Electrical Circuit-II

2nd Lecture (part 2)

Phase Relations

By:

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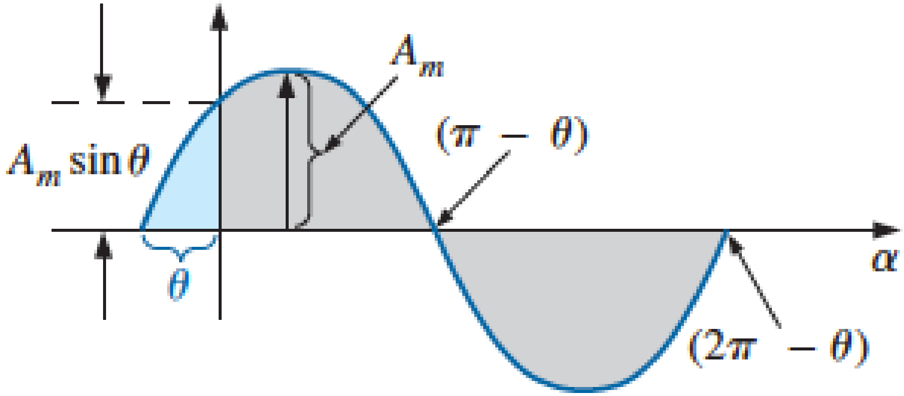
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Phase Relations

we have considered only sine waves that have ***maxima*** *at $\pi/2$ and $3\pi/2$, with a* ***zero*** *value at* ***0***, ***π***, *and* ***2π***, *as shown bel*



If the waveform is shifted to the ***right*** *or* ***left*** *of* ***0°***, *the expression becomes*

$$A_m \sin(\omega t \pm \theta)$$

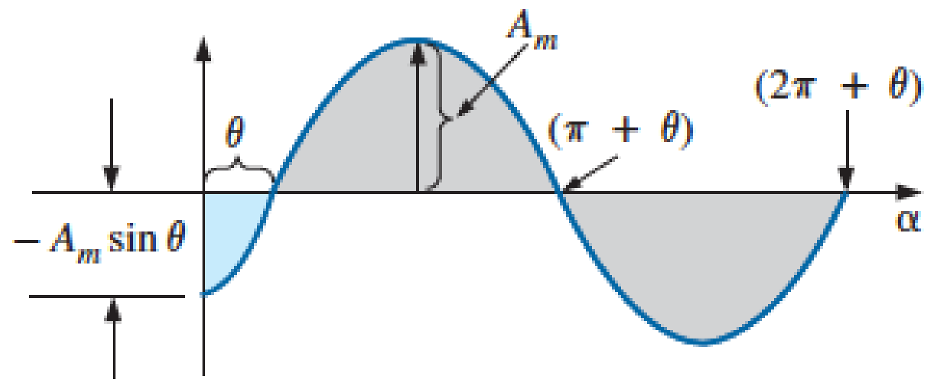
where ***θ*** *is the angle in degrees or radians that the waveform has been shifted.*

If the waveform passes through the horizontal axis with a positive going (increasing with time) slope before ***0°***, *as shown above, the expression is*

$$A_m \sin(\omega t + \theta)$$

At ωt = α = 0°, the magnitude is determined by Am sin θ. If the waveform passes through the horizontal axis with a positive-going slope after 0°, the expression is

$$A_m \sin(\omega t - \theta)$$

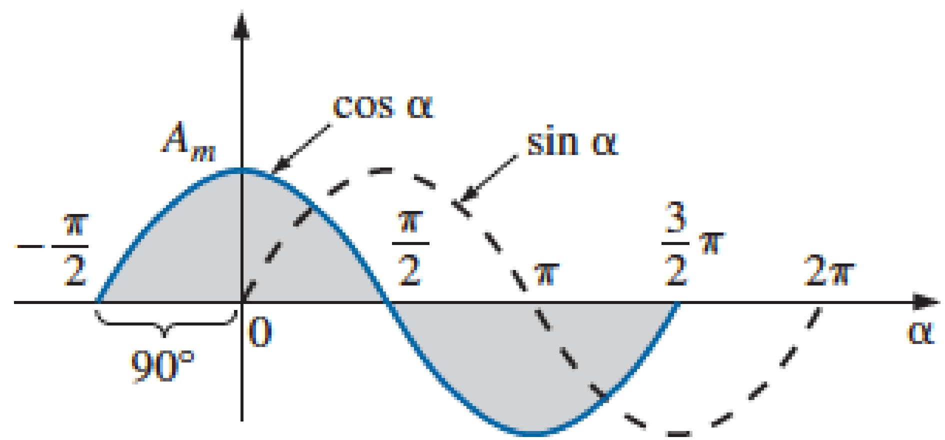


Finally, at ωt = α = 0°, the magnitude is Am sin (- θ), which, by a trigonometric identity, is -Am sinθ.

