Electrical Circuit-II

4^{th} Lecture (Part 2)

The Basic Elements and Phasors

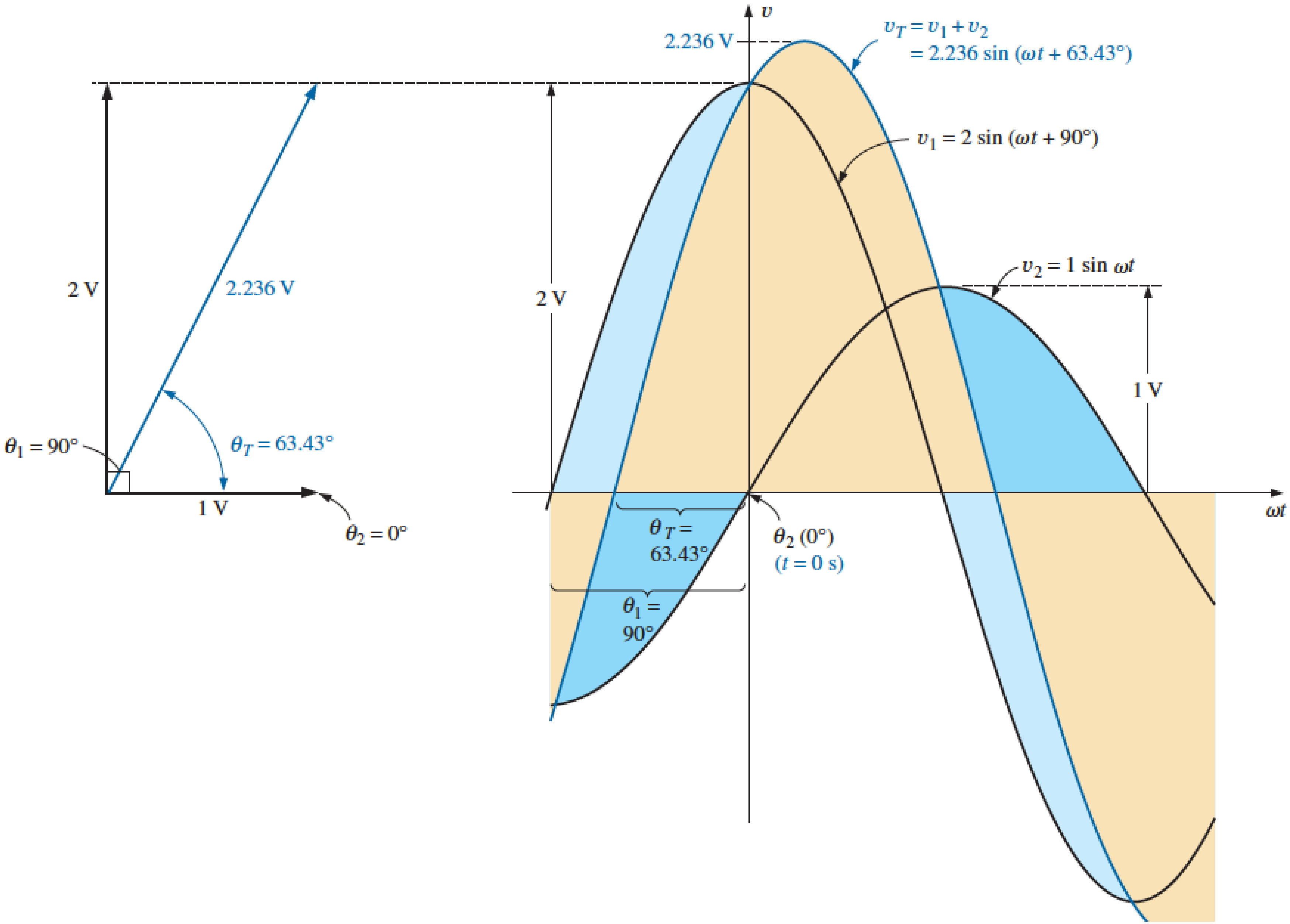
By:

