Electrical Circuit-II

5th Lecture-Tutorial

Series and Parallel AC Circuits

(Part 1)

By:

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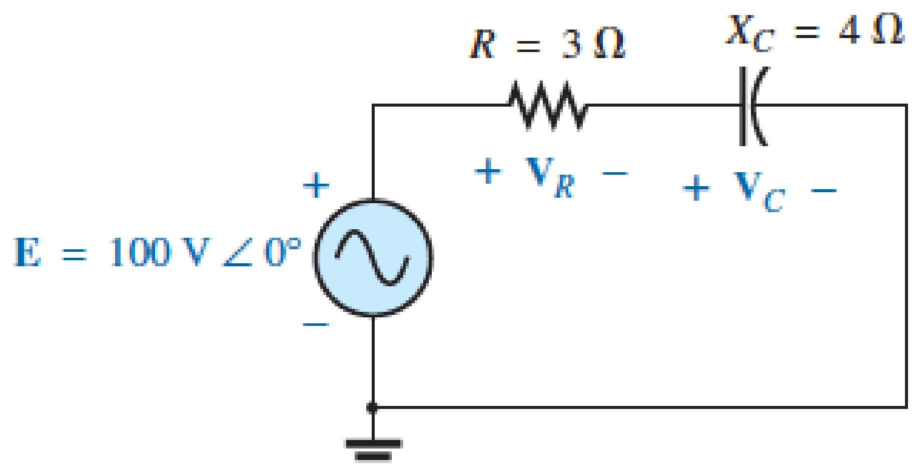
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Example

Using the voltage divider rule, find the voltage across each element of the circuit



Solution:

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega \angle -90°)(100\ \text{V} \angle 0°)}{4\ \Omega \angle -90° + 3\ \Omega \angle 0°} = \frac{400 \angle -90°}{3 - j4}$$

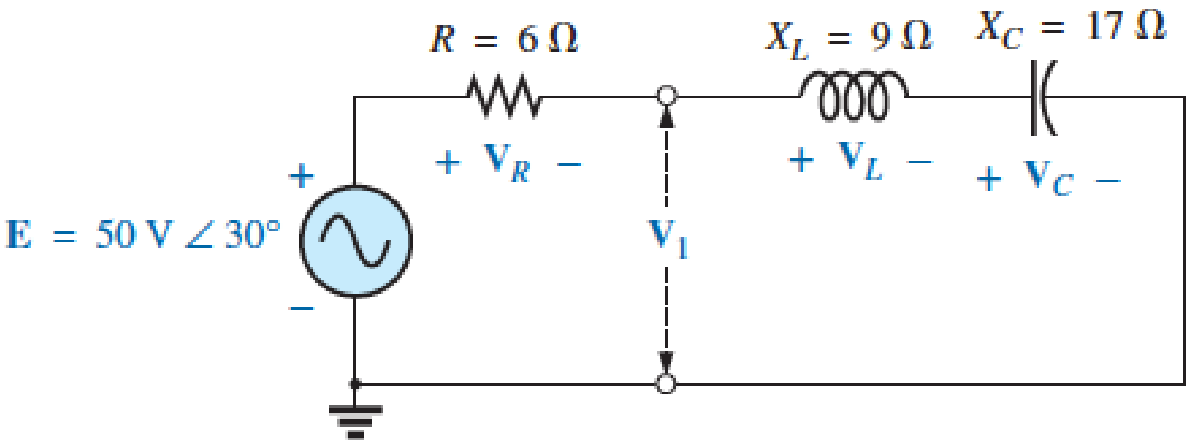
$$= \frac{400 \angle -90°}{5 \angle -53.13°} = \mathbf{80\ V \angle -36.87°}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega \angle 0°)(100\ \text{V} \angle 0°)}{5\ \Omega \angle -53.13°} = \frac{300 \angle 0°}{5 \angle -53.13°}$$

$$= \mathbf{60\ V \angle +53.13°}$$

Example

Using the voltage divider rule, find the unknown voltages VR , VL , VC , and V1 for the circuit