If the waveform crosses the horizontal axis with a positive-going slope 90° (π /2) sooner, it is called a cosine wave; that is,

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

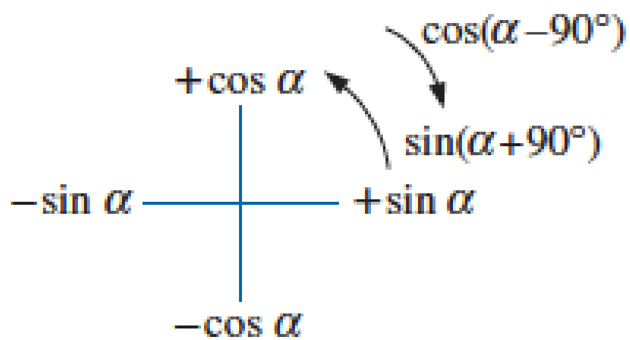
$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$



Phase Relations

*The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axes. The cosine curve is said to lead the sine curve by 90°, and the sine curve is said to lag the cosine curve by 90°. The 90° is referred to as the phase angle between the two waveforms*

$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.



In addition, note that

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\sin(\omega t \pm 180^\circ) = -\sin\omega t$
- $\cos(\omega t \pm 180^\circ) = -\cos\omega t$
- $\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$
- $\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$

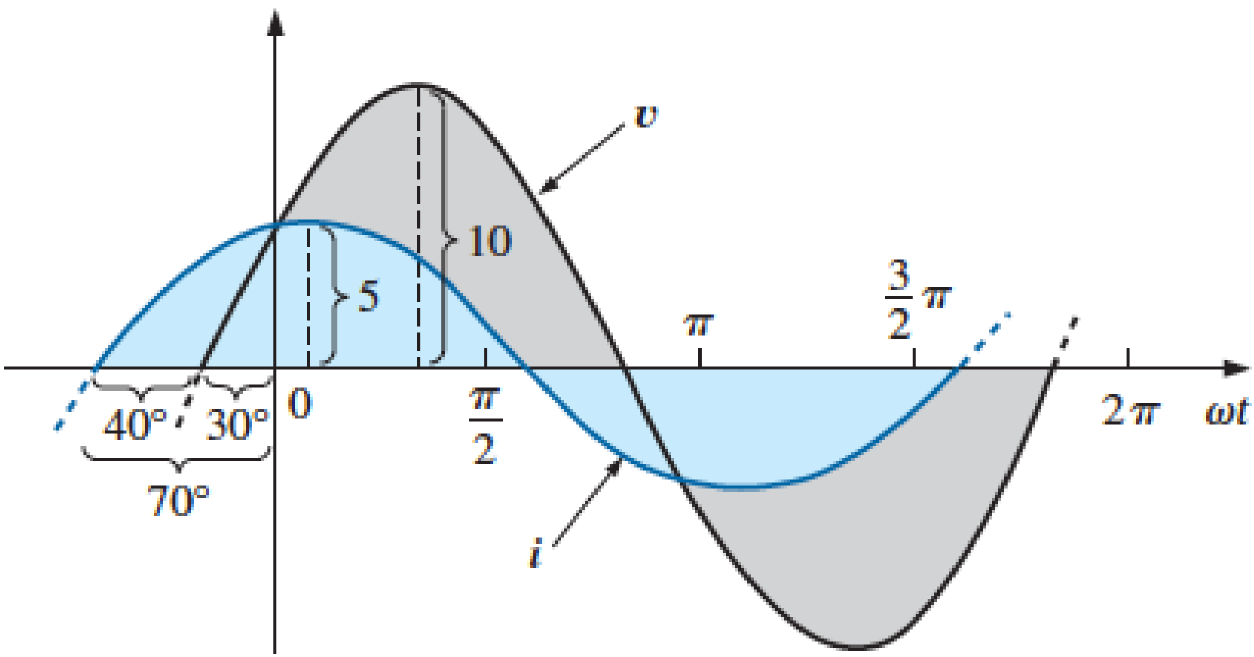
Example

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30°)$
   $i = 5 \sin(\omega t + 70°)$

Solution

i leads v by 40°, or v lags i by 40°.