Dr. Ali Albu-Rghaif

Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

PHASORS

Phasor is a radius vector, having a constant magnitude (length) at a fixed angle



EXAMPLE Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$\mathbf{50\angle 0^\circ}$
b. $69.6\sin(\omega t + 72^\circ)$	$(0.707)(69.6)\angle 72^\circ = \mathbf{49.21\angle 72^\circ}$
c. $45\cos\omega t$	$(0.707)(45)\angle 90^\circ = \mathbf{31.82\angle 90^\circ}$

EXAMPLE Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

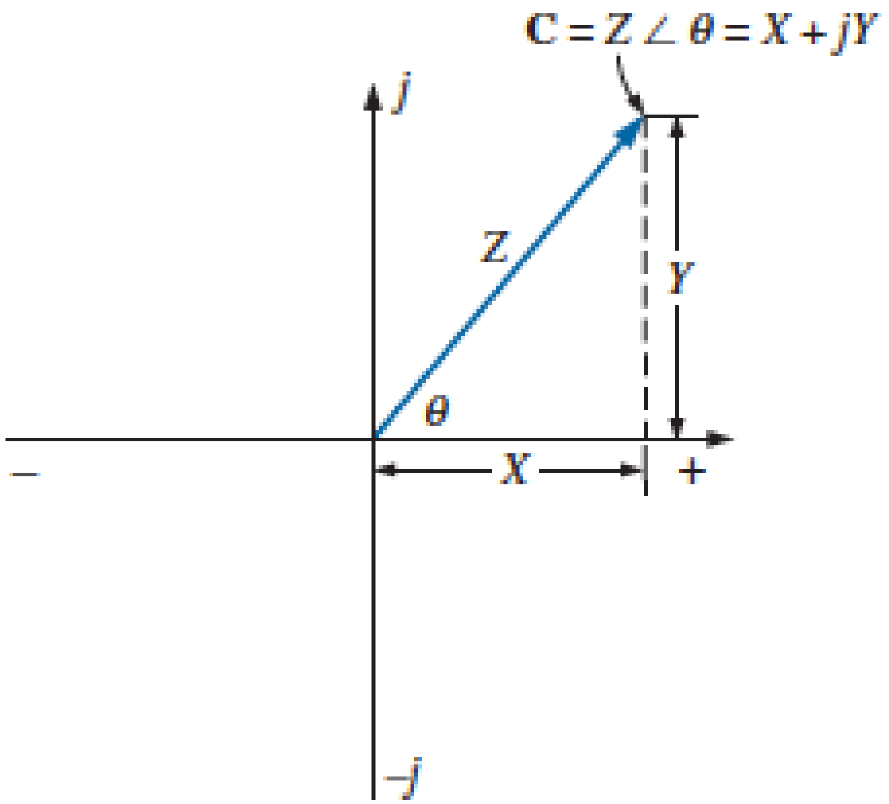
Phasor Domain	Time Domain
a. $\mathbf{I} = 10\angle 30^\circ$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30^\circ)$ and $i = \mathbf{14.14\sin(377}t\mathbf{ + 30^\circ)}$
b. $\mathbf{V} = 115\angle -70^\circ$	$v = \sqrt{2}(115)\sin(377t - 70^\circ)$ and $v = \mathbf{162.6\sin(377}t\mathbf{ - 70^\circ)}$

CONVERSION BETWEEN FORMS

Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1}\frac{Y}{X}$$



Polar to Rectangular

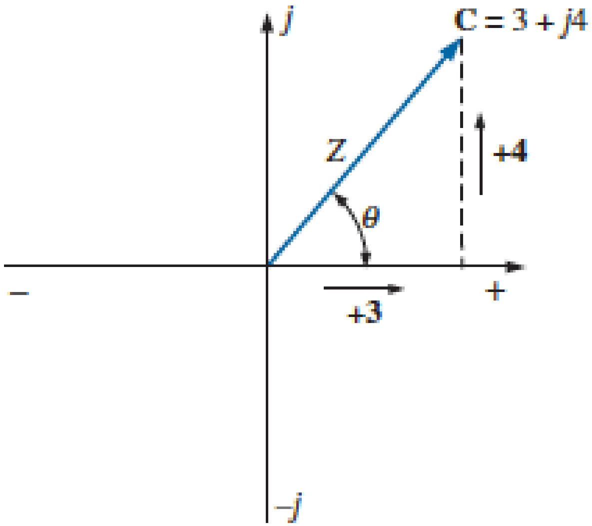
$$X = Z\cos\theta$$

$$Y = Z\sin\theta$$

Example:

