Electrical Circuit-II

5th Lecture

Series and Parallel AC Circuits

(Part 1)

By:

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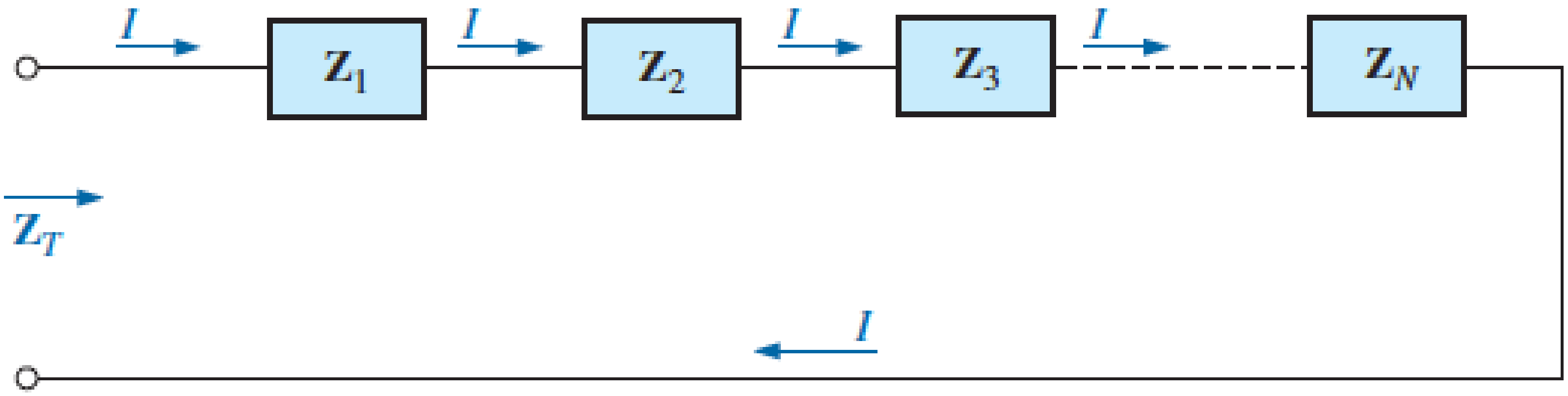
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Series Configuration

The overall properties of series AC circuits are the same as those for DC circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$$

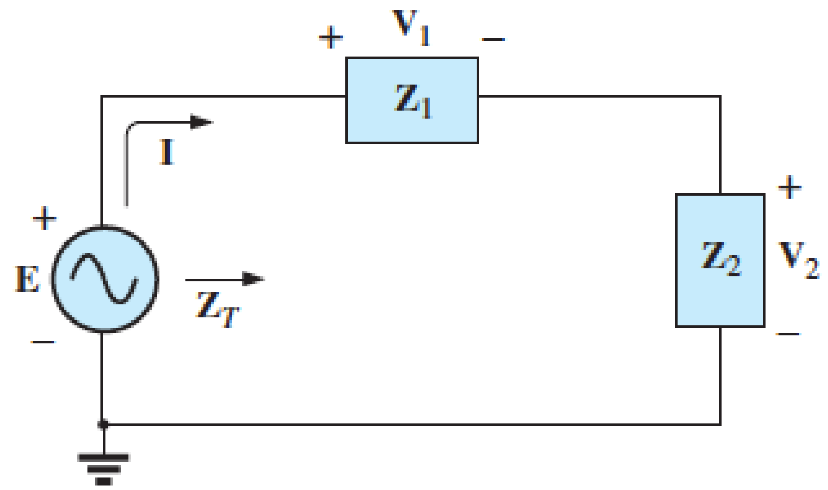


Series Configuration

For the representative series AC configuration having two impedances, the current is the same through each element (as it was for the series DC circuits) and is determined by Ohm's law:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}$$



The voltage across each element can then be found by another application of Ohm's law:

$$\mathbf{V}_1 = \mathbf{I}\mathbf{Z}_1$$

$$\mathbf{E} - \mathbf{V}_1 - \mathbf{V}_2 = 0$$

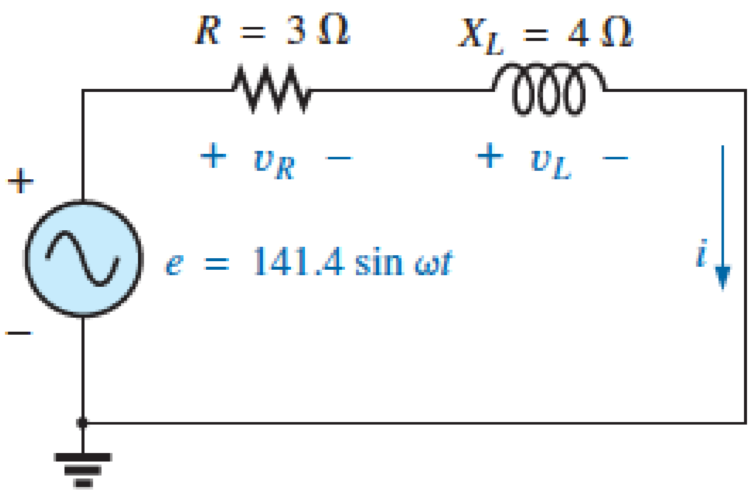
$$\mathbf{V}_2 = \mathbf{I}\mathbf{Z}_2$$

