Electrical Circuit-II

7th Lecture

Series and Parallel AC Circuits

(Part 3)

By:

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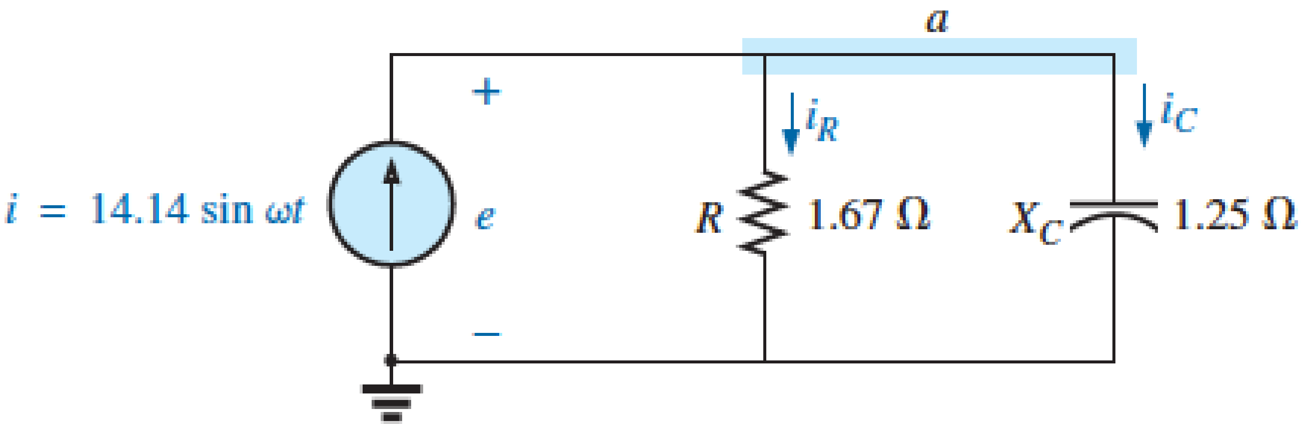
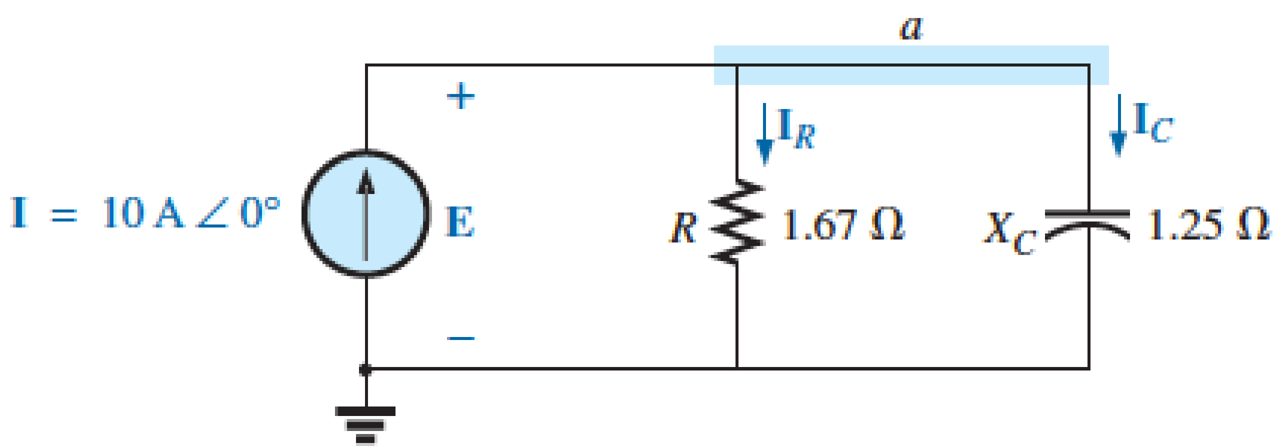
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

