Electrical Circuit-II

4th Lecture (Part 1)

The Basic Elements and Phasors

By:

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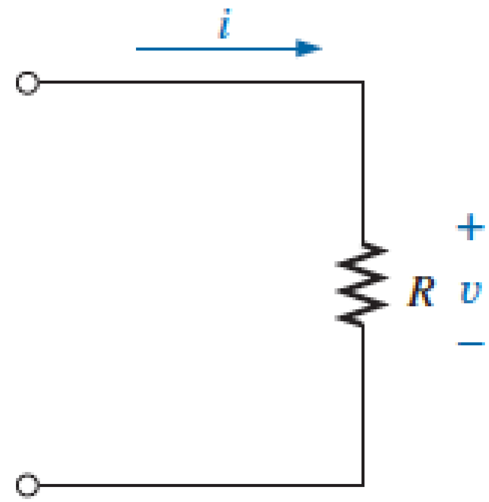
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

AC Through Pure Resistor

Resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current.



Ohm's law can be applied as follows: $\upsilon = V_m \sin \omega t,$

$$i = \frac{\upsilon}{R} = \frac{V_m \sin \omega t}{R}$$

$$= \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

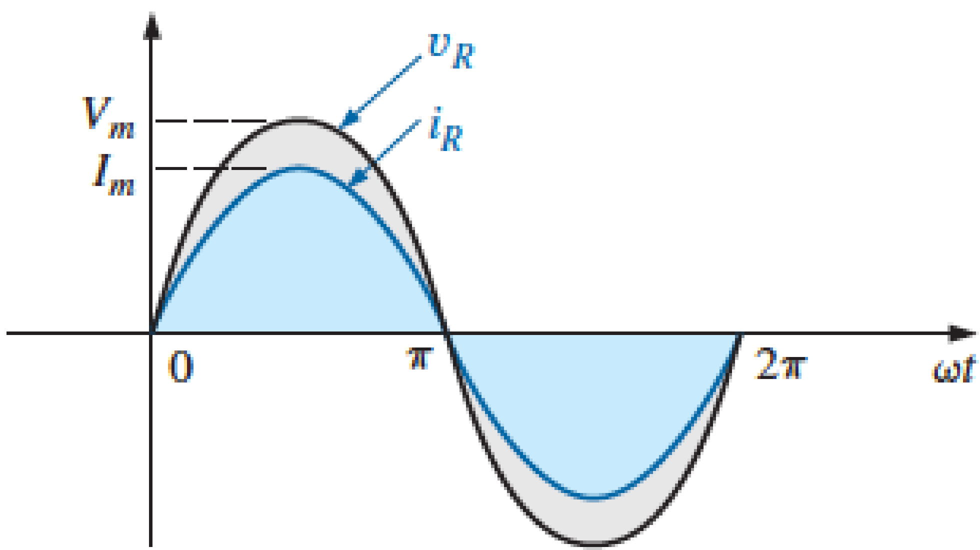
$$\boxed{I_m = \frac{V_m}{R}}$$

In addition, for a given i,

$$\upsilon = iR = (I_m \sin \omega t)R$$