$$\mathbf{E} = \mathbf{V}_1 + \mathbf{V}_2$$

The power to the circuit can be determined by

$$P = EI\cos\theta_T$$

R – L Circuit



Phasor Notation

$$e = 141.4 \sin \omega t \Rightarrow \mathbf{E} = 100\ \text{V} \angle 0°$$

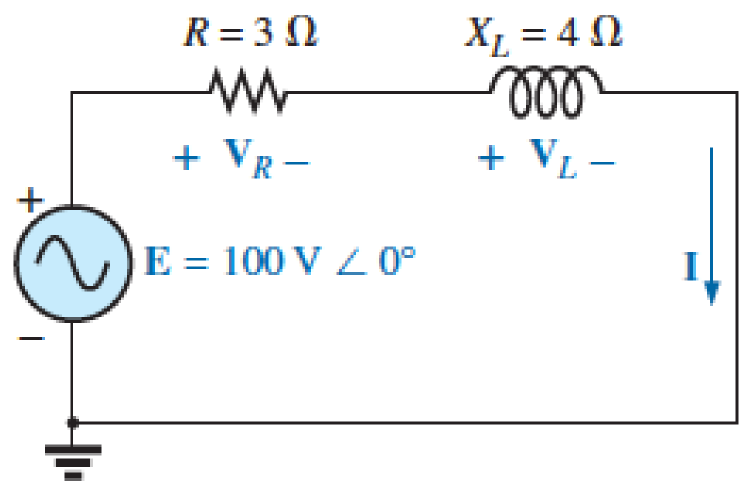
$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega \angle 0° + 4\ \Omega \angle 90° = 3\ \Omega + j4\ \Omega$$

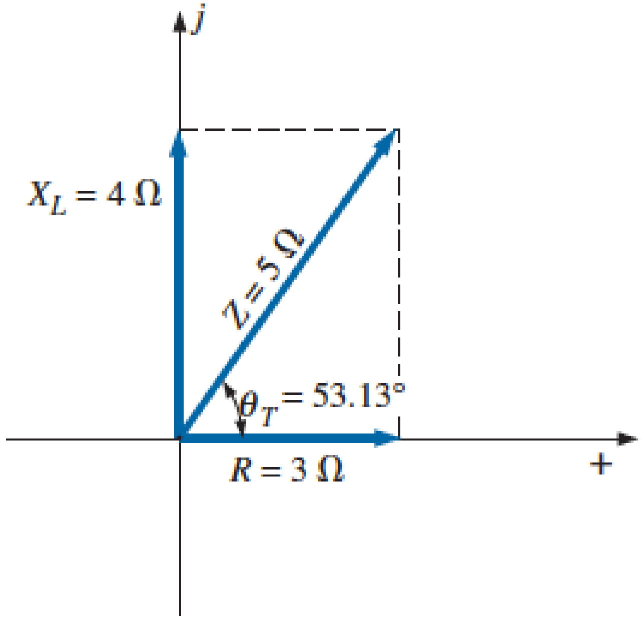
and
$$\mathbf{Z}_T = 5\ \Omega \angle 53.13°$$

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100\ \text{V} \angle 0°}{5\ \Omega \angle 53.13°} = 20\ \text{A} \angle -53.13°$$



Impedance diagram



R – L Circuit

V_R and V_L

Ohm's law:

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20\text{ A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ)$$
$$= \mathbf{60\text{ V}\angle -53.13^\circ}$$