Convert the following from rectangular to polar form

$$\mathbf{C} = 3 + j4$$



Solution:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

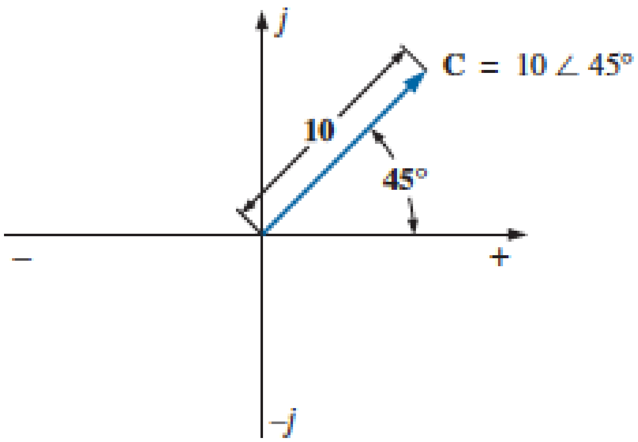
$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

and
$$\mathbf{C} = \mathbf{5 \angle 53.13°}$$

Example:

Convert the following from polar to rectangular form

$$\mathbf{C} = 10 \angle 45°$$



Solution:

$$X = 10 \cos 45° = (10)(0.707) = 7.07$$

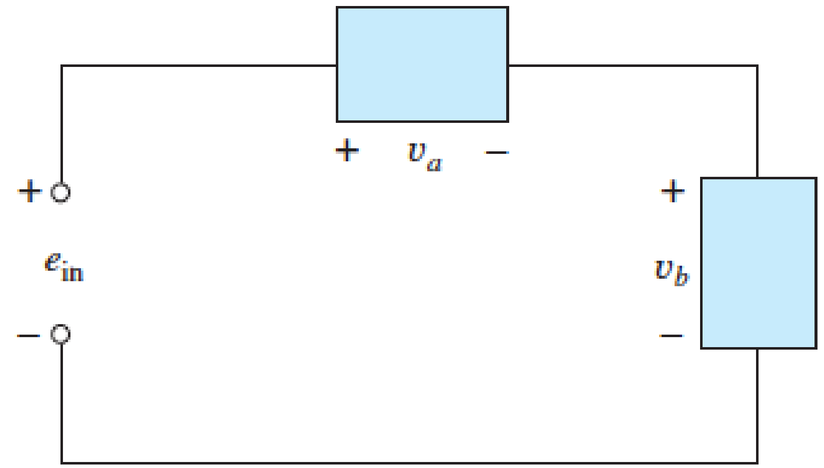
$$Y = 10 \sin 45° = (10)(0.707) = 7.07$$

and
$$\mathbf{C} = \mathbf{7.07 + j7.07}$$

Example

Find the input voltage of the circuit

$$\left.\begin{aligned} v_a &= 50 \sin(377t + 30°) \\ v_b &= 30 \sin(377t + 60°) \end{aligned}\right\} f = 60 \text{ Hz}$$



Solutions:

Applying Kirchhoff's voltage law, we have

$$e_{in} = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50 \sin(377t + 30°) \Rightarrow \mathbf{V}_a = 35.35 \text{ V} \angle 30°$$
$$v_b = 30 \sin(377t + 60°) \Rightarrow \mathbf{V}_b = 21.21 \text{ V} \angle 60°$$

Converting from polar to rectangular form for addition yields

$$x = z \cos \theta$$

$$\mathbf{V}_a = 35.35 \text{ V} \angle 30° = 30.61 \text{ V} + j17.68 \text{ V}$$
$$\mathbf{V}_b = 21.21 \text{ V} \angle 60° = 10.61 \text{ V} + j18.37 \text{ V}$$

$$y = z \sin \theta$$

Then

$$\mathbf{E}_{in} = \mathbf{V}_a + \mathbf{V}_b$$
$$= (30.61 \text{ V} + j17.68 \text{ V}) + (10.61 \text{ V} + j18.37 \text{ V})$$
$$= 41.22 \text{ V} + j36.05 \text{ V}$$