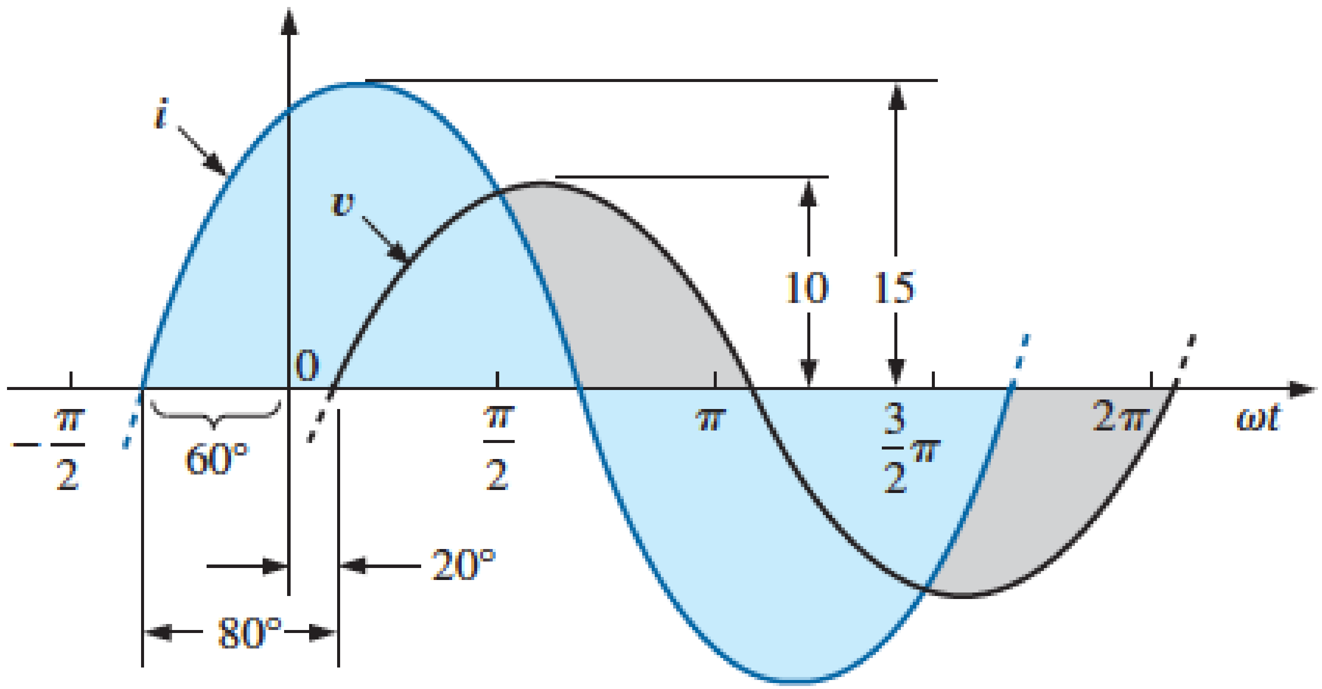
Example

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

b. $i = 15 \sin(\omega t + 60°)$
$v = 10 \sin(\omega t - 20°)$

Solution

i leads v by 80°, or v lags i by 80°.



Example

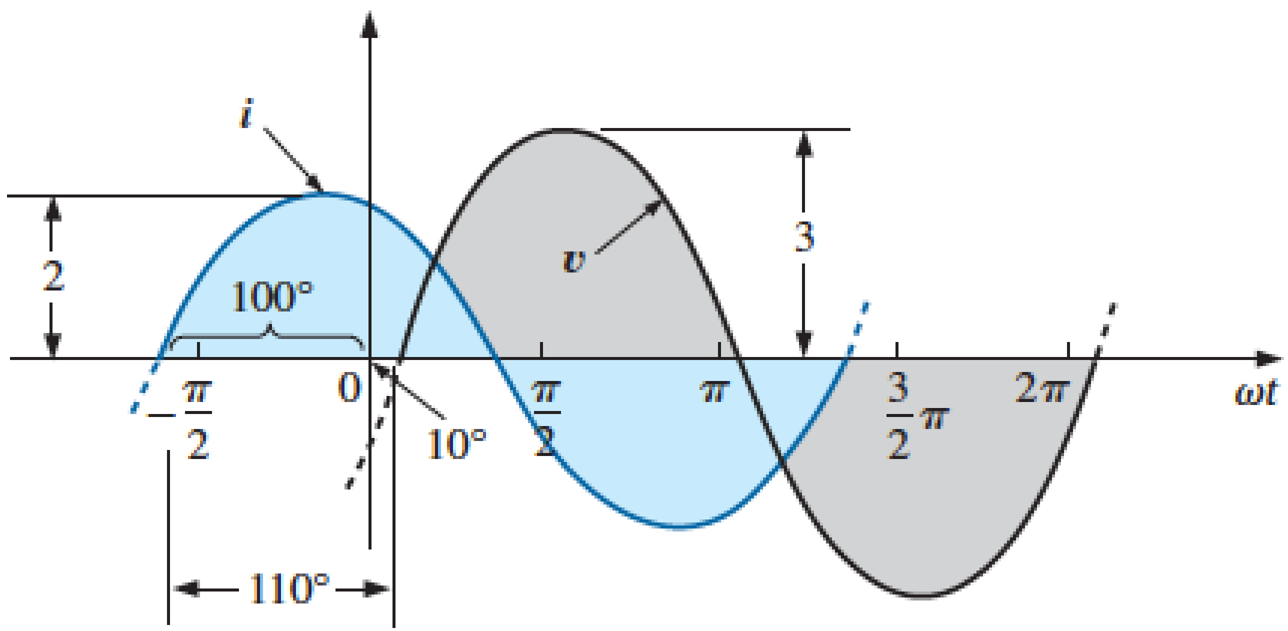
What is the phase relationship between the sinusoidal waveforms of each of the following sets?

