Electrical Circuit-II

3rd Lecture

Average & Effective Value

By:

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Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Average Value

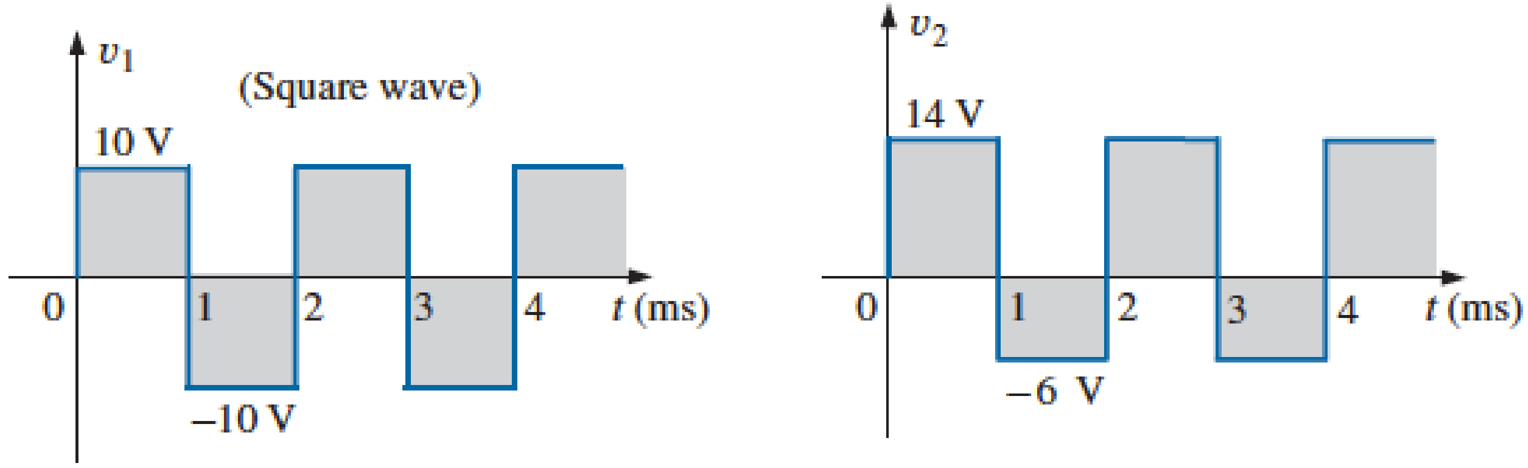
The average value of any current or voltage is the equivalent (dc) value over a complete cycle. In general the average value (G) of a waveform is given as:

$$G\text{ (average value)} = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

The algebraic sum of the areas must be determined, since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign, and those below, a negative sign.

Example

Determine the average value of the waveforms in figures below:



(a) (b)

Solution

a. **By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts**

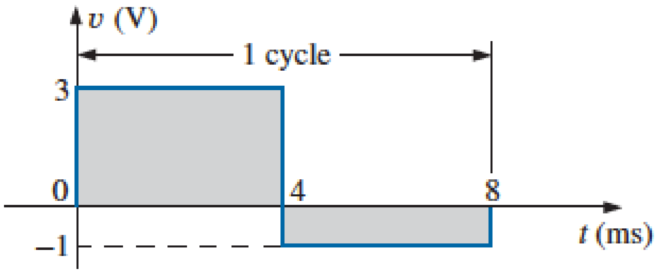
$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b.