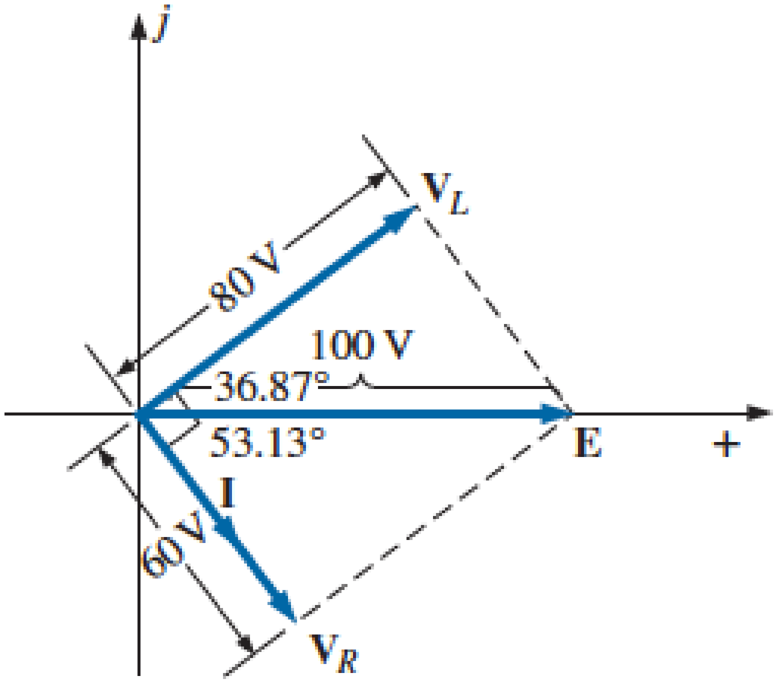
$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20\text{ A}\angle -53.13^\circ)(4\ \Omega\angle 90^\circ)$$
$$= \mathbf{80\text{ V}\angle 36.87^\circ}$$

Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright}\mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L$$



In rectangular form,

$$\mathbf{V}_R = 60\text{ V}\angle -53.13^\circ = 36\text{ V} - j48\text{ V}$$
$$\mathbf{V}_L = 80\text{ V}\angle +36.87^\circ = 64\text{ V} + j48\text{ V}$$

and

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L = (36\text{ V} - j48\text{ V}) + (64\text{ V} + j48\text{ V}) = 100\text{ V} + j0$$
$$= 100\text{ V}\angle 0^\circ$$

R – L Circuit

Power: The total power in watts delivered to the circuit is

$$\begin{aligned} P_T &= EI \cos \theta_T \\ &= (100\text{ V})(20\text{ A}) \cos 53.13^\circ = (2000\text{ W})(0.6) \\ &= \mathbf{1200\ W} \end{aligned}$$

where E and I are effective values and θ_T is the phase angle between E and I, or

$$\begin{aligned} P_T &= I^2R \\ &= (20\text{ A})^2(3\ \Omega) = (400)(3) \\ &= \mathbf{1200\ W} \end{aligned}$$

where I is the effective value, or, finally,

$$\begin{aligned} P_T = P_R + P_L &= V_R I \cos \theta_R + V_L I \cos \theta_L \\ &= (60\text{ V})(20\text{ A}) \cos 0^\circ + (80\text{ V})(20\text{ A}) \cos 90^\circ \\ &= 1200\text{ W} + 0 \\ &= \mathbf{1200\ W} \end{aligned}$$

where θ_R is the phase angle between $\mathbf{V}_R$ and $\mathbf{I}$, and θ_L is the phase angle between $\mathbf{V}_L$ and $\mathbf{I}$.

Power factor: The power factor F_p of the circuit is $\cos 53.13^\circ = \mathbf{0.6}$ **lagging**, where 53.13° is the phase angle between $\mathbf{E}$ and $\mathbf{I}$.

If we write the basic power equation $P = EI \cos \theta$ as follows:

$$\cos \theta = \frac{P}{EI}$$

$$\cos \theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

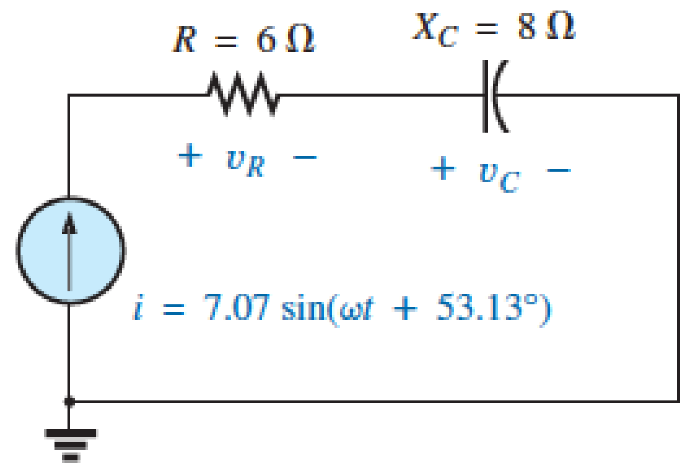
$$F_p = \cos \theta_T = \frac{R}{Z_T}$$

R – C Circuit

Phasor Notation

$$i = 7.07 \sin(\omega t + 53.13°)$$

$$\Rightarrow \mathbf{I} = 5 \text{ A} \angle 53.13°$$



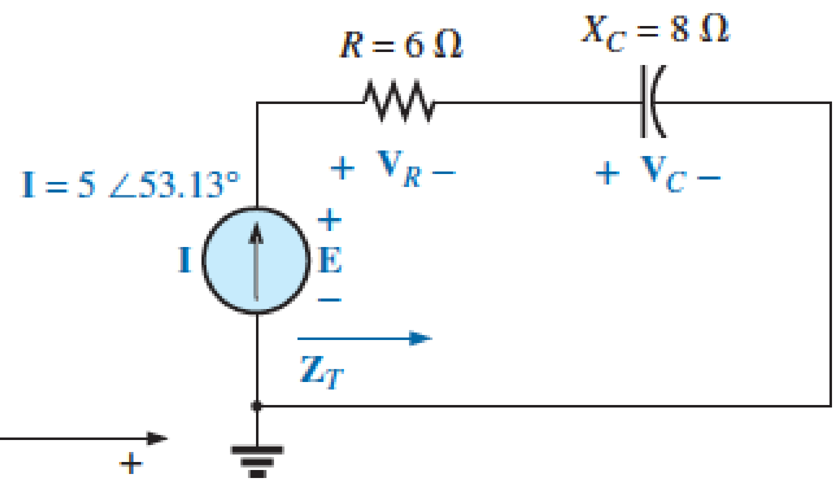
$\mathbf{Z}_T$

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$= 6\ \Omega \angle 0° + 8\ \Omega \angle -90°$$

$$= 6\ \Omega - j8\ \Omega$$

and $\mathbf{Z}_T = \mathbf{10\ \Omega \angle -53.13°}$



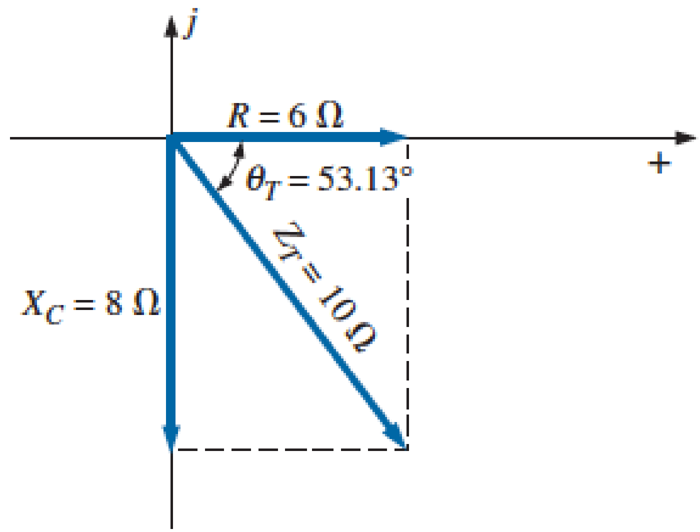
E

$$\mathbf{E} = \mathbf{I}\mathbf{Z}_T$$

$$= (5 \text{ A} \angle 53.13°)(10\ \Omega \angle -53.13°)$$

$$= \mathbf{50\ V \angle 0°}$$

Impedance diagram



R – C Circuit

V_R and V_C

$$\mathbf{V}_R = \mathbf{IZ}_R = (I\angle\theta)(R\angle 0^\circ) = (5\text{ A}\angle 53.13^\circ)(6\ \Omega\angle 0^\circ)$$
$$= \mathbf{30\text{ V}\angle 53.13^\circ}$$

$$\mathbf{V}_C = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90^\circ) = (5\text{ A}\angle 53.13^\circ)(8\ \Omega\angle -90^\circ)$$
$$= \mathbf{40\text{ V}\angle -36.87^\circ}$$

Kirchhoff's voltage law:

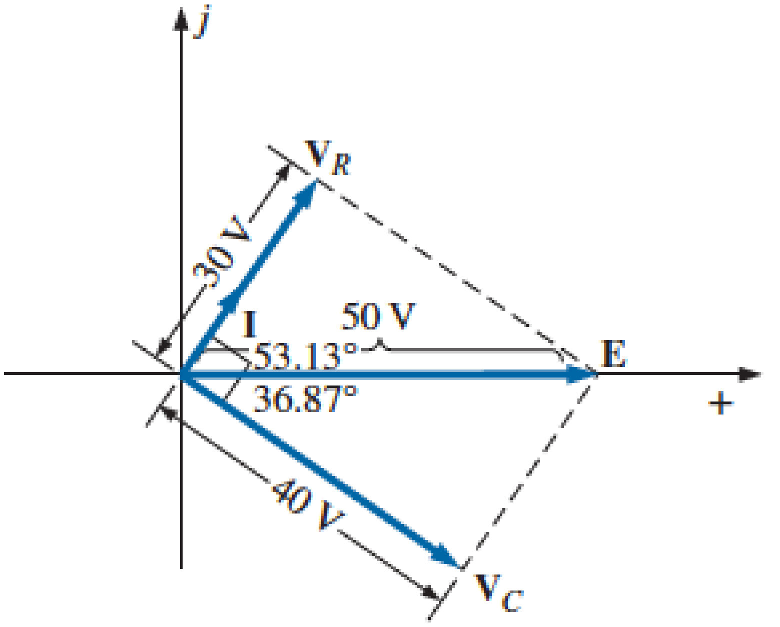
$$\Sigma_{\circlearrowright}\mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_C = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_C$$

which can be verified by vector algebra as demonstrated for the *R-L* circuit.