Converting from rectangular to polar form, we have

$$\mathbf{E}_{in} = 41.22 \text{ V} + j36.05 \text{ V}$$
$$= 54.76 \text{ V} \angle 41.17^\circ$$

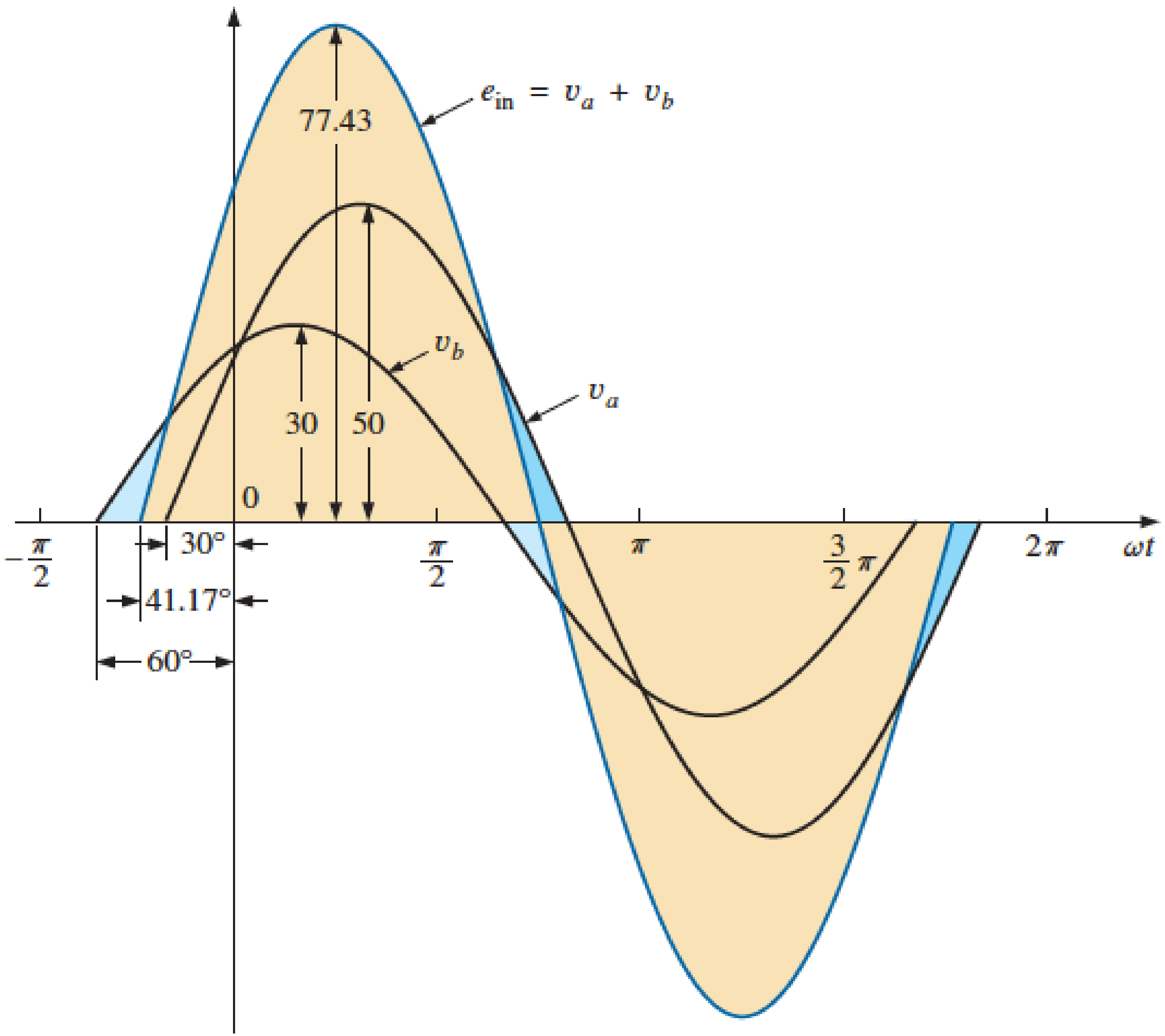
$$z = \sqrt{x^2 + y^2}$$
$$\theta = tan^{-1}\frac{y}{x}$$