R – C Circuit



Phasor Representation



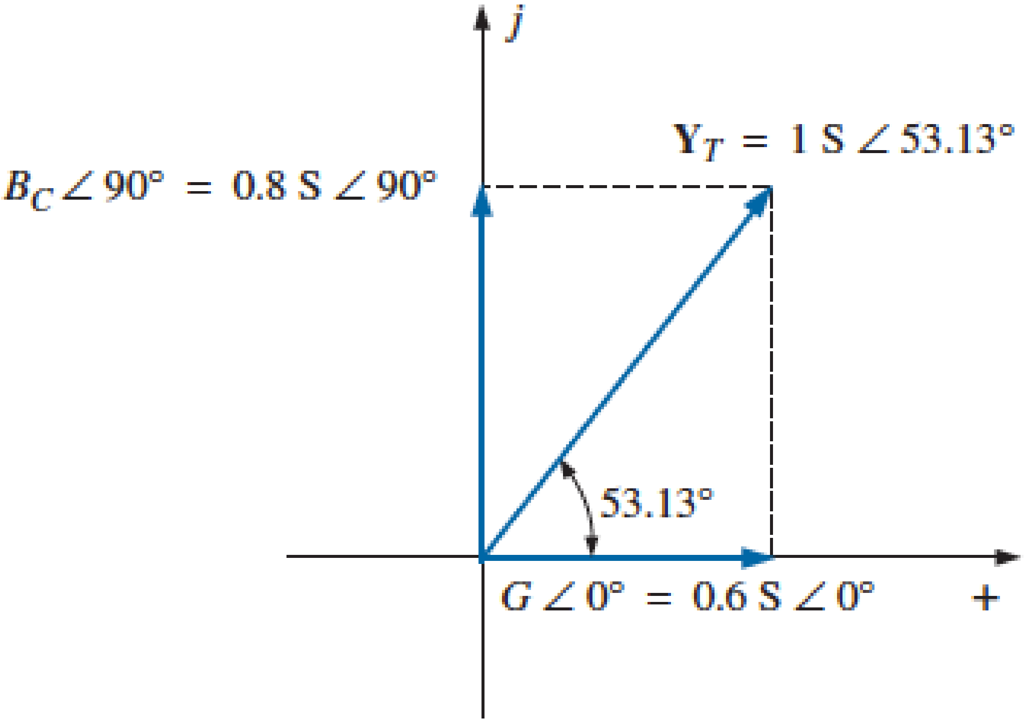
R – C Circuit

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_C = G\angle 0° + B_C\angle 90° = \frac{1}{1.67\ \Omega}\angle 0° + \frac{1}{1.25\ \Omega}\angle 90°$$

$$= 0.6\ \text{S}\angle 0° + 0.8\ \text{S}\angle 90° = 0.6\ \text{S} + j\,0.8\ \text{S} = \mathbf{1.0\ S\angle 53.13°}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{1.0\ \text{S}\angle 53.13°} = \mathbf{1\ \Omega\angle -53.13°}$$

Admittance diagram



R – C Circuit

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \frac{\mathbf{I}}{\mathbf{Y}_T} = \frac{10\ \text{A} \angle 0^\circ}{1\ \text{S} \angle 53.13^\circ} = \mathbf{10\ V \angle -53.13^\circ}$$

$$\begin{aligned}\mathbf{I}_R &= (E \angle \theta)(G \angle 0^\circ) \\ &= (10\ \text{V} \angle -53.13^\circ)(0.6\ \text{S} \angle 0^\circ) = \mathbf{6\ A \angle -53.13^\circ}\end{aligned}$$

$$\begin{aligned}\mathbf{I}_C &= (E \angle \theta)(B_C \angle 90^\circ) \\ &= (10\ \text{V} \angle -53.13^\circ)(0.8\ \text{S} \angle 90^\circ) = \mathbf{8\ A \angle 36.87^\circ}\end{aligned}$$

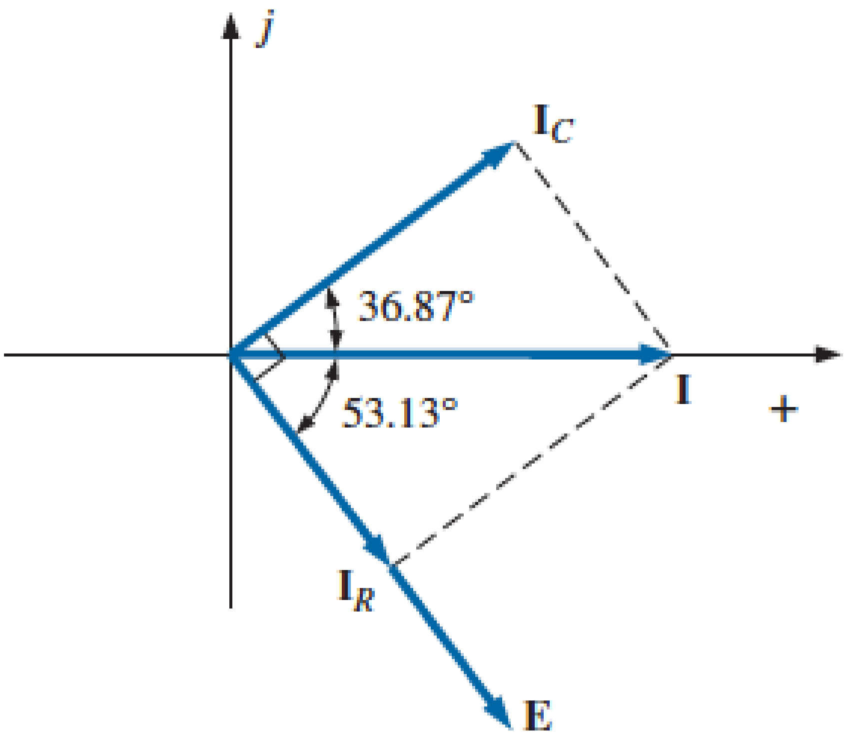
Kirchhoff's current law: At node *a*,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_C = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C$$