Solution:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0^\circ)(50\ \text{V}\ \angle 30^\circ)}{6\ \Omega\ \angle 0^\circ + 9\ \Omega\ \angle 90^\circ + 17\ \Omega\ \angle -90^\circ}$$

$$= \frac{300\ \angle 30^\circ}{6 + j\,9 - j\,17} = \frac{300\ \angle 30^\circ}{6 - j\,8}$$

$$= \frac{300\ \angle 30^\circ}{10\ \angle -53.13^\circ} = \mathbf{30\ V\ \angle 83.13^\circ}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°} = \frac{450\ \text{V} \angle 120°}{10 \angle -53.13°}$$

$$= \mathbf{45\ V \angle 173.13°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega \angle -90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°} = \frac{850\ \text{V} \angle -60°}{10 \angle -53°}$$

$$= \mathbf{85\ V \angle -6.87°}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega \angle 90° + 17\ \Omega \angle -90°)(50\ \text{V} \angle 30°)}{10\ \Omega \angle -53.13°}$$

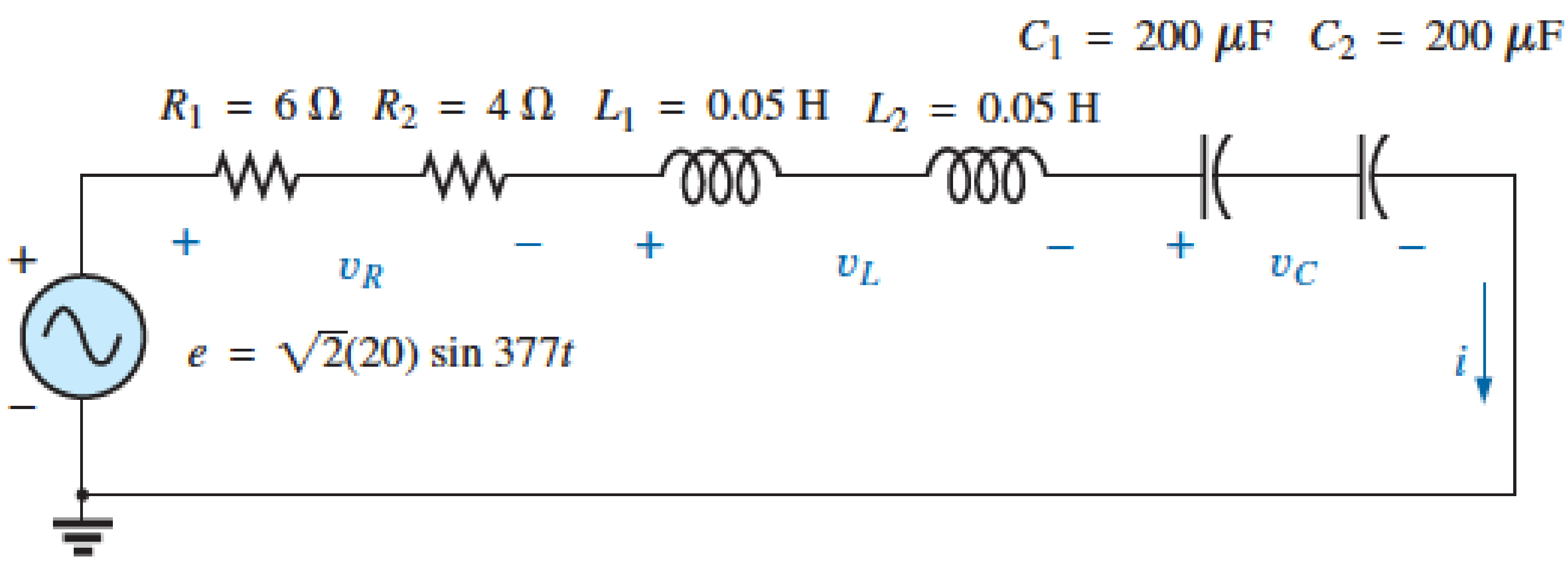
$$= \frac{(8 \angle -90°)(50 \angle 30°)}{10 \angle -53.13°}$$

$$= \frac{400 \angle -60°}{10 \angle -53.13°} = \mathbf{40\ V \angle -6.87°}$$

Example

For the circuit calculate:

a. Calculate I, VR , VL , and VC in phasor form.
b. Calculate the total power factor.
c. Calculate the average power delivered to the circuit.
d. Draw the phasor diagram.
e. Obtain the phasor sum of VR , VL , and VC, and show that it equals the input voltage E.
f. Find VR and VC using the voltage divider rule.