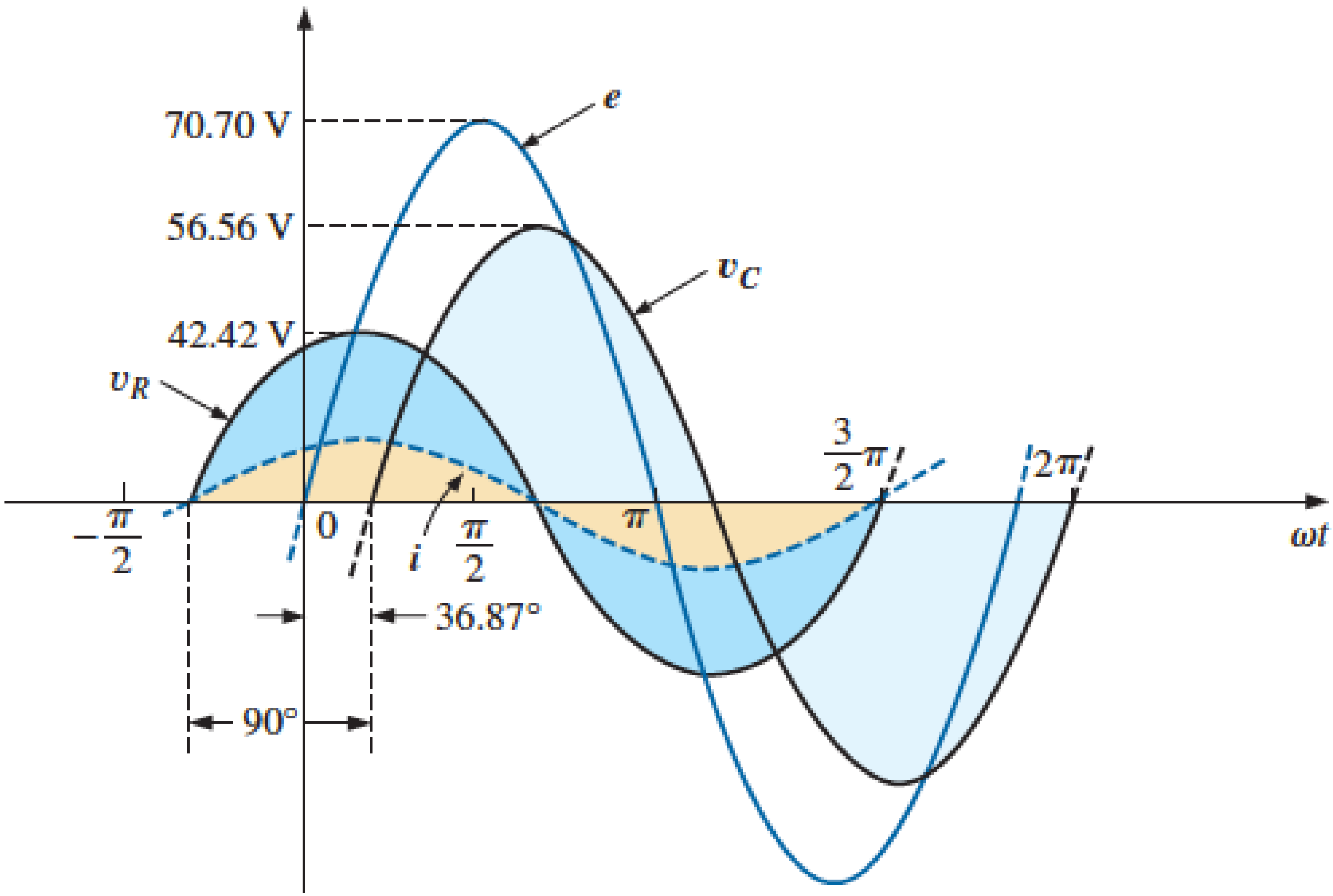
Phasor diagram: Note on the phasor diagram that the current I is in phase with the voltage across the resistor and leads the voltage across the capacitor by 90°.