Phasor diagram

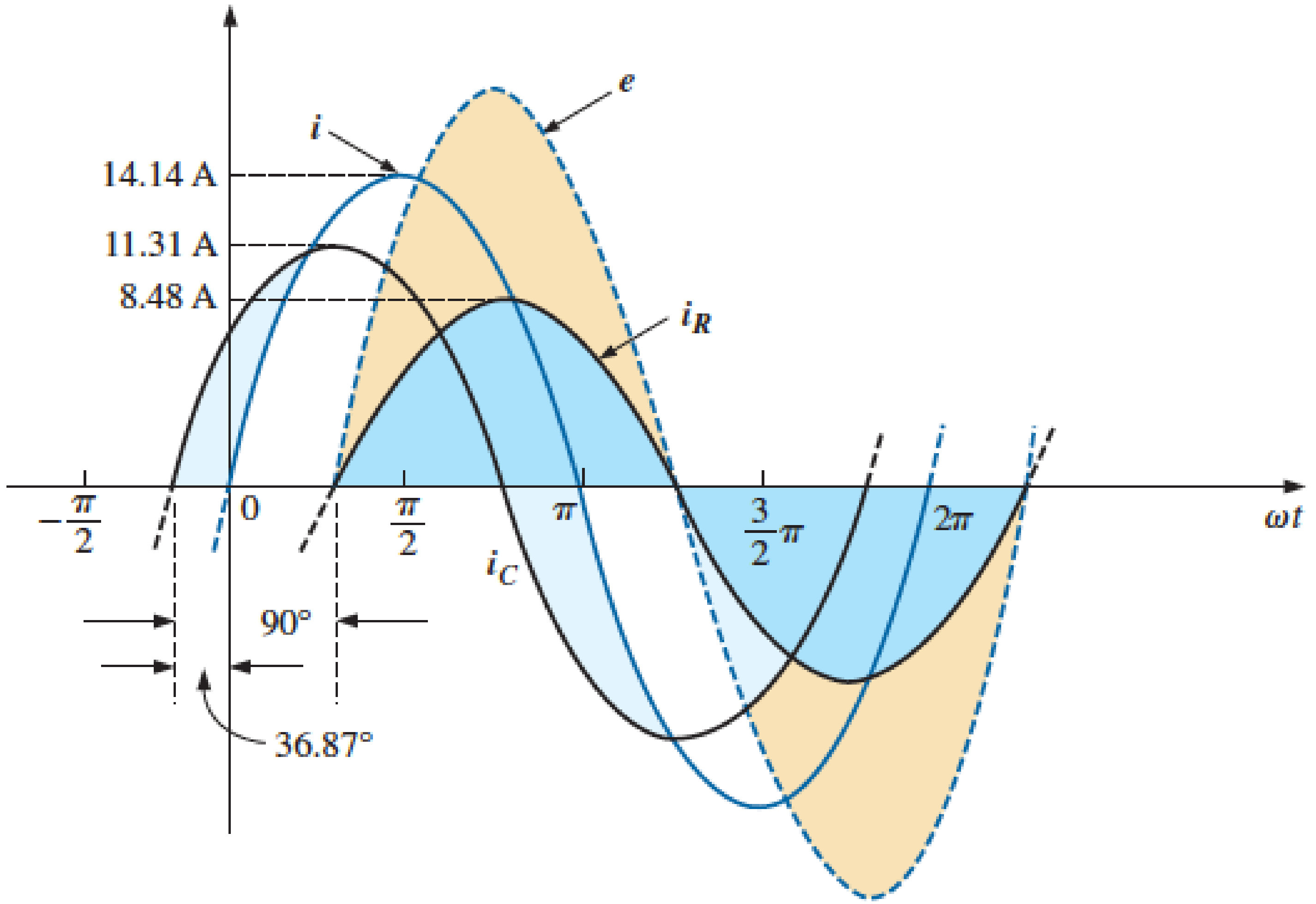


R – C Circuit

$$e = \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = \mathbf{14.14\sin(\omega t - 53.13^\circ)}$$

$$i_R = \sqrt{2}(6)\sin(\omega t - 53.13^\circ) = \mathbf{8.48\sin(\omega t - 53.13^\circ)}$$

$$i_C = \sqrt{2}(8)\sin(\omega t + 36.87^\circ) = \mathbf{11.31\sin(\omega t + 36.87^\circ)}$$



R – C Circuit

Power:

$$P_T = EI\cos\theta = (10\text{ V})(10\text{ A})\cos 53.13^\circ = (10)^2(0.6)$$
$$= \mathbf{60\ W}$$