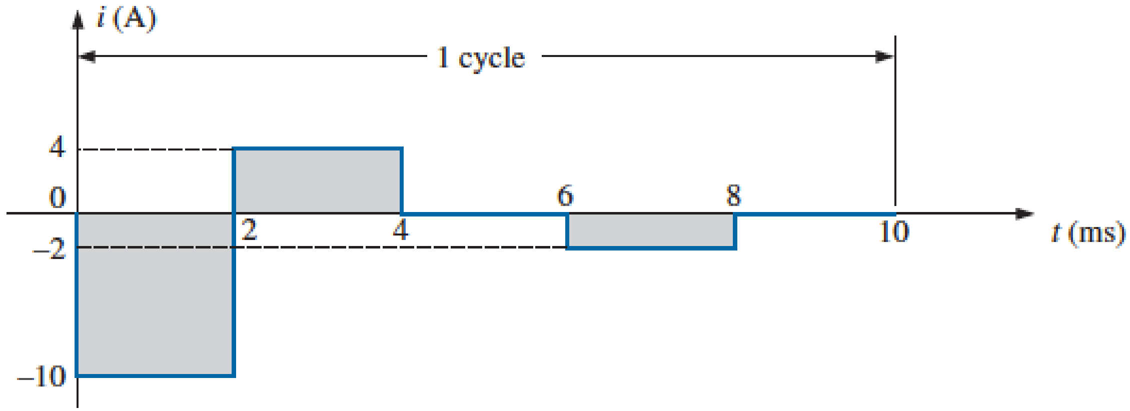
$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

Example

Find the average values of the following waveforms over one full cycle



(a)



(b)

Solution

a.

$$G = \frac{+(3\text{ V})(4\text{ ms}) - (1\text{ V})(4\text{ ms})}{8\text{ ms}} = \frac{12\text{ V} - 4\text{ V}}{8} = \mathbf{1\text{ V}}$$

b.

$$G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$$

$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\text{ V}}$$

Average Value for sinusoidal waveform

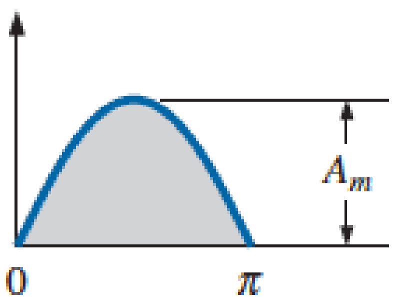
Finding the area under the positive pulse of a sine wave using integration, we have:

$$\text{Area} = \int_0^{\pi} A_m \sin \alpha \, d\alpha$$

where ∫ is the sign of integration, 0 and π are the limits of integration, *Am* sin α is the function to be integrated, *d*α indicates that we are integrating with respect to α.

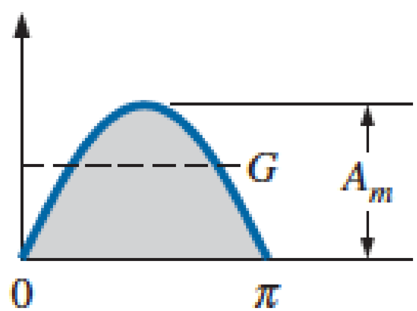
$$\begin{aligned}\text{Area} &= A_m[-\cos \alpha]_0^{\pi} \\ &= -A_m(\cos \pi - \cos 0°) \\ &= -A_m[-1 - (+1)] = -A_m(-2)\end{aligned}$$