Solutions:

a. **Combining common elements and finding the reactance of the inductor and capacitor, we obtain**

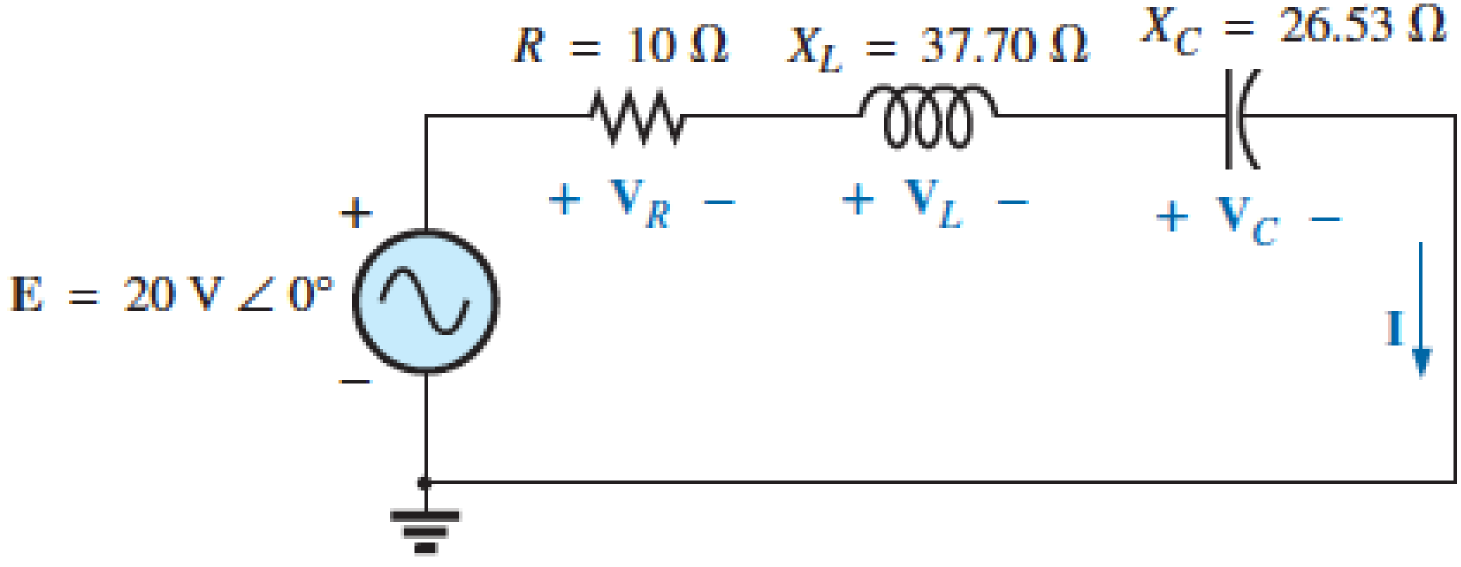
$$R_T = 6\ \Omega + 4\ \Omega = 10\ \Omega$$

$$L_T = 0.05\ \text{H} + 0.05\ \text{H} = 0.1\ \text{H}$$

$$C_T = \frac{200\ \mu\text{F}}{2} = 100\ \mu\text{F}$$

$$X_L = \omega L = (377\ \text{rad/s})(0.1\ \text{H}) = 37.70\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\ \text{rad/s})(100 \times 10^{-6}\ \text{F})} = \frac{10^6\ \Omega}{37{,}700} = 26.53\ \Omega$$



Solutions:

$$\mathbf{Z}_T = R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= 10\ \Omega + j37.70\ \Omega - j26.53\ \Omega$$
$$= 10\ \Omega + j11.17\ \Omega = \mathbf{15\ \Omega\angle 48.16^\circ}$$

The current **I** is

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{20\text{ V}\angle 0^\circ}{15\ \Omega\angle 48.16^\circ} = \mathbf{1.33\text{ A}\angle -48.16^\circ}$$

The voltage across the resistor, inductor, and capacitor can be found using Ohm's law:

$$\mathbf{V}_R = \mathbf{IZ}_R = (I\angle\theta)(R\angle 0^\circ) = (1.33\text{ A}\angle -48.16^\circ)(10\ \Omega\angle 0^\circ)$$
$$= \mathbf{13.30\text{ V}\angle -48.16^\circ}$$

$$\mathbf{V}_L = \mathbf{IZ}_L = (I\angle\theta)(X_L\angle 90^\circ) = (1.33\text{ A}\angle -48.16^\circ)(37.70\ \Omega\angle 90^\circ)$$
$$= \mathbf{50.14\text{ V}\angle 41.84^\circ}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90^\circ) = (1.33\text{ A}\angle -48.16^\circ)(26.53\ \Omega\angle -90^\circ)$$
$$= \mathbf{35.28\text{ V}\angle -138.16^\circ}$$