or $$P_T = E^2G = (10\text{ V})^2(0.6\text{ S}) = \mathbf{60\ W}$$

or, finally,

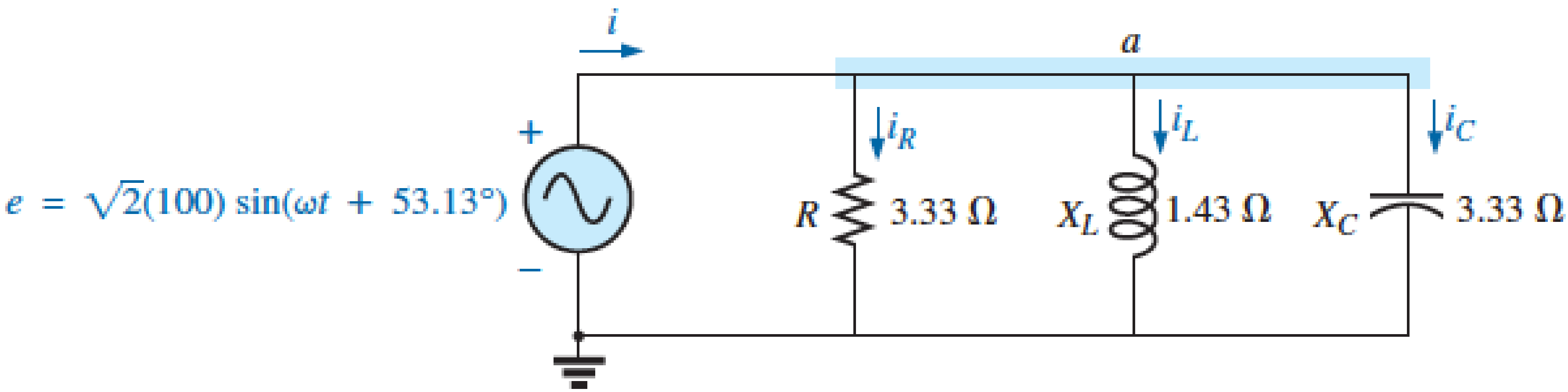
$$P_T = P_R + P_C = EI_R\cos\theta_R + EI_C\cos\theta_C$$
$$= (10\text{ V})(6\text{ A})\cos 0^\circ + (10\text{ V})(8\text{ A})\cos 90^\circ$$
$$= \mathbf{60\ W}$$

Power factor: The power factor of the circuit is

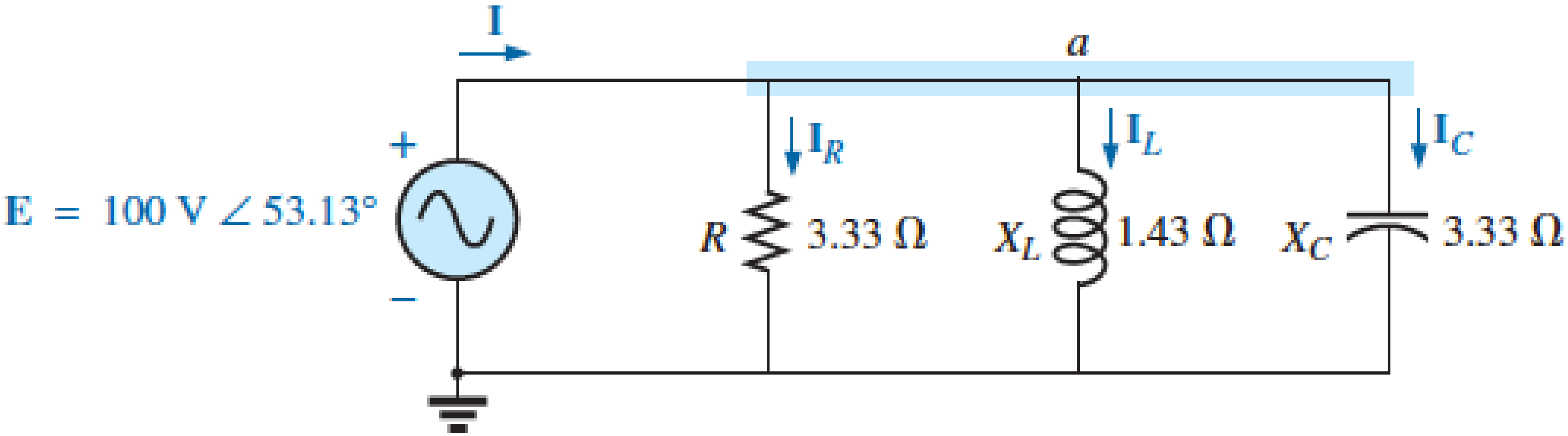
$$F_p = \cos 53.13^\circ = \mathbf{0.6\ leading}$$

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.6\text{ S}}{1.0\text{ S}} = \mathbf{0.6\ leading}$$

