Electrical Circuit-I

10th Lecture

MESH ANALYSIS

By:

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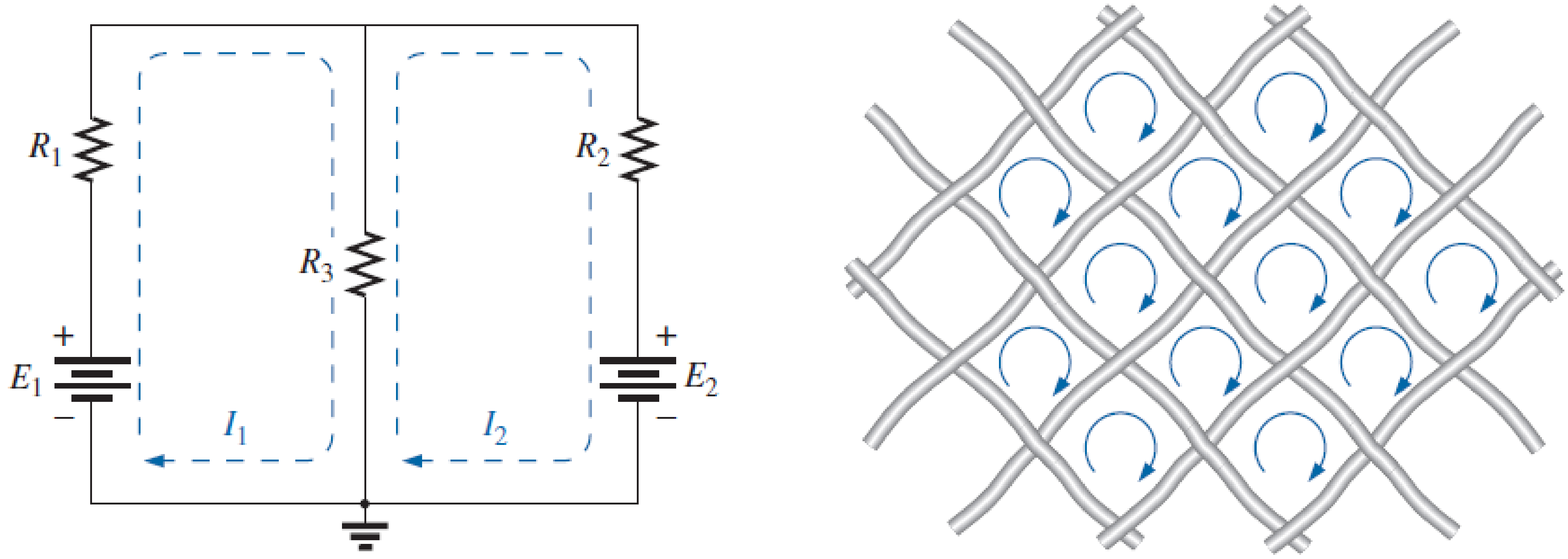
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

MESH ANALYSIS

The number of mesh currents required to analyze a network will equal the number of "windows" of the configuration.

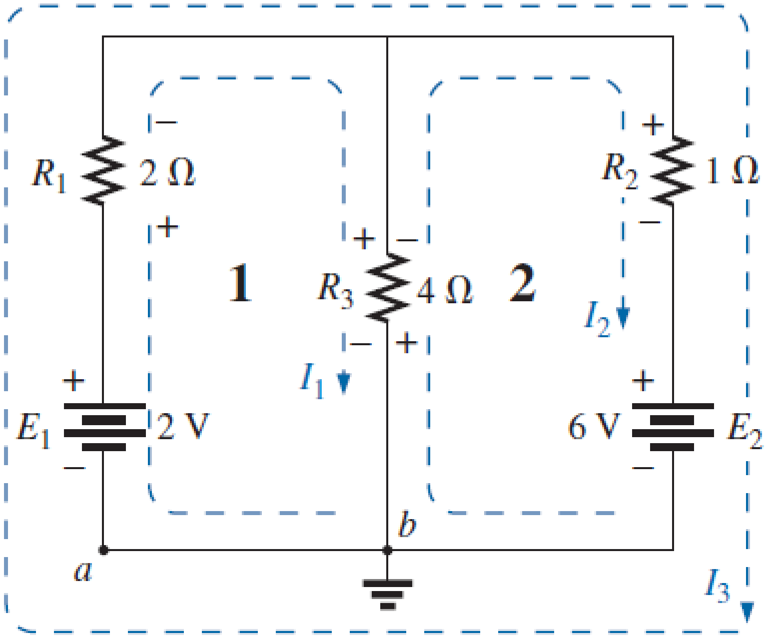


(a) (b)

Mesh Analysis Procedure

1. *Assign a distinct current in the **clockwise direction** to each independent **closed loop** of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. In fact, any direction can be chosen for each loop current with no loss in accuracy, as long as the remaining steps are followed properly. However, by choosing the clockwise direction as a standard, we can develop a shorthand method for writing the required equations that will save time and possibly prevent some common errors.*

2. *Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Figure below, that the **4Ω resistor** have two sets of polarities across it.*



Mesh Analysis Procedure

3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity.***

a) If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

b) The polarity of a voltage source is unaffected by the direction of the assigned loop currents.