$$\boxed{\text{Area} = 2A_m}$$



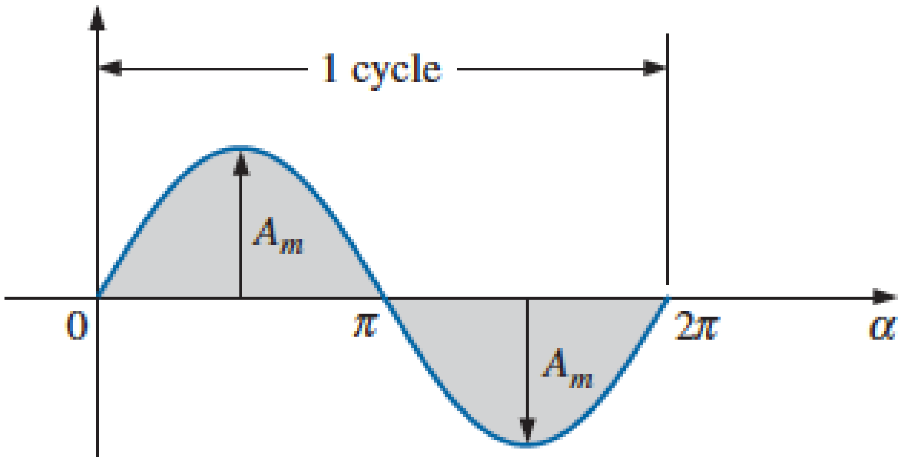
$$G = \frac{2A_m}{\pi}$$

$$\boxed{G = \frac{2A_m}{\pi} = 0.637A_m}$$



Example

Determine the average value of the sinusoidal waveform



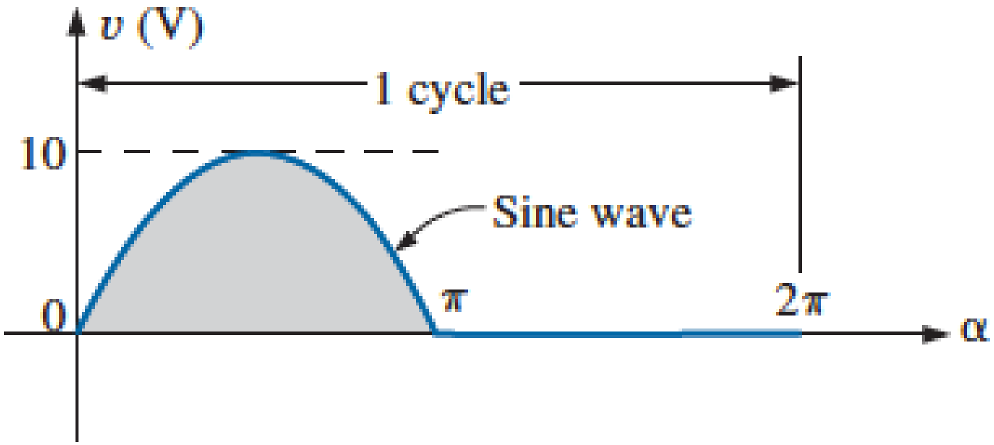
Solution

The average value of a pure sinusoidal waveform over one full cycle is zero.

$$G = \frac{+2A_m - 2A_m}{2\pi} = 0 \text{ V}$$

Example

Determine the average value of the waveform



Solution

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\ \text{V})}{2\pi} \cong \mathbf{3.18\ V}$$

Effective Value

The equivalent dc value of a sinusoidal current or voltage is $1/\sqrt{2}$ *or 0.707 of its peak value.*

The equivalent dc value is called the rms or effective value of the sinusoidal quantity

$$I_{\text{rms}} = \sqrt{\frac{\int_0^T i^2(t)\, dt}{T}}$$

which means

$$I_{\text{rms}} = \sqrt{\frac{\text{area}\,(i^2(t))}{T}}$$

The algebraic sum of the areas must be determined, since some area contributions are from below the horizontal axis. Areas above the axis are assigned a positive sign, and those below, a negative sign.