R – L – C Circuit



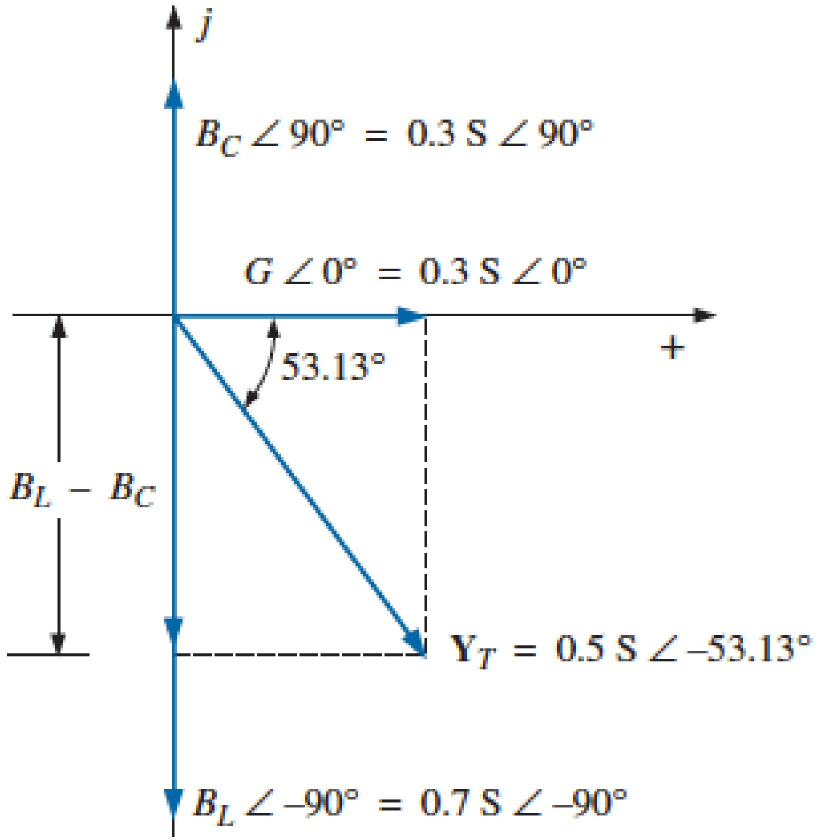
Phasor Representation



R – L – C Circuit

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C = G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle 90^\circ$$
$$= \frac{1}{3.33\ \Omega}\angle 0^\circ + \frac{1}{1.43\ \Omega}\angle -90^\circ + \frac{1}{3.33\ \Omega}\angle 90^\circ$$
$$= 0.3\ \text{S}\angle 0^\circ + 0.7\ \text{S}\angle -90^\circ + 0.3\ \text{S}\angle 90^\circ$$
$$= 0.3\ \text{S} - j\,0.7\ \text{S} + j\,0.3\ \text{S}$$
$$= 0.3\ \text{S} - j\,0.4\ \text{S} = \mathbf{0.5\ S\angle -53.13^\circ}$$
$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5\ \text{S}\angle -53.13^\circ} = \mathbf{2\ \Omega\angle 53.13^\circ}$$

Admittance diagram



R – L – C Circuit

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100\text{ V}\angle 53.13^\circ)(0.5\text{ S}\angle -53.13^\circ) = \mathbf{50\text{ A}\angle 0^\circ}$$

$$\begin{aligned}
\mathbf{I}_R &= (E\angle\theta(G\angle 0^\circ) \\
&= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle 0^\circ) = \mathbf{30\text{ A}\angle 53.13^\circ} \\
\mathbf{I}_L &= (E\angle\theta)(B_L\angle -90^\circ) \\
&= (100\text{ V}\angle 53.13^\circ)(0.7\text{ S}\angle -90^\circ) = \mathbf{70\text{ A}\angle -36.87^\circ} \\
\mathbf{I}_C &= (E\angle\theta)(B_C\angle 90^\circ) \\
&= (100\text{ V}\angle 53.13^\circ)(0.3\text{ S}\angle +90^\circ) = \mathbf{30\text{ A}\angle 143.13^\circ}
\end{aligned}$$

Kirchhoff's current law: At node *a*,

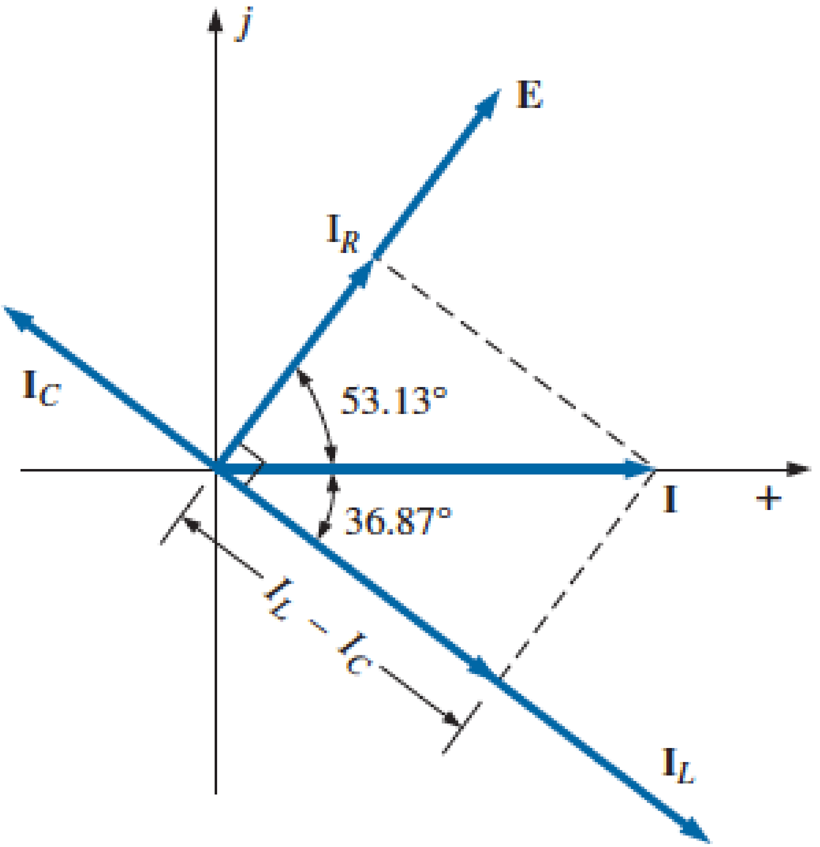
$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

