Electrical Circuit-II

7th Lecture-Tutorial

Series and Parallel AC Circuits

By:

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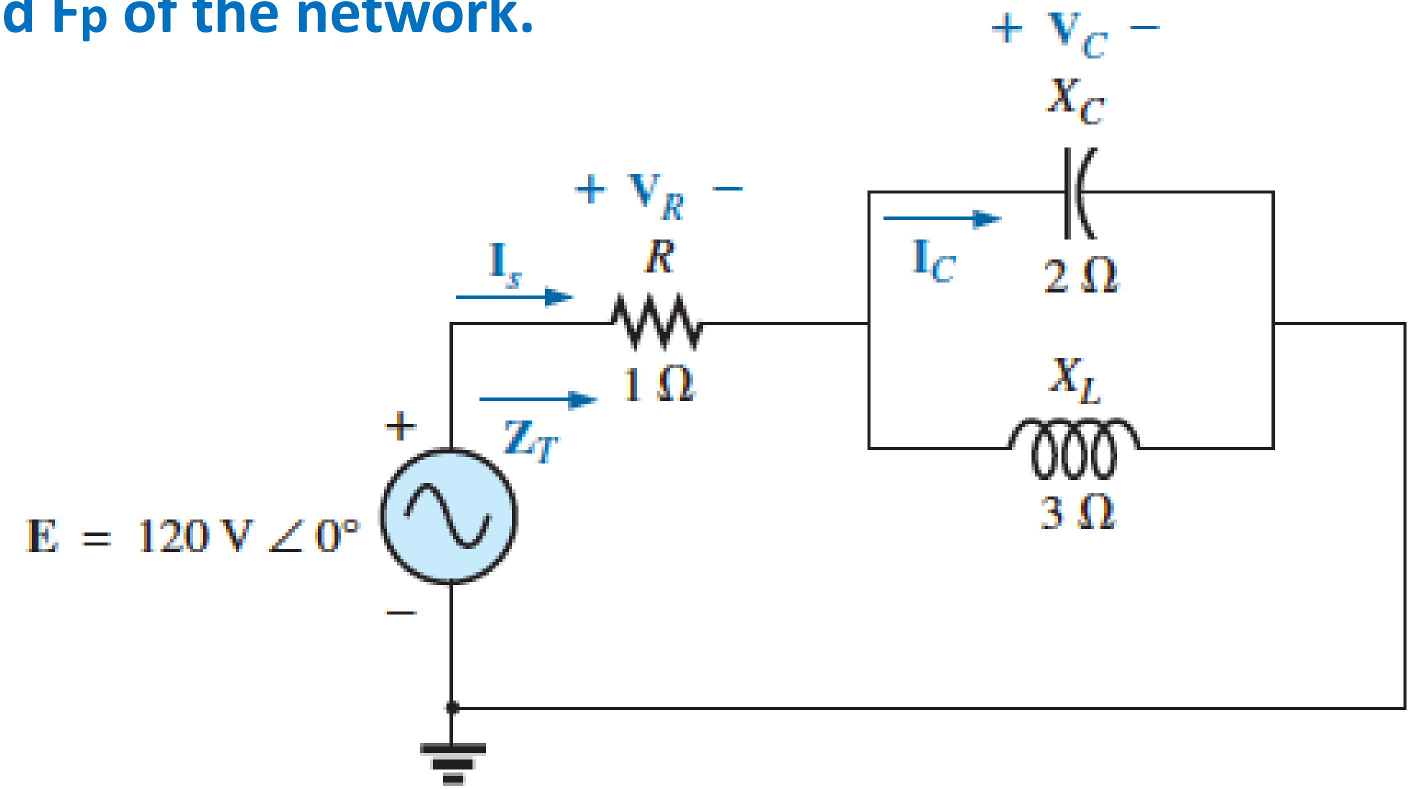
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