Solutions:

b. The total power factor, determined by the angle between the applied voltage **E** and the resulting current **I**, is 48.16°:

$$F_p = \cos\theta = \cos 48.16° = \mathbf{0.667\ lagging}$$

or

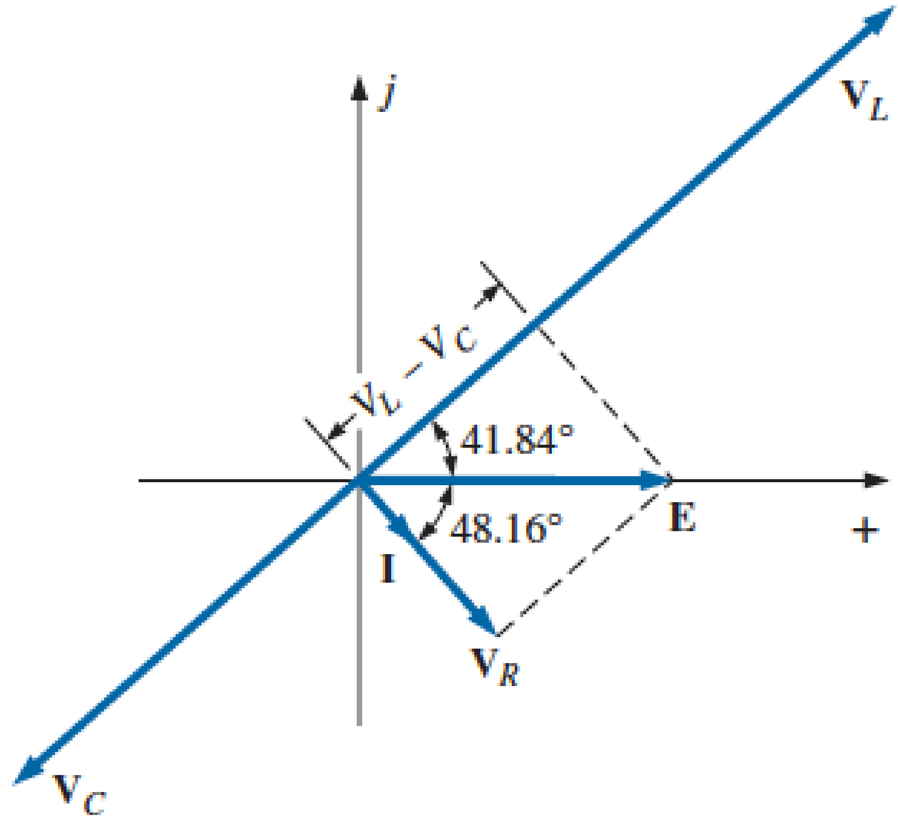
$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{10\ \Omega}{15\ \Omega} = \mathbf{0.667\ lagging}$$

c. The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta = (20\ \text{V})(1.33\ \text{A})(0.667) = \mathbf{17.74\ W}$$

Solutions:

d. The phasor diagram



Solutions:

e. The phasor sum of $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ is

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$
$$= 13.30\text{ V}\angle -48.16^\circ + 50.14\text{ V}\angle 41.84^\circ + 35.28\text{ V}\angle -138.16^\circ$$
$$\mathbf{E} = 13.30\text{ V}\angle -48.16^\circ + 14.86\text{ V}\angle 41.84^\circ$$

Therefore,

$$E = \sqrt{(13.30\text{ V})^2 + (14.86\text{ V})^2} = \mathbf{20\ V}$$

and $\theta_E = \mathbf{0^\circ}$ (from phasor diagram)

and $\mathbf{E} = 20\angle 0^\circ$

f. $$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_T} = \frac{(10\ \Omega\angle 0^\circ)(20\text{ V}\angle 0^\circ)}{15\ \Omega\angle 48.16^\circ} = \frac{200\text{ V}\angle 0^\circ}{15\angle 48.16^\circ}$$
$$= \mathbf{13.3\ V\angle -48.16^\circ}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(26.5\ \Omega\angle -90^\circ)(20\text{ V}\angle 0^\circ)}{15\ \Omega\angle 48.16^\circ} = \frac{530.6\text{ V}\angle -90^\circ}{15\angle 48.16^\circ}$$
$$= \mathbf{35.37\ V\angle -138.16^\circ}$$