R – C Circuit

Time domain: In the time domain,

$$e = \sqrt{2}(50)\sin\omega t = \mathbf{70.70\sin\omega t}$$
$$v_R = \sqrt{2}(30)\sin(\omega t + 53.13^\circ = \mathbf{42.42\sin(\omega t + 53.13^\circ)}$$
$$v_C = \sqrt{2}(40)\sin(\omega t - 36.87^\circ = \mathbf{56.56\ \sin(\omega t - 36.87^\circ)}$$



R – C Circuit

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(5\text{ A})\cos 53.13^\circ$$
$$= (250)(0.6) = \mathbf{150\ W}$$

or
$$P_T = I^2R = (5\text{ A})^2(6\ \Omega) = (25)(6)$$
$$= \mathbf{150\ W}$$

or, finally,

$$P_T = P_R + P_C = V_R I\cos\theta_R + V_C I\cos\theta_C$$
$$= (30\text{ V})(5\text{ A})\cos 0^\circ + (40\text{ V})(5\text{ A})\cos 90^\circ$$
$$= 150\text{ W} + 0$$
$$= \mathbf{150\ W}$$

Power factor: The power factor of the circuit is

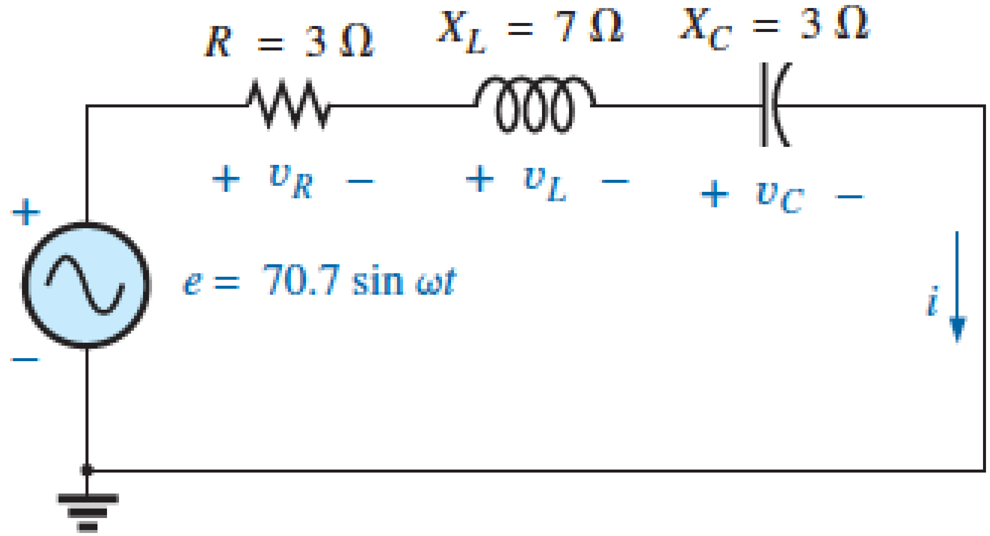
$$F_p = \cos\theta = \cos 53.13^\circ = \mathbf{0.6\ leading}$$

Using Eq. (15.9), we obtain

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{6\ \Omega}{10\ \Omega}$$
$$= \mathbf{0.6\ leading}$$

R – L – C Circuit

Phasor Notation $\mathbf{E} = 50\text{ V}\angle 0^\circ$



$\mathbf{Z}_T$

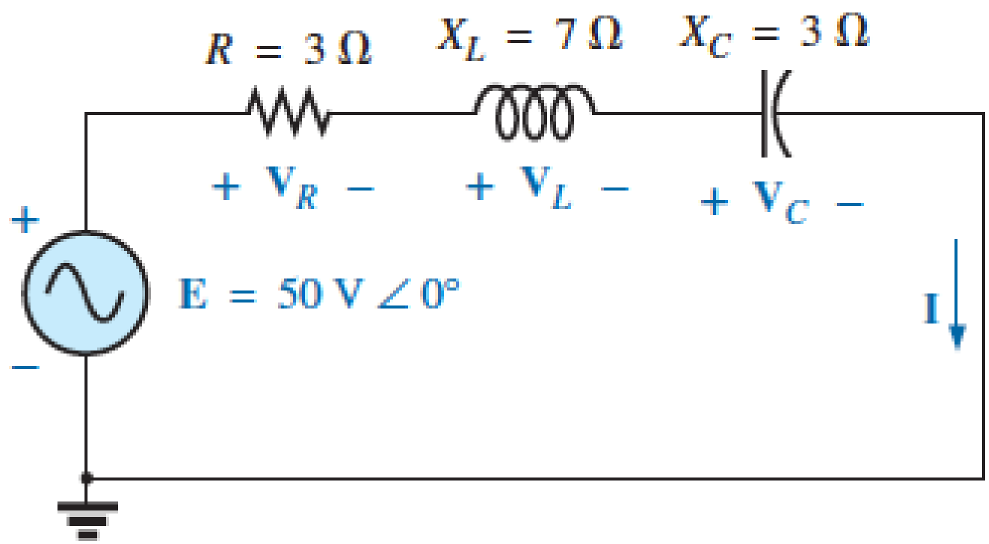
$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \\
&= R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ \\
&= 3\,\Omega + j7\,\Omega - j3\,\Omega \\
&= 3\,\Omega + i4\,\Omega
\end{aligned}
$$