$$I_{\text{rms}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$$

$$E_{\text{rms}} = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$$

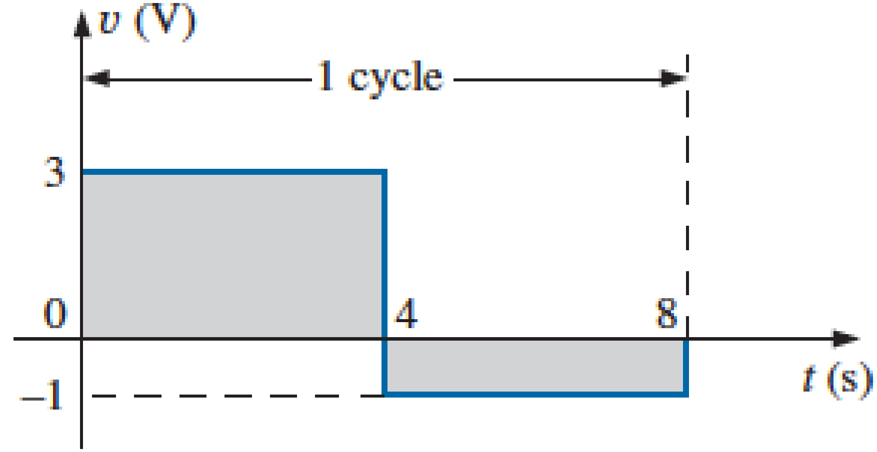
Similarly

$$I_m = \sqrt{2} I_{\text{rms}} = 1.414 I_{\text{rms}}$$

$$E_m = \sqrt{2} E_{\text{rms}} = 1.414 E_{\text{rms}}$$

Example

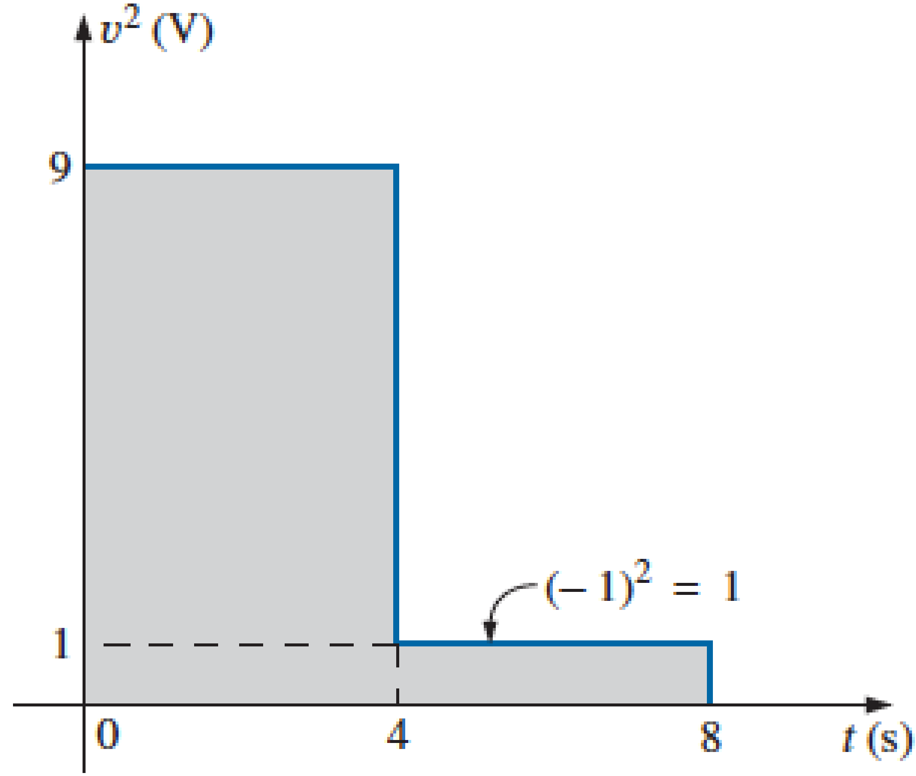
Find the rms value of the waveform



Solution

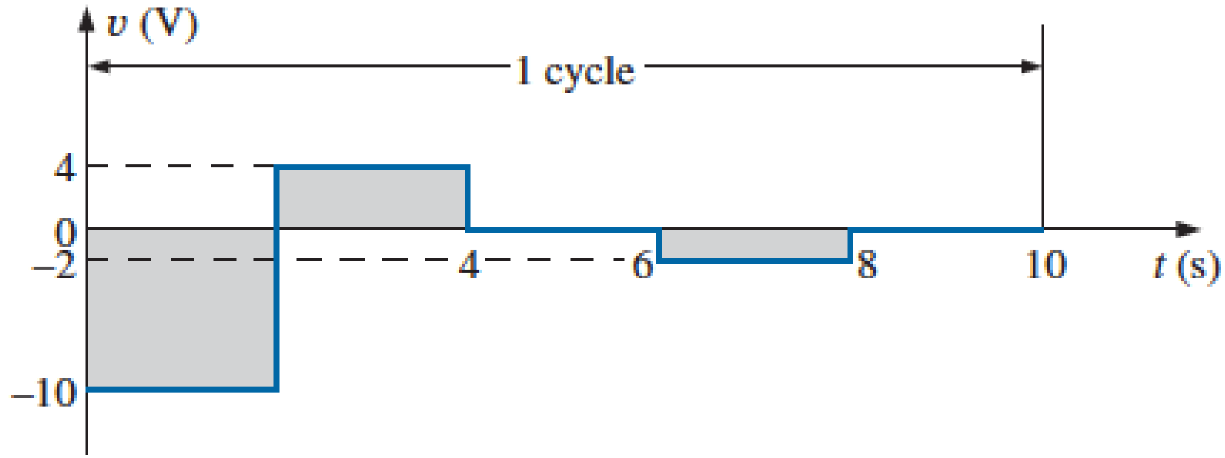
$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}}$$