or

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

R – L – C Circuit

Phasor diagram



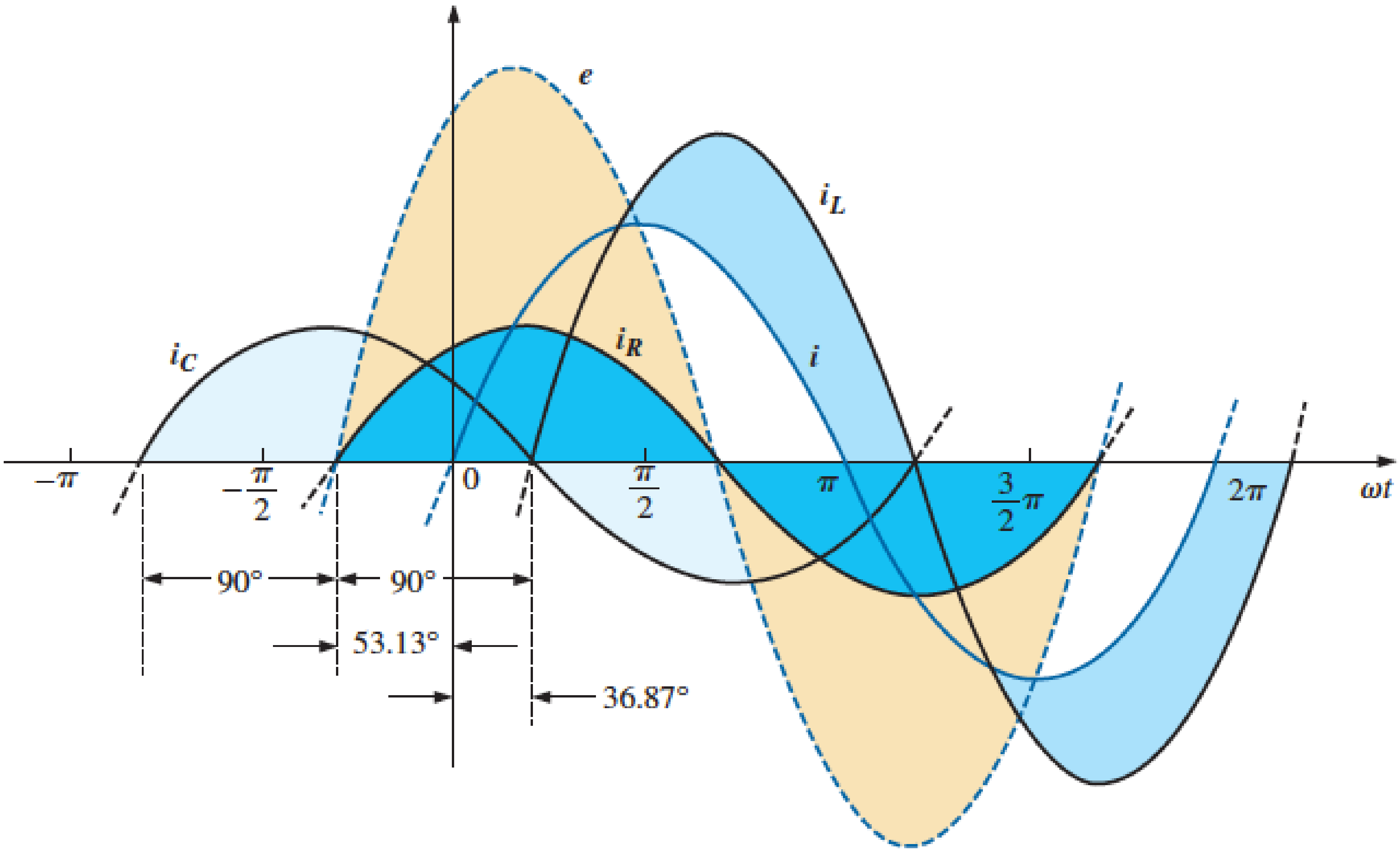
R – L – C Circuit

$$i = \sqrt{2}(50)\sin\omega t = \mathbf{70.70 \sin \omega t}$$

$$i_R = \sqrt{2}(30)\sin(\omega t + 53.13^\circ) = \mathbf{42.42 \sin(\omega t + 53.13^\circ)}$$

$$i_L = \sqrt{2}(70)\sin(\omega t - 36.87^\circ) = \mathbf{98.98 \sin(\omega t - 36.87^\circ)}$$

$$i_C = \sqrt{2}(30)\sin(\omega t + 143.13^\circ) = \mathbf{42.42 \sin(\omega t + 143.13^\circ)}$$



R – L – C Circuit

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (100\text{ V})(50\text{ A})\cos 53.13° = (5000)(0.6)$$
$$= \mathbf{3000\ W}$$

or

$$P_T = E^2G = (100\text{ V})^2(0.3\text{ S}) = \mathbf{3000\ W}$$

or, finally,

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= EI_R\cos\theta_R + EI_L\cos\theta_L + EI_C\cos\theta_C \\ &= (100\text{ V})(30\text{ A})\cos 0° + (100\text{ V})(70\text{ A})\cos 90° \\ &\qquad + (100\text{ V})(30\text{ A})\cos 90° \\ &= 3000\text{ W} + 0 + 0 \\ &= \mathbf{3000\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

$$F_p = \cos\theta_T = \frac{G}{Y_T} = \frac{0.3\text{ S}}{0.5\text{ S}} = \mathbf{0.6\ lagging}$$