$$= I_m R \sin \omega t = V_m \sin \omega t$$

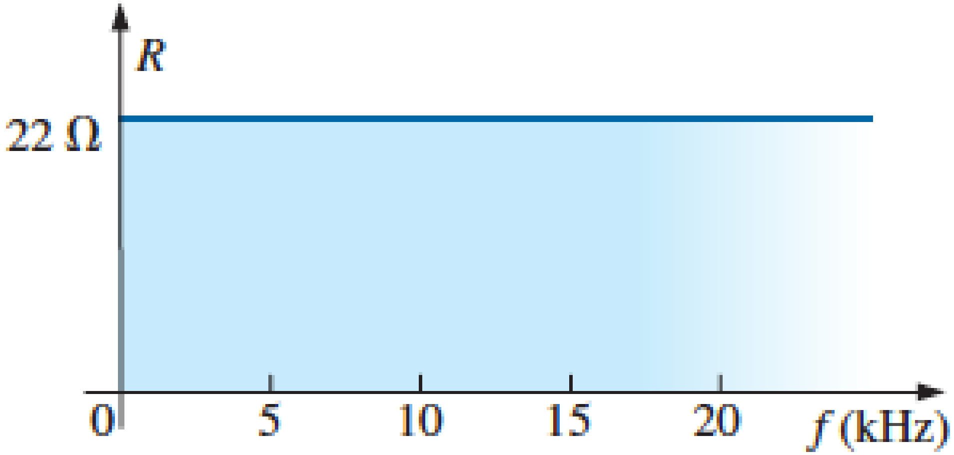
$$\boxed{V_m = I_m R}$$



Frequency Response of the Resistor

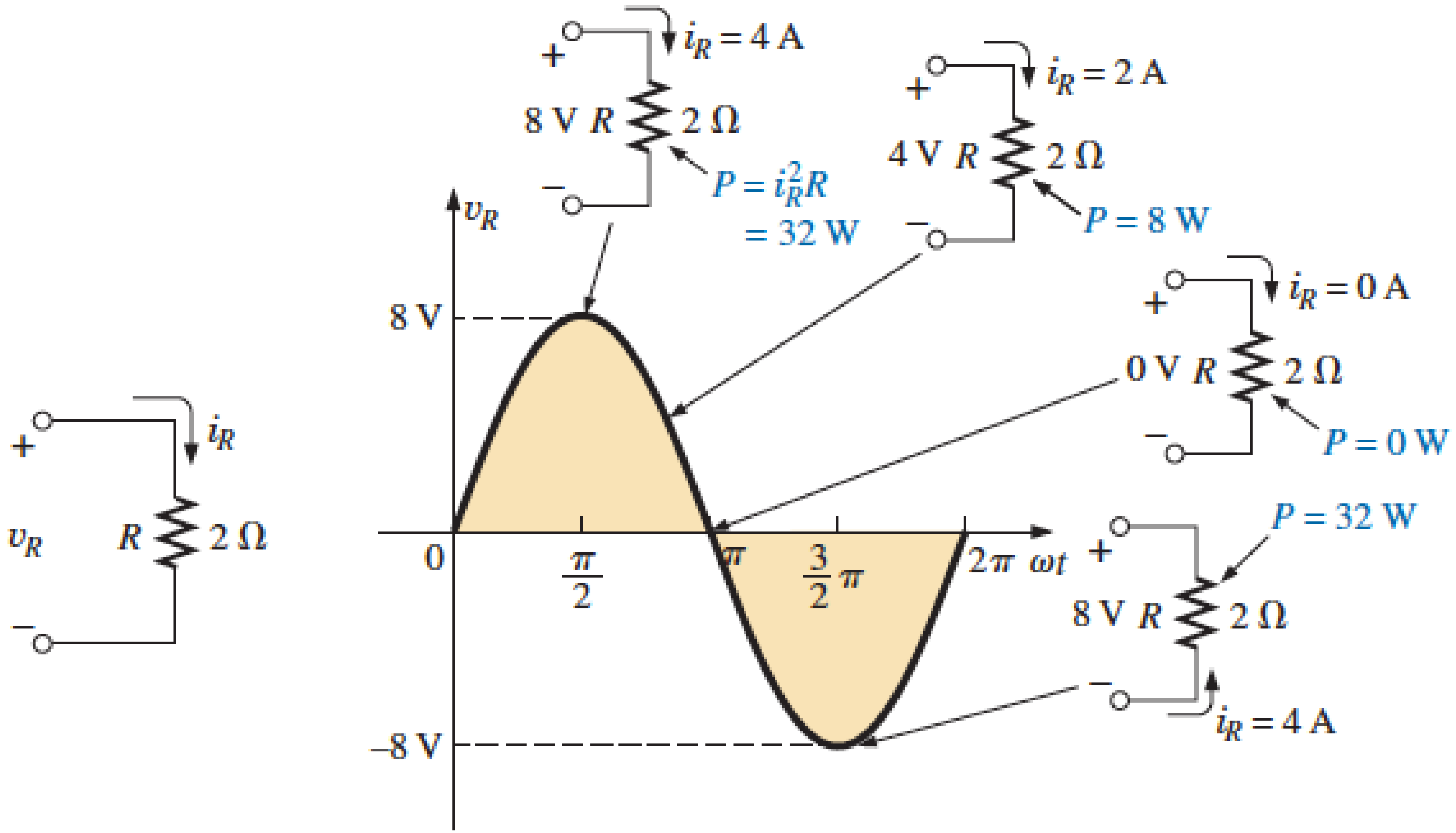
Resistor R For an ideal resistor, you can assume that frequency will have absolutely no effect on the impedance level

Note that at 5 kHz or 20 kHz, the resistance of the resistor remains at 22 Ω



Average Power & Power Factor

A common question is, How can a sinusoidal voltage or current deliver power to a load if it seems to be delivering power during one part of its cycle and taking it back during the negative part of the sinusoidal cycle?