$$V_{rms} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$



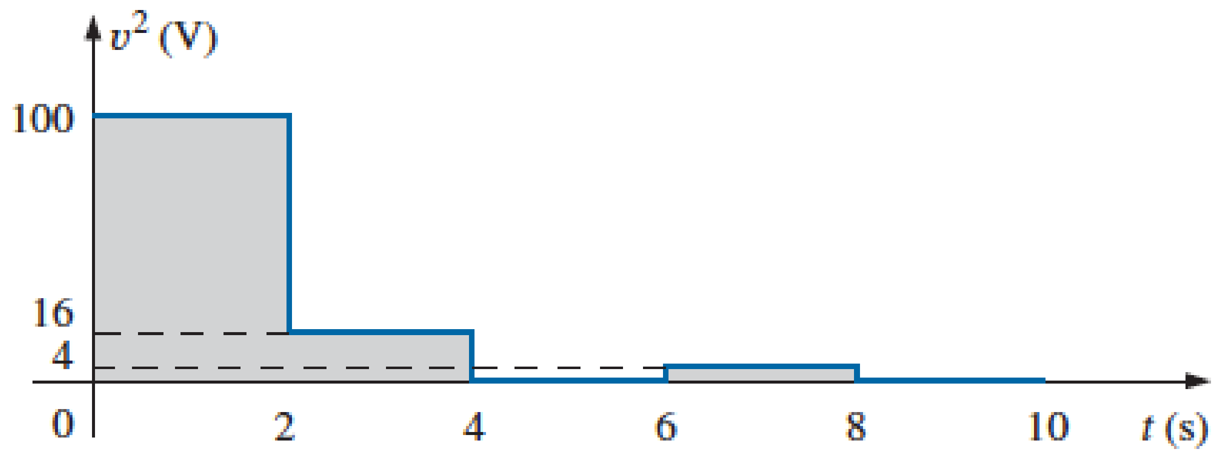
Example

Find the rms value of the waveform



Solution

$$V_{rms} = \sqrt{\frac{(100)(2) + (16)(2) + (4)(2)}{10}}$$



$$V_{rms} = \sqrt{\frac{240}{10}}$$

$$V_{rms} = 4.898\ v$$

Example

Find the rms value of the waveform given by ($i = I_m \sin wt$)

Solution

$$I_{eff} = I_{rms} = \sqrt{\int_0^{2\pi} \frac{i^2}{2\pi}\, d\theta}$$

$$i = I_m \sin wt = I_m \sin \theta$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \theta)^2\, d\theta} = \sqrt{\frac{{I_m}^2}{2\pi} \int_0^{2\pi} (\sin \theta)^2\, d\theta}$$

$$I_{rms} = \sqrt{\frac{{I_m}^2}{2\pi} \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2}\, d\theta} = \sqrt{\frac{{I_m}^2}{4\pi} \left| \theta - \frac{\sin 2\theta}{2} \right|_0^{2\pi}}$$

$$I_{rms} = \sqrt{\frac{{I_m}^2}{4\pi} 2\pi} = \sqrt{\frac{{I_m}^2}{2}}$$

$$I_{rms} = \frac{1}{\sqrt{2}} I_m = 0.707\, I_m$$

The rms value of sinusoidal waveform (voltage or current)