$$\mathbf{Z}_T = 5\,\Omega\angle 53.13^\circ$$

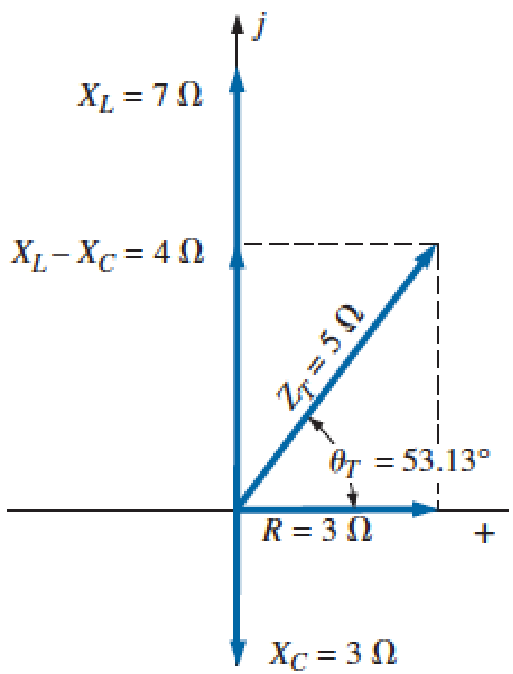
I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\angle 0^\circ}{5\,\Omega\angle 53.13^\circ}$$

$$= 10\text{ A}\angle -53.13^\circ$$



Impedance diagram



R – L – C Circuit

V_R, V_L, and V_C

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0^\circ) = (10\text{ A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ)$$
$$= \mathbf{30\ V\angle -53.13^\circ}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90^\circ) = (10\text{ A}\angle -53.13^\circ)(7\ \Omega\angle 90^\circ)$$
$$= \mathbf{70\ V\angle 36.87^\circ}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90^\circ) = (10\text{ A}\angle -53.13^\circ)(3\ \Omega\angle -90^\circ)$$
$$= \mathbf{30\ V\angle -143.13^\circ}$$

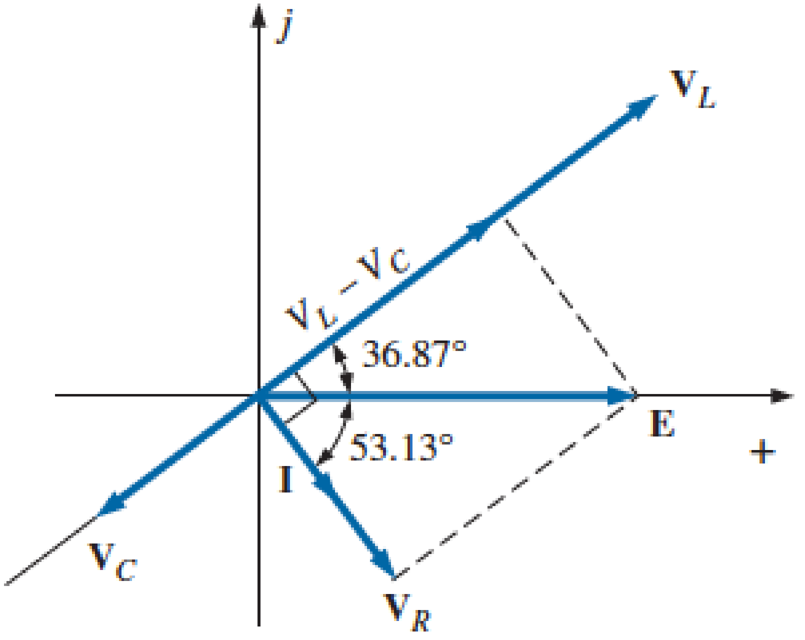
Kirchhoff's voltage law:

$$\Sigma_{\circlearrowright} \mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