Example

For the network calculate:

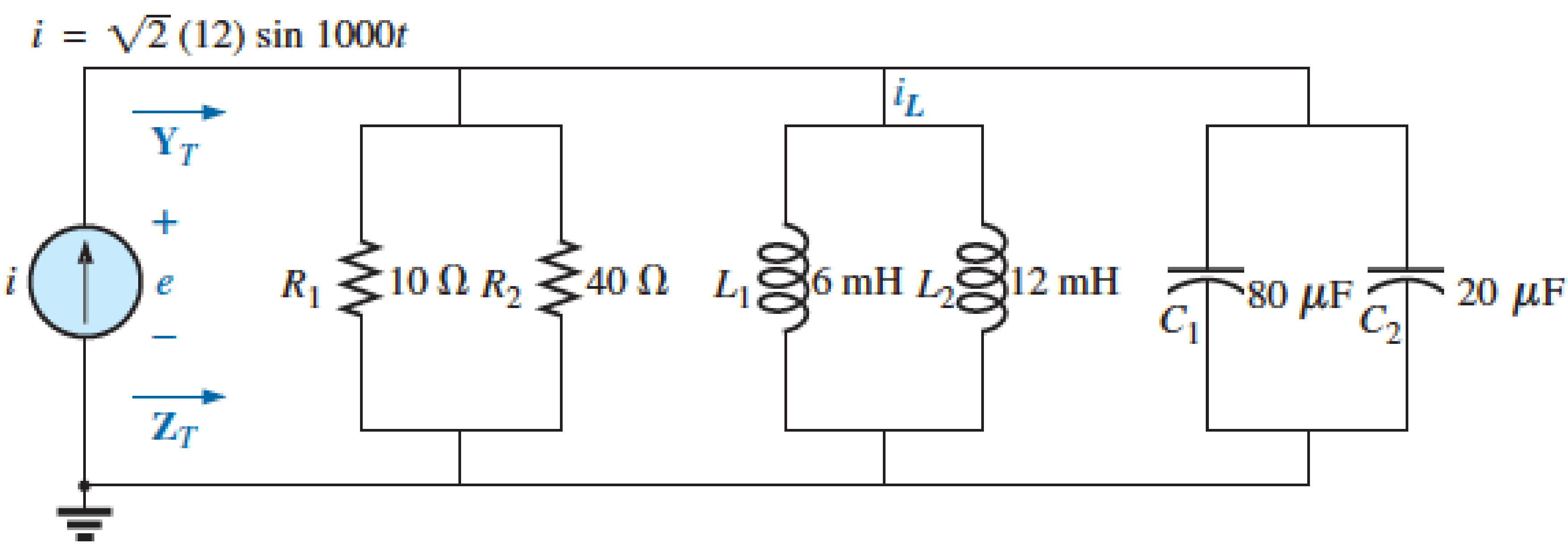
a. **Determine $\mathbf{Y}_T$ and $\mathbf{Z}_T$.**

b. **Sketch the admittance diagram.**

c. **Find E and $\mathbf{I}_L$.**

d. **Compute the power factor of the network and the power delivered to the network.**

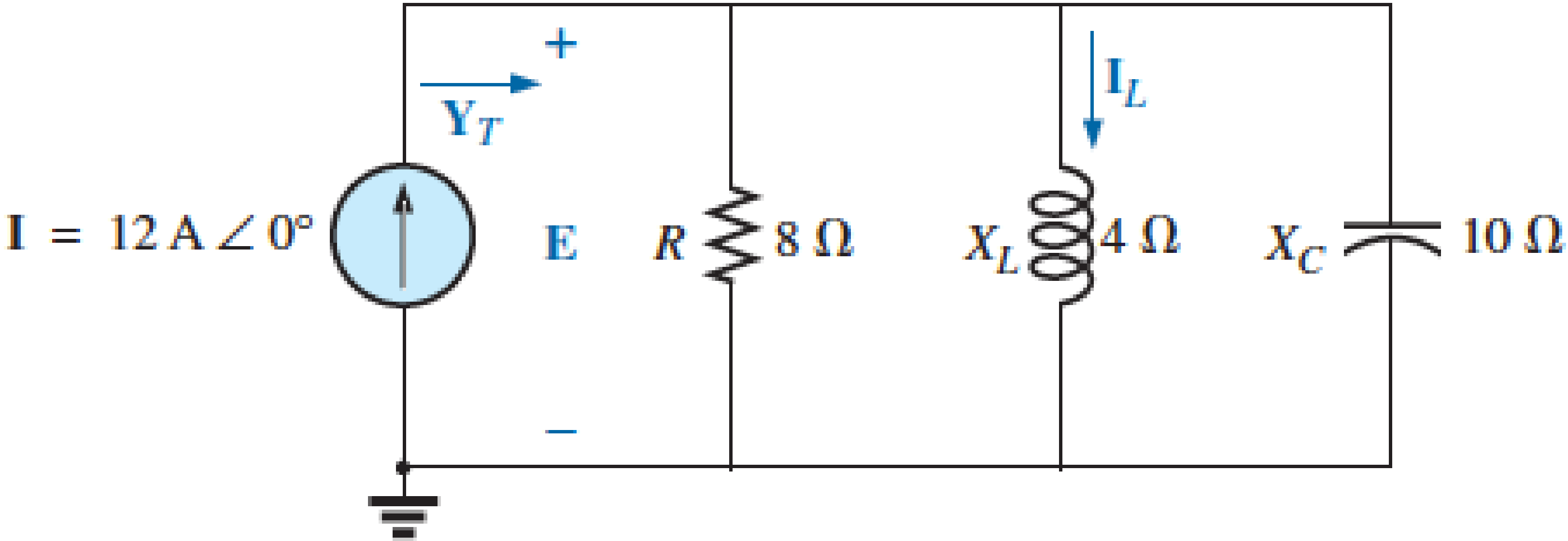
e. **Determine the power delivered to the network, and compare it with the solution of part (d).**