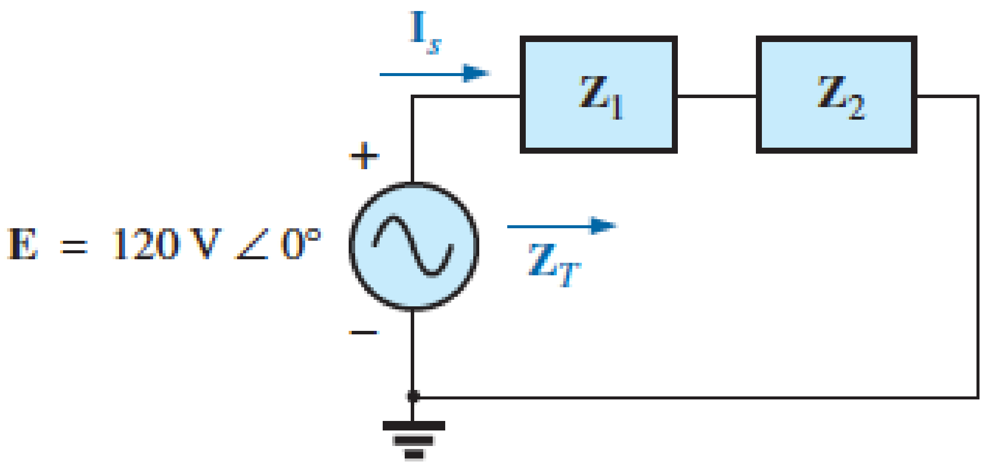
Example

For the network calculate:

a. Calculate Z_T .
b. Determine I_s .
c. Calculate V_R and V_C.
d. Find I_C.
e. Compute the power delivered.
f. Find F_p of the network.



Solution



a.

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

with

$$\mathbf{Z}_1 = R\angle 0° = 1\ \Omega \angle 0°$$

$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-j X_C + j X_L} = \frac{(2\ \Omega \angle -90°)(3\ \Omega \angle 90°)}{-j\,2\ \Omega + j\,3\ \Omega}$$

$$= \frac{6\ \Omega \angle 0°}{j\,1} = \frac{6\ \Omega \angle 0°}{1\angle 90°} = 6\ \Omega \angle -90°$$

and

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega \angle -80.54°}$$

b. $$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\text{ V} \angle 0°}{6.08\ \Omega \angle -80.54°} = \mathbf{19.74\ A \angle 80.54°}$$

c. $\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\text{ A}\angle 80.54°)(1\ \Omega\angle 0°) = \mathbf{19.74\ V\angle 80.54°}$

$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\text{ A}\angle 80.54°)(6\ \Omega\angle -90°)$$
$$= \mathbf{118.44\ V\angle -9.46°}$$

d. Now that $\mathbf{V}_C$ is known, the current $\mathbf{I}_C$ can also be found using Ohm's law.

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\text{ V}\angle -9.46°}{2\ \Omega\angle -90°} = \mathbf{59.22\ A\angle 80.54°}$$