4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

Example (1)

Apply the MESH analysis method to the network in Fig. 1

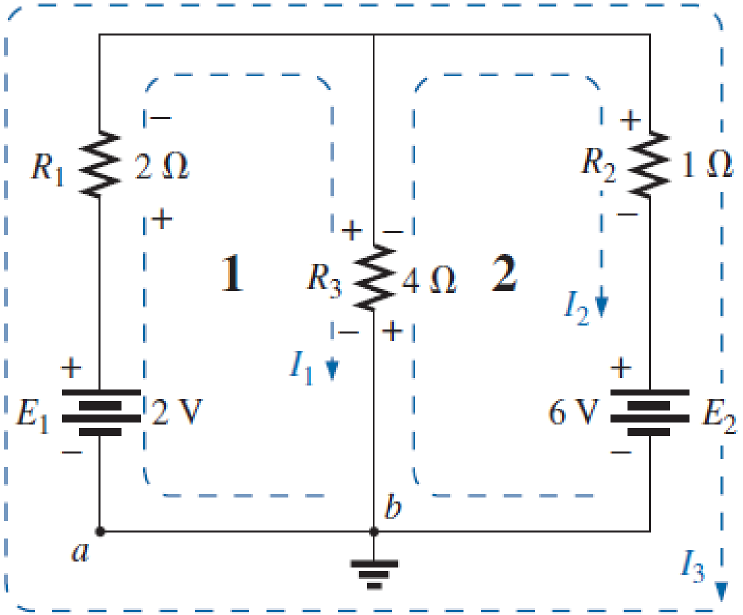


FIG. 1

Solution:

Step 1:

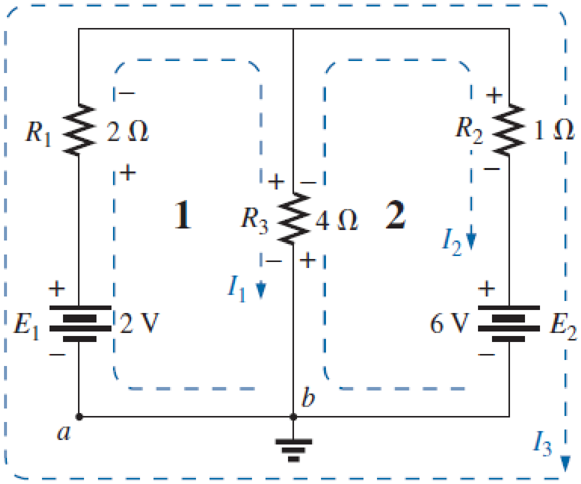
Two loop currents (I1 and I2) are assigned in the clockwise direction in the windows of the network. A third loop (I3) could have been included around the entire network, but the information carried by this loop is already included in the other two.

Step 2:

Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4Ω resistor are the opposite for each loop current.

Step 3:

Kirchhoff's voltage law is applied around each loop in the clockwise direction.



loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\,\Omega)\,I_1 - \overbrace{(4\,\Omega)\underbrace{(I_1 - I_2)}_{\text{Total current through } 4\,\Omega \text{ resistor}}}^{\text{Voltage drop across } 4\,\Omega \text{ resistor}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\,\Omega)(I_2 - I_1) - (1\,\Omega)I_2 - 6\text{ V} = 0$$

Step 4:

The equations are then rewritten as follows (without units for clarity):

loop 1: $+2 - 2I_1 - 4I_1 + 4I_2 = 0$

loop 2: $-4I_2 + 4I_1 - 1I_2 - 6 = 0$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current.

and loop 1: $+2 - 6I_1 + 4I_2 = 0$
loop 2: $-5I_2 + 4I_1 - 6 = 0$

or loop 1: $-6I_1 + 4I_2 = -2$
loop 2: $+4I_1 - 5I_2 = +6$

Applying determinants results in

$$I_1 = \mathbf{-1\ A} \quad \text{and} \quad I_2 = \mathbf{-2\ A}$$

The current through the 4 Ω resistor is determined by the following equation from the original network:

loop 1: $I_{4\Omega} = I_1 - I_2 = -1\text{ A} - (-2\text{ A}) = -1\text{ A} + 2\text{ A}$
$= \mathbf{1\ A}$ (in the direction of I_1)

Example (2)

Apply the MESH analysis method to find the current through each branch of the network in Fig.2