or

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

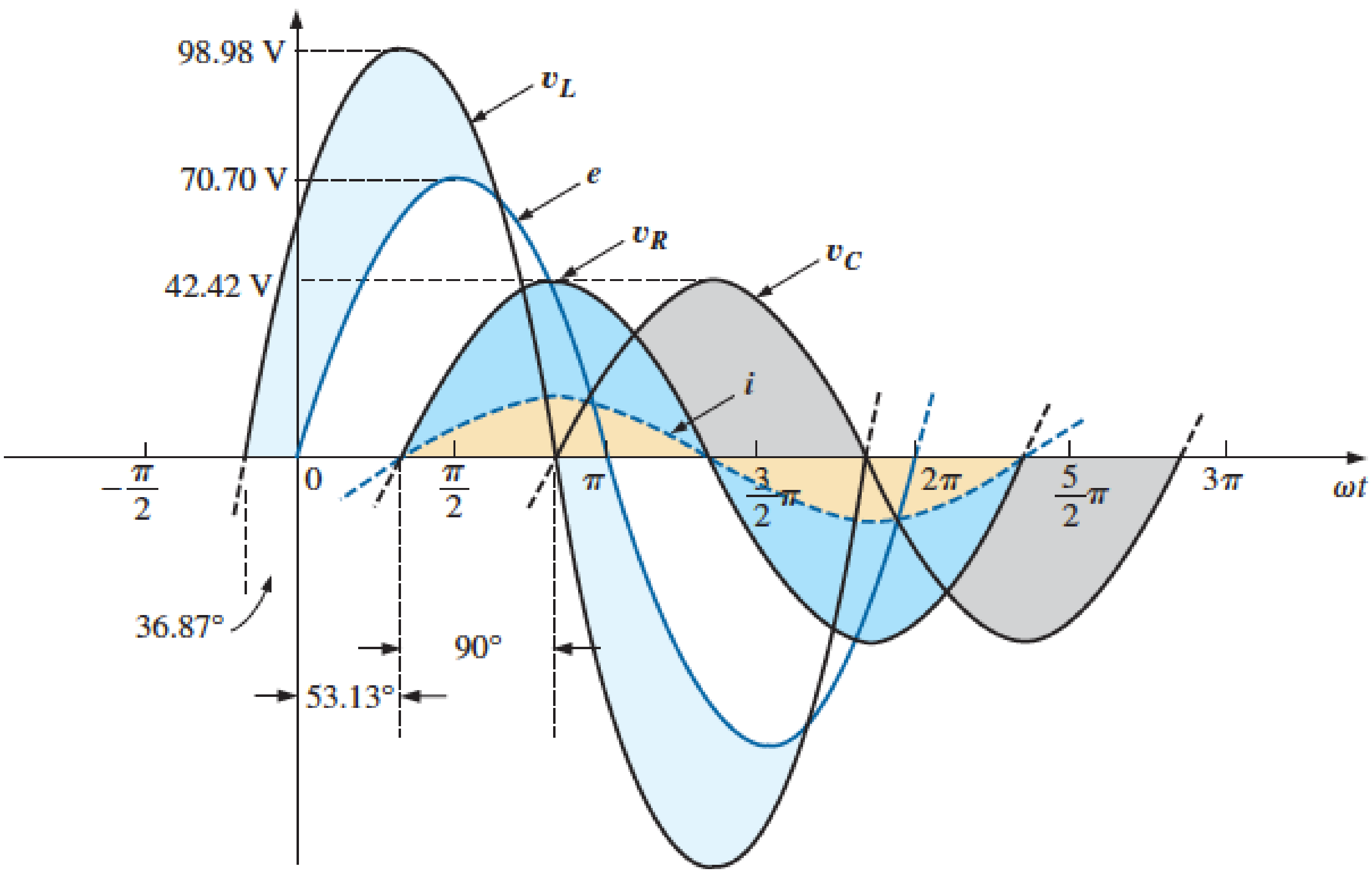
Phasor diagram: The phasor diagram indicates that the current **I** is in phase with the voltage across the resistor, lags the voltage across the inductor by 90°, and leads the voltage across the capacitor by 90°.



R – L – C Circuit

Time domain:

$$i = \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = \mathbf{14.14\sin(\omega t - 53.13^\circ)}$$
$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13^\circ) = \mathbf{42.42\sin(\omega t - 53.13^\circ)}$$
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87^\circ) = \mathbf{98.98\sin(\omega t + 36.87^\circ)}$$
$$v_C = \sqrt{2}(30)\sin(\omega t - 143.13^\circ) = \mathbf{42.42\sin(\omega t - 143.13^\circ)}$$



R – L – C Circuit

Power: The total power in watts delivered to the circuit is

$$P_T = EI\cos\theta_T = (50\text{ V})(10\text{ A})\cos 53.13° = (500)(0.6) = \mathbf{300\ W}$$

or

$$P_T = I^2R = (10\text{ A})^2(3\ \Omega) = (100)(3) = \mathbf{300\ W}$$

or

$$\begin{aligned} P_T &= P_R + P_L + P_C \\ &= V_R I\cos\theta_R + V_L I\cos\theta_L + V_C I\cos\theta_C \\ &= (30\text{ V})(10\text{ A})\cos 0° + (70\text{ V})(10\text{ A})\cos 90° + (30\text{ V})(10\text{ A})\cos 90° \\ &= (30\text{ V})(10\text{ A}) + 0 + 0 = \mathbf{300\ W} \end{aligned}$$

Power factor: The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

or

$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$