Average Power & Power Factor

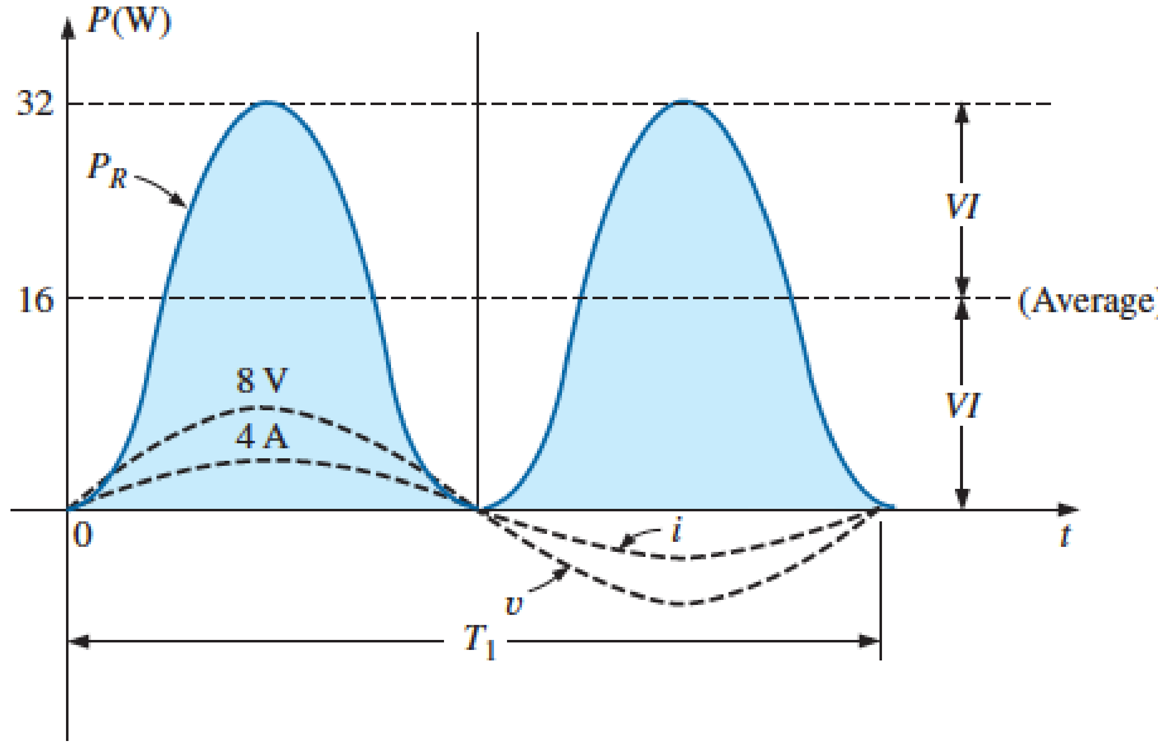
$$P = \frac{V_m I_m}{2} \cos\theta \qquad \text{(watts, W)}$$

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

$$P_{\text{av}} = V_{\text{rms}}\, I_{\text{rms}}$$



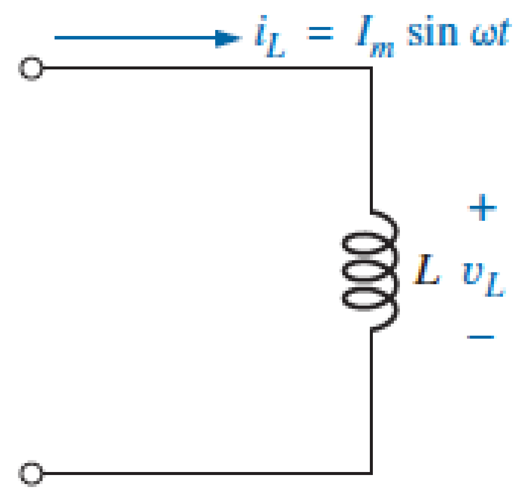
***The Power Factor** is defined as the cosine of the phase angle between the voltage and current: Power Factor = P.F = cos Φ*