Solutions:

a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$$
$$L_T = 6\text{ mH} \parallel 12\text{ mH} = 4\text{ mH}$$
$$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$$
$$X_L = \omega L = (1000\text{ rad/s})(4\text{ mH}) = 4\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ rad/s})(100\ \mu\text{F})} = 10\ \Omega$$

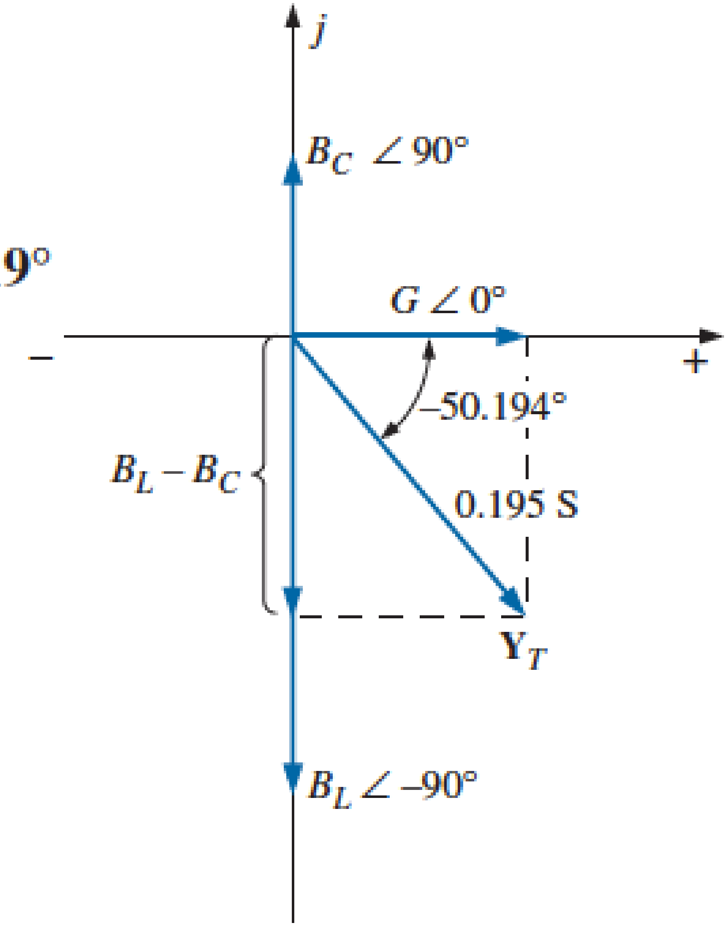


The network is redrawn with phasor notation.
The total admittance is

$$\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C$$
$$= G \angle 0° + B_L \angle -90° + B_C \angle +90°$$
$$= \frac{1}{8\ \Omega} \angle 0° + \frac{1}{4\ \Omega} \angle -90° + \frac{1}{10\ \Omega} \angle +90°$$
$$= 0.125\ \text{S} \angle 0° + 0.25\ \text{S} \angle -90° + 0.1\ \text{S} \angle +90°$$
$$= 0.125\ \text{S} - j\,0.25\ \text{S} + j\,0.1\ \text{S}$$
$$= 0.125\ \text{S} - j\,0.15\ \text{S} = \mathbf{0.195\ S \angle -50.194°}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.195\ \text{S}} \angle -50.194° = \mathbf{5.13\ \Omega \angle 50.19°}$$

b.



c. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \dfrac{\mathbf{I}}{\mathbf{Y}_T} = \dfrac{12\ \text{A}\ \angle 0^\circ}{0.195\ \text{S}\ \angle -50.194^\circ} = \mathbf{61.54\ V\ \angle 50.19^\circ}$

$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{\mathbf{E}}{\mathbf{Z}_L} = \frac{61.538\ \text{V}\ \angle 50.194^\circ}{4\ \Omega\ \angle 90^\circ} = \mathbf{15.39\ A\ \angle -39.81^\circ}$$

d. $F_p = \cos\theta = \dfrac{G}{Y_T} = \dfrac{0.125\ \text{S}}{0.195\ \text{S}} = \mathbf{0.641\ lagging\ (E\ leads\ I)}$

$$P = EI\cos\theta = (61.538\ \text{V})(12\ \text{A})\cos 50.194^\circ$$
$$= \mathbf{472.75\ W}$$

e. $P = I^2R = (12\ \text{A})^2(3.28\ \Omega) = \mathbf{472.32\ W}$