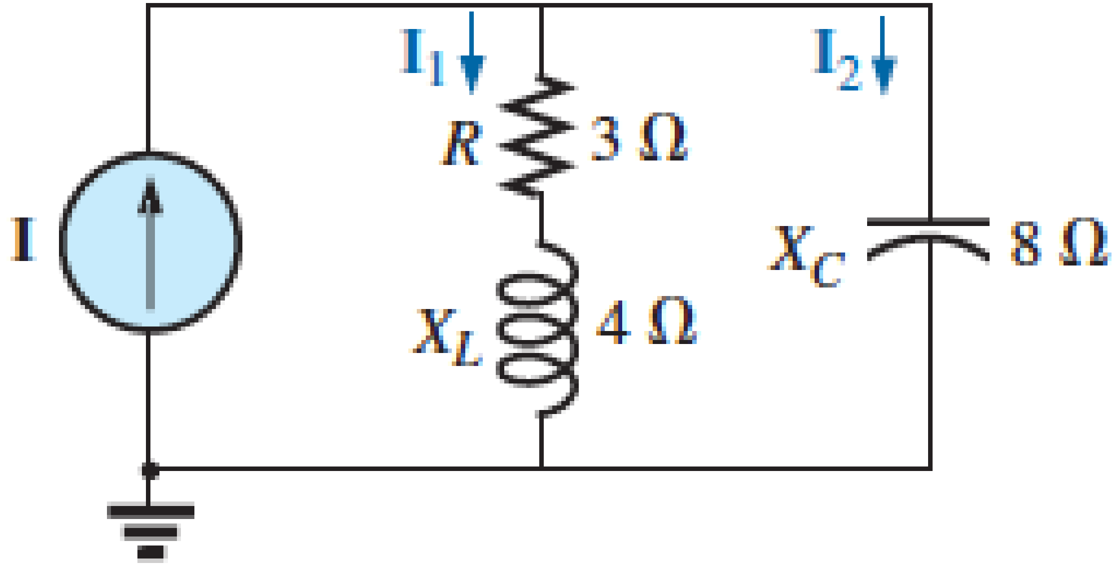
e. $P_{\text{del}} = I_s^2R = (19.74\text{ A})^2(1\ \Omega) = \mathbf{389.67\ W}$

f. $F_p = \cos\theta = \cos 80.54° = \mathbf{0.164\ leading}$

Example

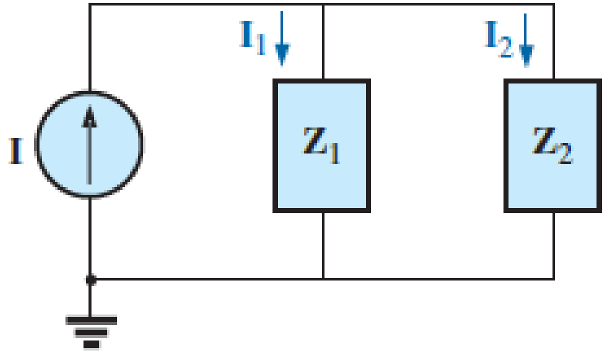
For the network calculate:

a. If $\mathbf{I}$ is 50 A $\angle 30°$, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.



Solution

a. Redrawing the circuit as



$$\mathbf{Z}_1 = R + j X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega \angle 53.13^\circ$$
$$\mathbf{Z}_2 = -j X_C = -j\,8\ \Omega = 8\ \Omega \angle -90^\circ$$

Using the current divider rule yields

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega \angle -90^\circ)(50\ \text{A} \angle 30^\circ)}{(-j\,8\ \Omega) + (3\ \Omega + j\,4\ \Omega)} = \frac{400 \angle -60^\circ}{3 - j\,4}$$
$$= \frac{400 \angle -60^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ A \angle -6.87^\circ}$$

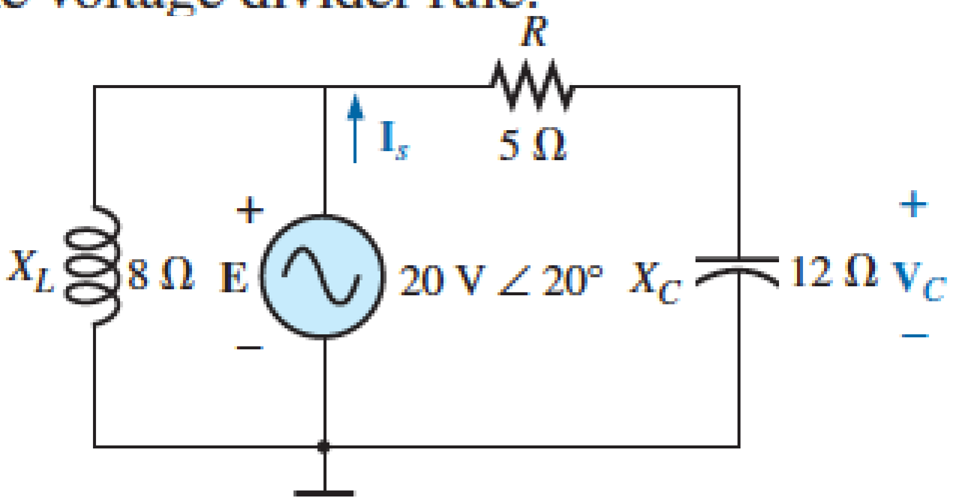
b. $$\mathbf{I}_2 = \frac{\mathbf{Z}_1 \mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5\ \Omega \angle 53.13^\circ)(50\ \text{A} \angle 30^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{250 \angle 83.13^\circ}{5 \angle -53.13^\circ}$$
$$= \mathbf{50\ A \angle 136.26^\circ}$$