Converting from the phasor to the time domain, we obtain

$$\mathbf{E}_{in} = 54.76 \text{ V} \angle 41.17^\circ \Rightarrow$$
$$e_{in} = \sqrt{2}(54.76) \sin(377t + 41.17^\circ)$$

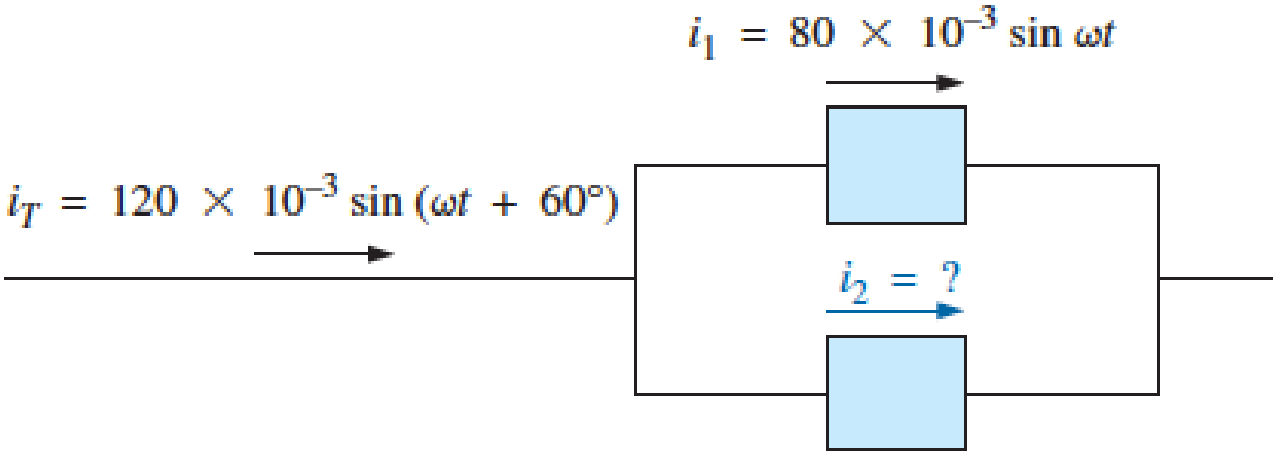
and

$$e_{in} = \mathbf{77.43 \sin(377}t \mathbf{+ 41.17^\circ)}$$


$e_{in} = v_a + v_b$
77.43
v_b
v_a
30
50
0
$-\frac{\pi}{2}$
30°
41.17°
60°
$\frac{\pi}{2}$
π
$\frac{3}{2}\pi$
2π
ωt

Example

Determine the current i_2 for the network



Solutions:

Applying Kirchhoff's current law, we obtain

$$i_T = i_1 + i_2 \quad \text{or} \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120 \times 10^{-3} \sin(\omega t + 60°) \Rightarrow 84.84 \text{ mA} \angle 60°$$
$$i_1 = 80 \times 10^{-3} \sin \omega t \Rightarrow 56.56 \text{ mA} \angle 0°$$

Converting from polar to rectangular form for subtraction yields

$$\mathbf{I}_T = 84.84 \text{ mA} \angle 60° = 42.42 \text{ mA} + j73.47 \text{ mA}$$
$$\mathbf{I}_1 = 56.56 \text{ mA} \angle 0° = 56.56 \text{ mA} + j0$$

Then

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{I}_T - \mathbf{I}_1 \\ &= (42.42 \text{ mA} + j73.47 \text{ mA}) - (56.56 \text{ mA} + j0)\end{aligned}$$

