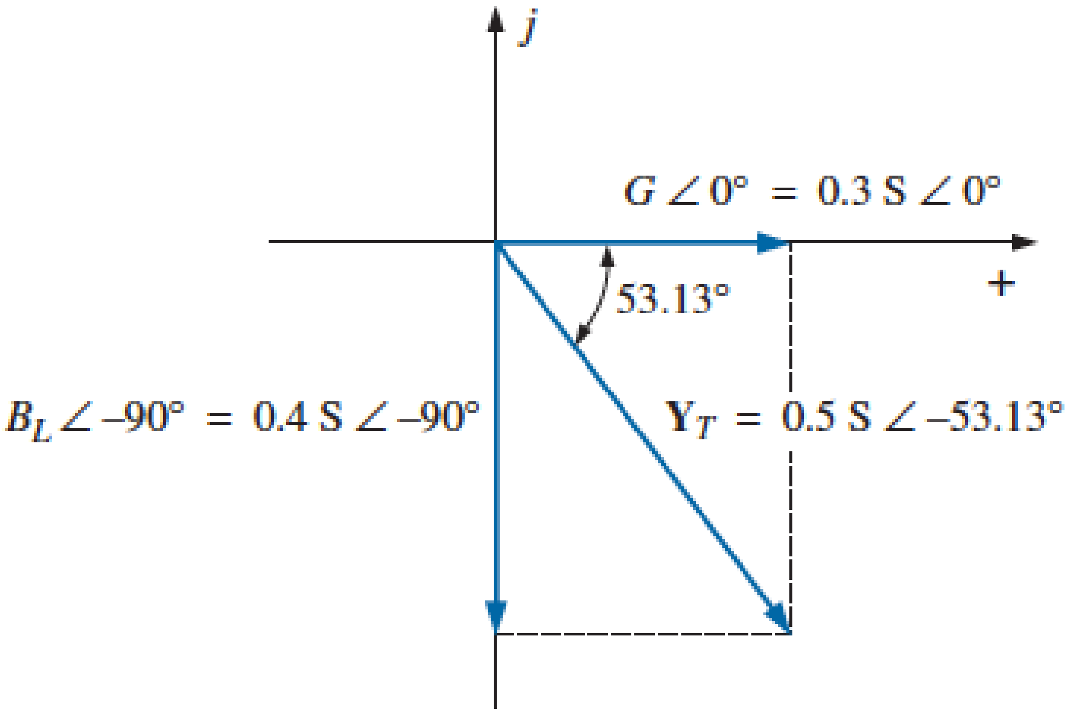
Phasor Representation



R – L Circuit

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L$$
$$= G\angle 0^\circ + B_L\angle -90^\circ = \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{2.5\ \Omega}\angle -90^\circ$$
$$= 0.3\ \text{S}\angle 0^\circ + 0.4\ \text{S}\angle -90^\circ = 0.3\ \text{S} - j\,0.4\ \text{S}$$
$$= \mathbf{0.5\ S\angle -53.13^\circ}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13^\circ} = \mathbf{2\ \Omega\angle 53.13^\circ}$$

Admittance diagram



$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\text{ V}\angle 53.13^\circ}{2\ \Omega\angle 53.13^\circ} = \mathbf{10\ A\angle 0^\circ}$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (20\text{ V}\angle 53.13^\circ)(0.5\text{ S}\angle -53.13^\circ) = \mathbf{10\ A\angle 0^\circ}$$

$$\mathbf{I}_R = \frac{E\angle\theta}{R\angle 0^\circ} = (E\angle\theta)(G\angle 0^\circ)$$
$$= (20\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle 0^\circ) = \mathbf{6\ A\angle 53.13^\circ}$$

$$\mathbf{I}_L = \frac{E\angle\theta}{X_L\angle 90^\circ} = (E\angle\theta)(B_L\angle -90^\circ)$$
$$= (20\text{ V}\angle 53.13^\circ)(0.4\text{ S}\angle -90^\circ)$$
$$= \mathbf{8\ A\angle -36.87^\circ}$$

Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L$$

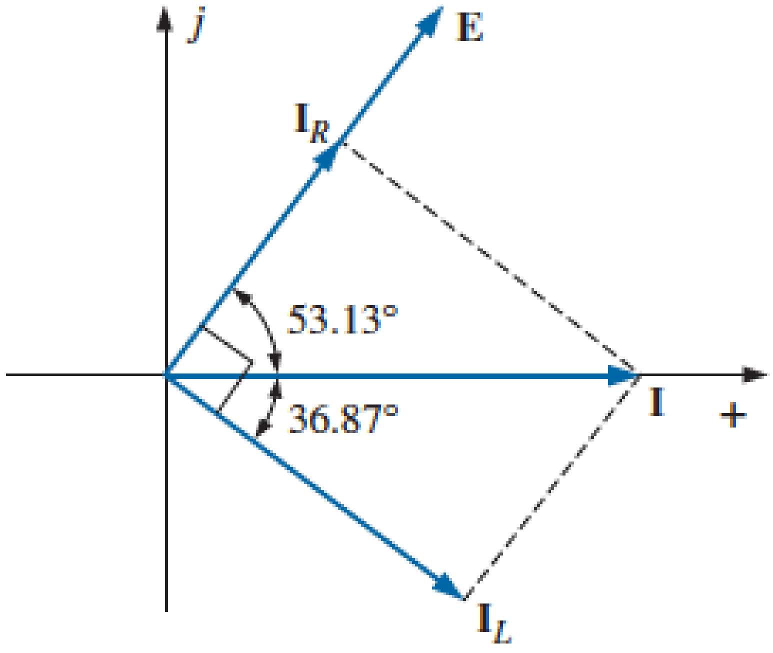
$$10\text{ A}\angle 0^\circ = 6\text{ A}\angle 53.13^\circ + 8\text{ A}\angle -36.87^\circ$$
$$10\text{ A}\angle 0^\circ = (3.60\text{ A} + j\,4.80\text{ A}) + (6.40\text{ A} - j\,4.80\text{ A}) = 10\text{ A} + j\,0$$

and

$$\mathbf{10\ A\angle 0^\circ = 10\ A\angle 0^\circ} \qquad \text{(checks)}$$

R – L Circuit

The phasor diagram indicates that the applied voltage **E** is **in phase** with the current I_R and **leads** the current I_L by **90°**.



Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T$$
$$= (20\text{ V})(10\text{ A})\cos 53.13° = (200\text{ W})(0.6)$$
$$= \mathbf{120\text{ W}}$$

or $$P_T = I^2R = \frac{V_R^2}{R} = V_R^2G = (20\text{ V})^2(0.3\text{ S}) = \mathbf{120\text{ W}}$$

R – L Circuit

$$P_T = P_R + P_L = EI_R \cos \theta_R + EI_L \cos \theta_L$$
$$= (20 \text{ V})(6 \text{ A}) \cos 0° + (20 \text{ V})(8 \text{ A}) \cos 90° = 120 \text{ W} + 0$$
$$= \mathbf{120 \text{ W}}$$

Power factor: The power factor of the circuit is

$$F_p = \cos \theta_T = \cos 53.13° = \mathbf{0.6 \text{ lagging}}$$

or, through an analysis similar to that used for a series ac circuit,

$$\cos \theta_T = \frac{P}{EI} = \frac{E^2/R}{EI} = \frac{EG}{I} = \frac{G}{I/V} = \frac{G}{Y_T}$$

$$\boxed{F_p = \cos \theta_T = \frac{G}{Y_T}}$$

$$F_p = \cos \theta_T = \frac{0.3 \text{ S}}{0.5 \text{ S}} = \mathbf{0.6 \text{ lagging}}$$