c. $$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$
$$50\ \text{A} \angle 30^\circ = 80\ \text{A} \angle -6.87^\circ + 50\ \text{A} \angle 136.26^\circ$$
$$= (79.43 - j\,9.57) + (-36.12 + j\,34.57)$$
$$= 43.31 + j\,25.0$$
$$50\ \text{A} \angle 30^\circ = 50\ \text{A} \angle 30^\circ \qquad \text{(checks)}$$

Example

For the network calculate:

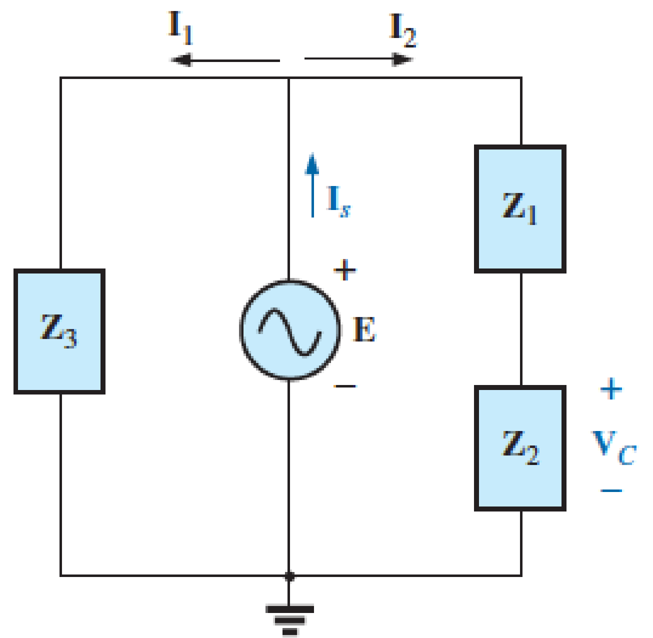
a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.



Solution

a. The network is redrawn

$$\mathbf{Z}_1 = 5\ \Omega = 5\ \Omega \angle 0^\circ$$
$$\mathbf{Z}_2 = -j\,12\ \Omega = 12\ \Omega \angle -90^\circ$$
$$\mathbf{Z}_3 = +j\,8\ \Omega = 8\ \Omega \angle 90^\circ$$



$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$
$$= \frac{(12\ \Omega \angle -90^\circ)(20\ \text{V} \angle 20^\circ)}{5\ \Omega - j\,12\ \Omega}$$
$$= \frac{240\ \text{V} \angle -70^\circ}{13 \angle -67.38^\circ}$$
$$= \mathbf{18.46\ V \angle -2.62^\circ}$$

Solution

b. $$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\text{ V}\angle 20^\circ}{8\ \Omega\angle 90^\circ} = 2.5\text{ A}\angle -70^\circ$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\text{ V}\angle 20^\circ}{13\ \Omega\angle -67.38^\circ} = 1.54\text{ A}\angle 87.38^\circ$$

and

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= 2.5\text{ A}\angle -70^\circ + 1.54\text{ A}\angle 87.38^\circ \\ &= (0.86 - j\,2.35) + (0.07 + j\,1.54) \\ \mathbf{I}_s &= 0.93 - j\,0.81 = \mathbf{1.23\text{ A}\angle -41.05^\circ}\end{aligned}$$