and
$$\mathbf{I}_2 = -14.14 \text{ mA} + j73.47 \text{ mA}$$

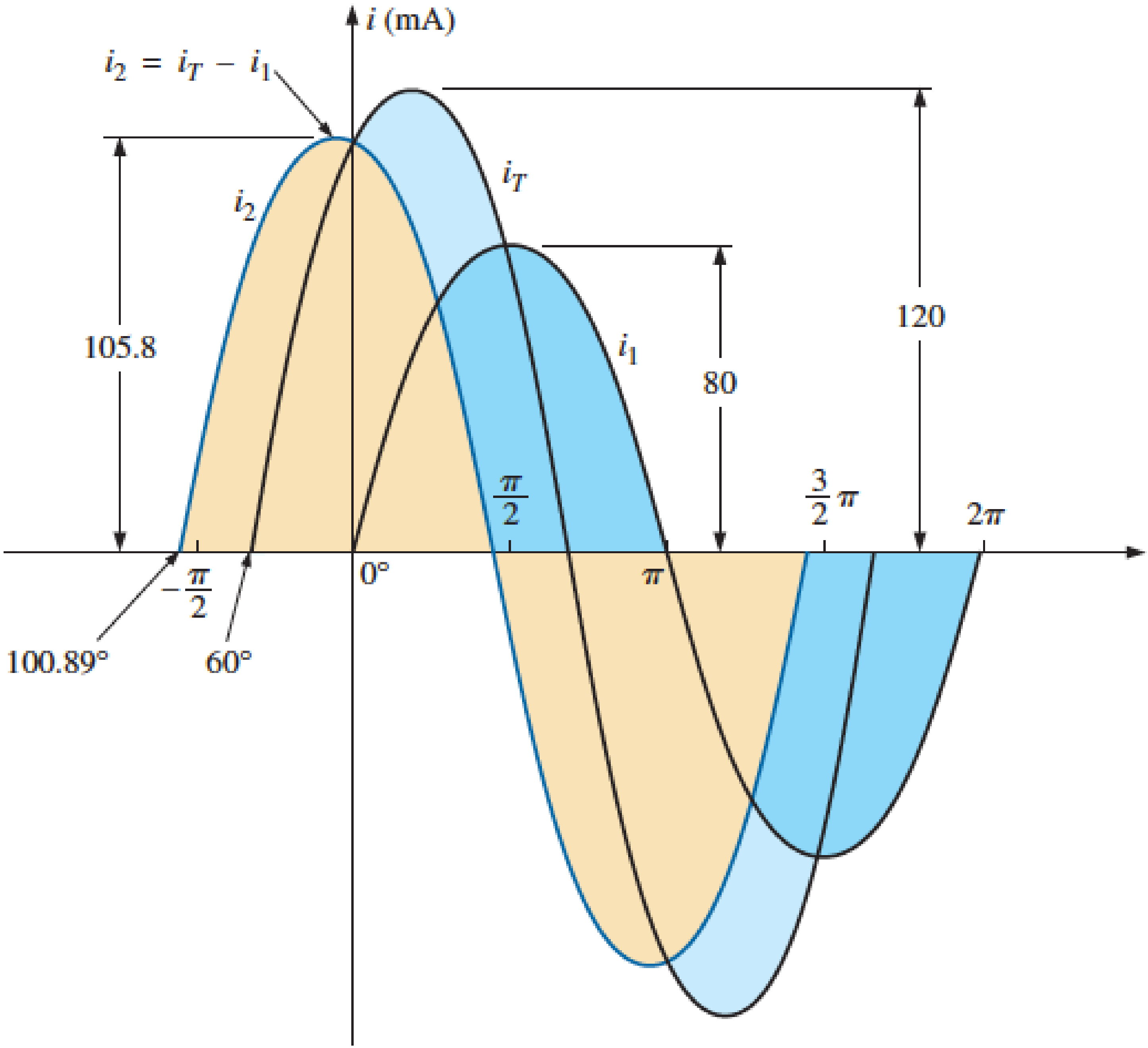
Converting from rectangular to polar form, we have

$$\mathbf{I}_2 = 74.82 \text{ mA} \angle 100.89^\circ$$

Converting from the phasor to the time domain, we have

$$\begin{aligned}\mathbf{I}_2 &= 74.82 \text{ mA} \angle 100.89^\circ \Rightarrow \\ i_2 &= \sqrt{2}(74.82 \times 10^{-3}) \sin(\omega t + 100.89^\circ)\end{aligned}$$

and
$$i_2 = \mathbf{105.8 \times 10^{-3} \sin(\omega t + 100.89^\circ)}$$



i (mA)
i2 = iT − i1
iT
i2
i1
105.8
120
80
π/2
3/2 π
2π
−π/2
0°
π
100.89°
60°