*Where ∅ is the phase difference angle between **(v & i)**. Since v & i are **in-phase**, then ∅=0*

Power Factor = 1

AC Through Pure Inductance

$$v_L = L\frac{di_L}{dt}$$



and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt}(I_m \sin \omega t) = \omega I_m \cos \omega t$$

Therefore, $v_L = L\frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$

or $$v_L = V_m \sin(\omega t + 90°)$$

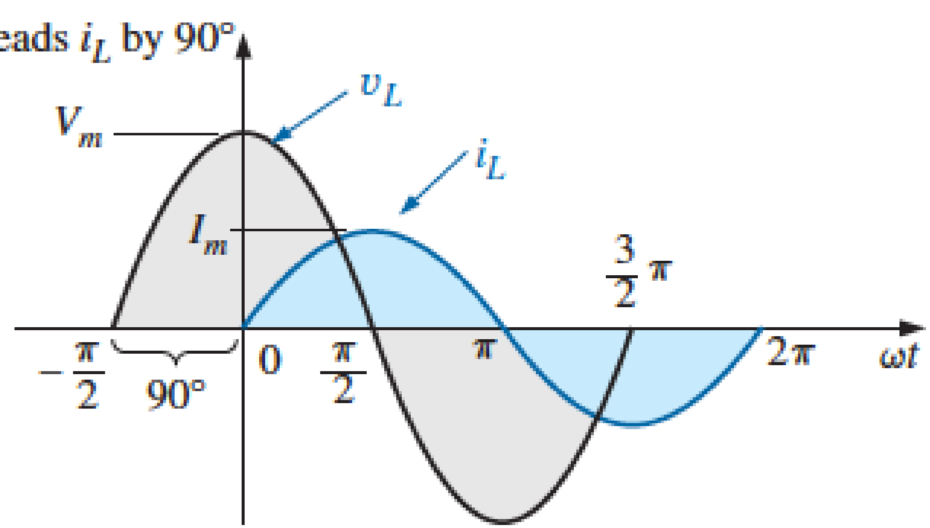
where $$V_m = \omega L I_m$$

for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.

$$\textbf{reactance} = \frac{\text{cause}}{\text{effect}} = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

$$X_L = \omega L \quad \text{(ohms, } \Omega\text{)}$$

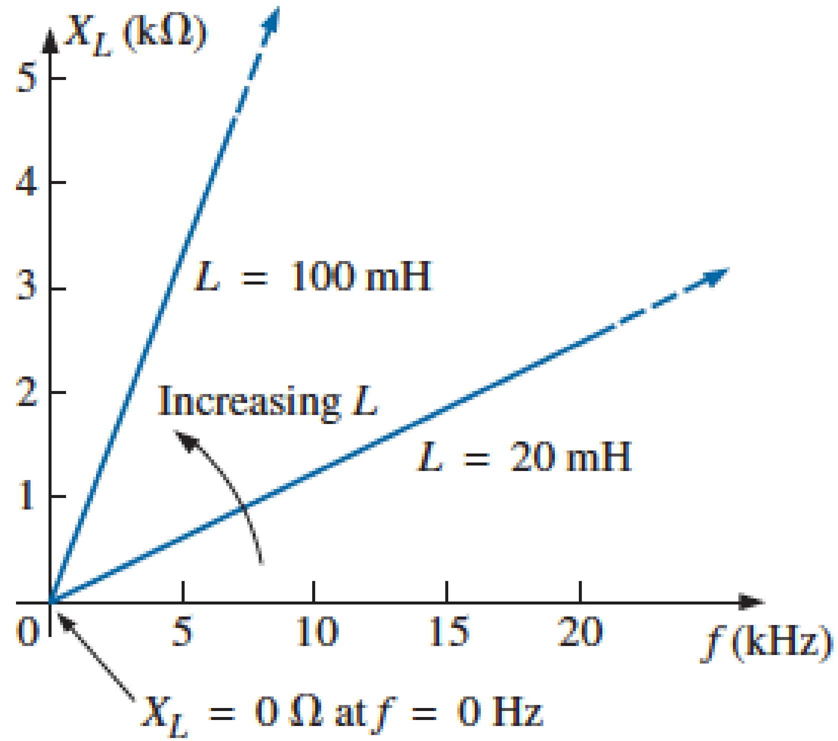
$$X_L = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$