c. $i = 2\cos(\omega t + 10°)$
$\quad v = 3\sin(\omega t - 10°)$

Solution

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

i leads v by 110°, or v lags i by 110°.



Example

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

d. $i = -\sin(\omega t + 30^\circ)$

$v = 2\sin(\omega t + 10^\circ)$

Solution

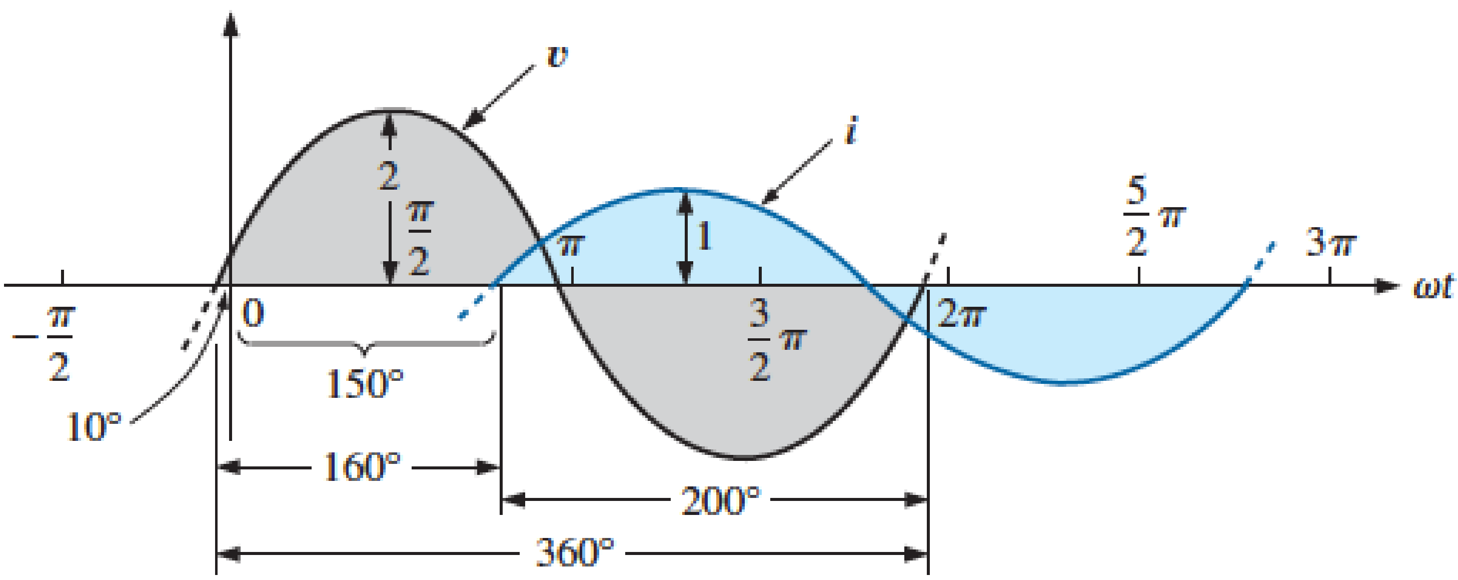
$$-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ \overset{\text{Note}}{-} 180^\circ) = \sin(\omega t - 150^\circ)$$

v leads i by 160°, or i lags v by 160°.

Or using

$$-\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ \overset{\text{Note}}{+} 180^\circ) = \sin(\omega t + 210^\circ)$$

i leads v by 200°, or v lags i by 200°.



Example

What is the phase relationship between the sinusoidal waveforms of each of the following sets?

e. $i = -2\cos(\omega t - 60°)$

$v = 3\sin(\omega t - 150°)$

Solution

By choice

$$i = -2\cos(\omega t - 60°) = 2\cos(\omega t - 60° - 180°)$$
$$= 2\cos(\omega t - 240°)$$

However, $\cos\alpha = \sin(\alpha + 90°)$

so that $2\cos(\omega t - 240°) = 2\sin(\omega t - 240° + 90°)$
$$= 2\sin(\omega t - 150°)$$

v and i are in phase.