Frequency Response of the Inductance

Inductor L For the ideal inductor, the equation for the reactance can be written as follows:

$$X_L = \omega L = 2\pi f L$$



Power Factor

Power Factor = P.F = cos ***Φ = cos 90 = 0***

Average Power

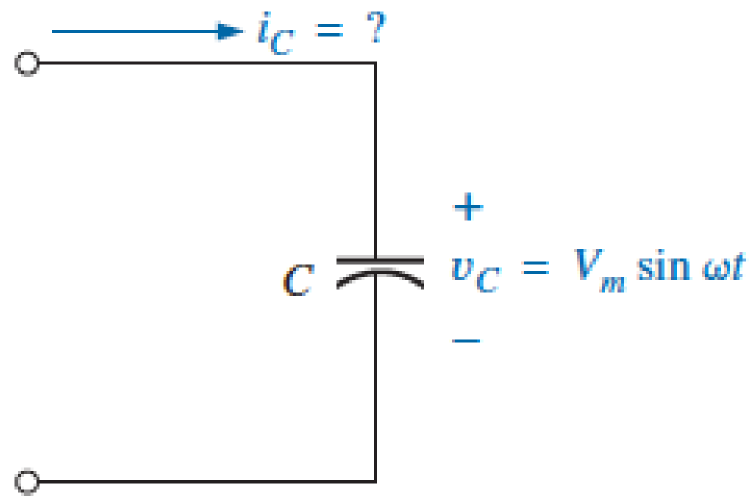
In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2}\cos 90° = \frac{V_m I_m}{2}(0) = 0 \text{ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

AC Through Pure Capacitor

$$i_C = C\frac{dv_C}{dt}$$



and, applying differentiation,

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

Therefore,

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or $$i_C = I_m \sin(\omega t + 90^\circ)$$

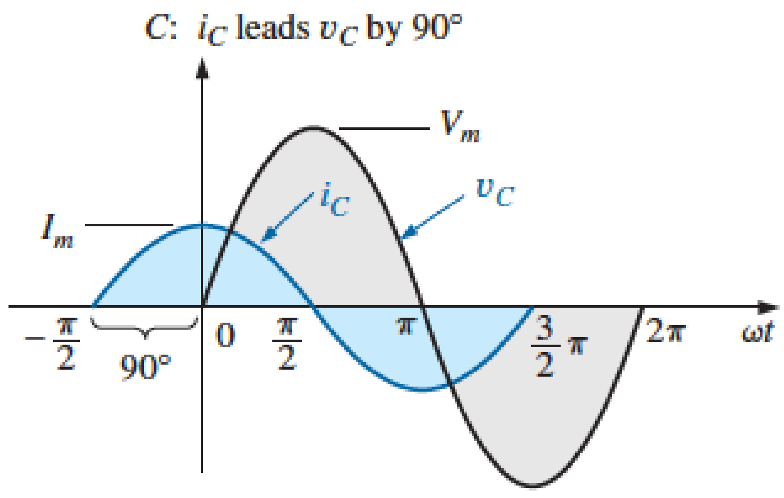
where $$I_m = \omega C V_m$$

for a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.

$$\textbf{reactance} = \frac{\text{cause}}{\text{effect}} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega)$$

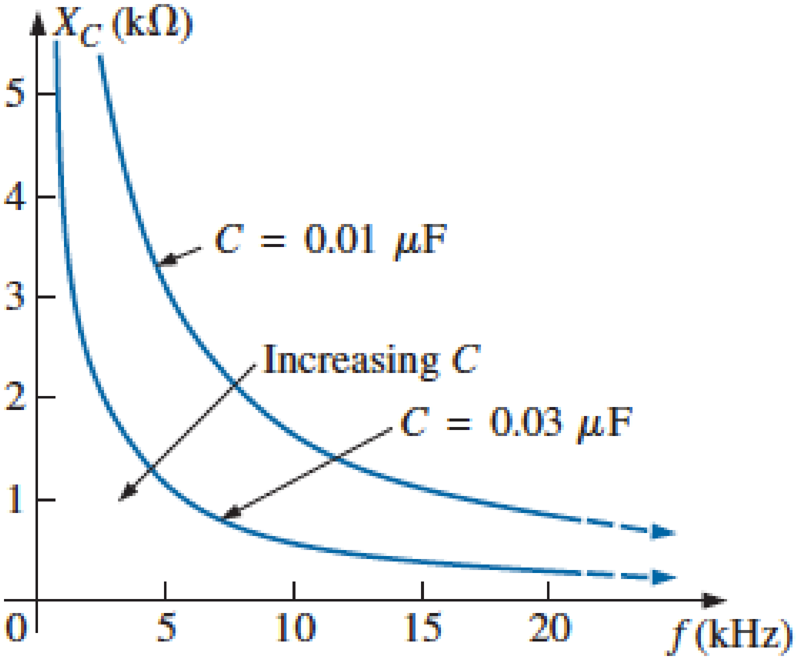
$$X_C = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega)$$



Frequency Response of the Capacitor

Capacitor C For the capacitor, the equation for the reactance:

$$X_C = \frac{1}{2\pi f C}$$



Power Factor

Power Factor = P.F = cos ***Φ = cos 90 = 0***

Average Power

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2}\cos(90°) = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

Frequency Response Summary

	R	L	C
Power Factor cosΦ	1	0	0
Average Power $P_{av} = P$	$\frac{V_m I_m}{2} = V_{rms} I_{rms}$	0	0
Impedance Z	R	$X_L = wL$	$X_C = \frac{1}{wc}$
Phase Difference between V & I Φ	0	v_L Leads i_L Or i_L Lags v_L 90^o	v_c Lags i_c Or i_c Leads v_c 90^o
Frequency Response	Constant	Linear	Non-Linear

Example

For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C, L, or R if sufficient data are provided

a. $v = 100 \sin(\omega t + 40°)$
   $i = 20 \sin(\omega t + 40°)$

b. $v = 1000 \sin(377t + 10°)$
   $i = 5 \sin(377t - 80°)$

Solutions:

a. Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{100\ \text{V}}{20\ \text{A}} = \mathbf{5\ \Omega}$$

b. Since v *leads* i by 90°, the element is an *inductor*, and

$$X_L = \frac{V_m}{I_m} = \frac{1000\ \text{V}}{5\ \text{A}} = 200\ \Omega$$

so that $X_L = \omega L = 200\ \Omega$ or

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377\ \text{rad/s}} = \mathbf{0.53\ H}$$

Example

c. $v = 500 \sin(157t + 30°)$
$i = 1 \sin(157t + 120°)$

d. $v = 50 \cos(\omega t + 20°)$
$i = 5 \sin(\omega t + 110°)$

Solutions:

c. Since i *leads* v by 90°, the element is a *capacitor*, and

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500 \ \Omega$$

so that $X_C = \frac{1}{\omega C} = 500 \ \Omega$ or

$$C = \frac{1}{\omega 500 \ \Omega} = \frac{1}{(157 \text{ rad/s})(500 \ \Omega)} = \mathbf{12.74 \ \mu F}$$

d. $v = 50 \cos(\omega t + 20°) = 50 \sin(\omega t + 20° + 90°)$
$= 50 \sin(\omega t + 110°)$

Since v and i are *in phase*, the element is a *resistor*, and

$$R = \frac{V_m}{I_m} = \frac{50 \text{ V}}{5 \text{ A}} = \mathbf{10 \ \Omega}$$

Example

Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$
$i = 20 \sin(\omega t + 70°)$

b. $v = 150 \sin(\omega t - 70°)$
$i = 3 \sin(\omega t - 50°)$

Solutions:

a. $V_m = 100, \quad \theta_v = 40°$
$I_m = 20\text{ A}, \quad \theta_i = 70°$
$\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(100\text{ V})(20\text{ A})}{2}\cos(30°) = (1000\text{ W})(0.866)$$
$$= \mathbf{866\ W}$$

b. $V_m = 150\text{ V}, \quad \theta_v = -70°$
$I_m = 3\text{ A}, \quad \theta_i = -50°$
$\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
$= |-70° + 50°| = |-20°| = 20°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(150\text{ V})(3\text{ A})}{2}\cos(20°) = (225\text{ W})(0.9397)$$
$$= \mathbf{211.43\ W}$$