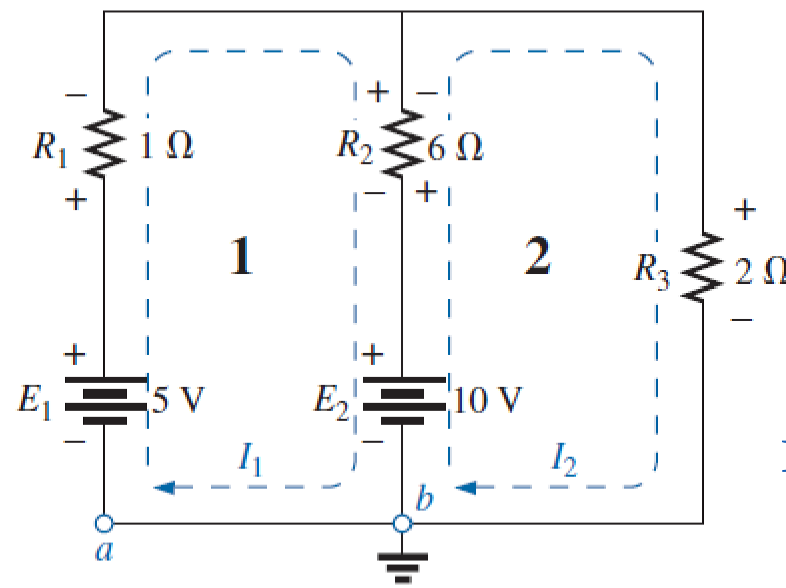


FIG. 2

Solution:

Step 1 and 2:

These are as indicated in the circuit. Note that the polarities of the 6 Ω resistor are different for each loop current.

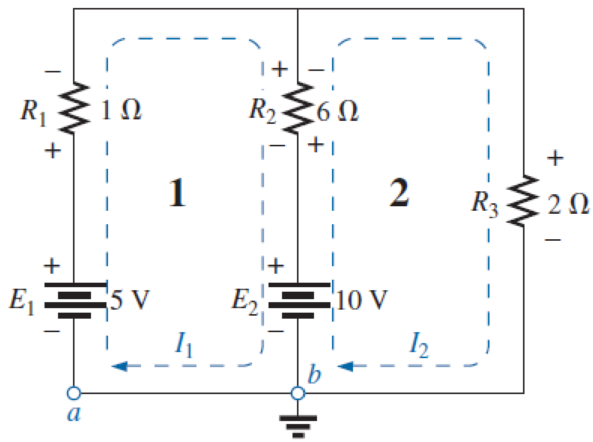
Step 3:

Kirchhoff's voltage law is applied around each loop in the clockwise direction.

loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

↑ I_2 flows through the 6 Ω resistor in the direction opposite to I_1.



loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$

$I_2 > I_1$ (2 A > 1 A)

Therefore,

$$I_{R_2} = I_2 - I_1 = 2\text{ A} - 1\text{ A} = \mathbf{1\ A}$$

in the direction of I_2

Example (3)

Find the branch currents of the networks in Fig. 3

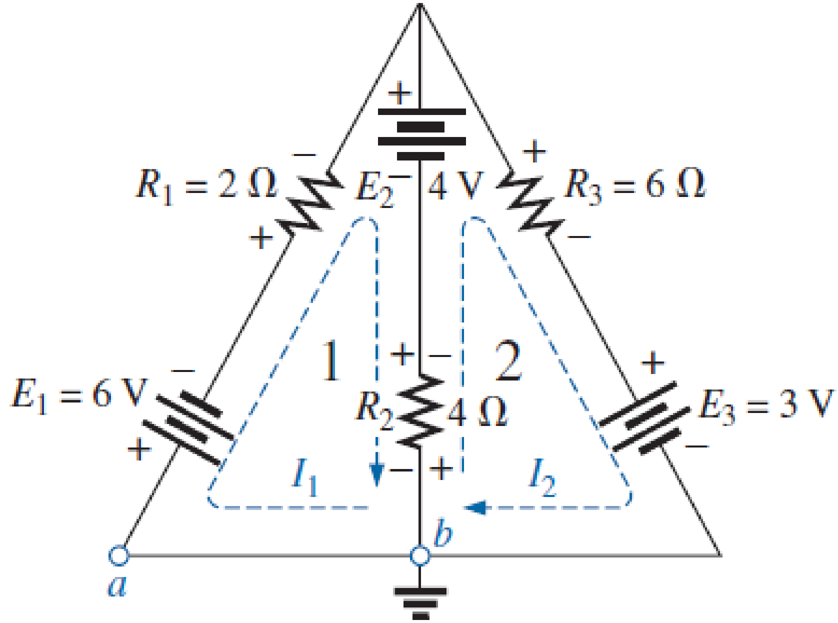


FIG. 3

Solution:

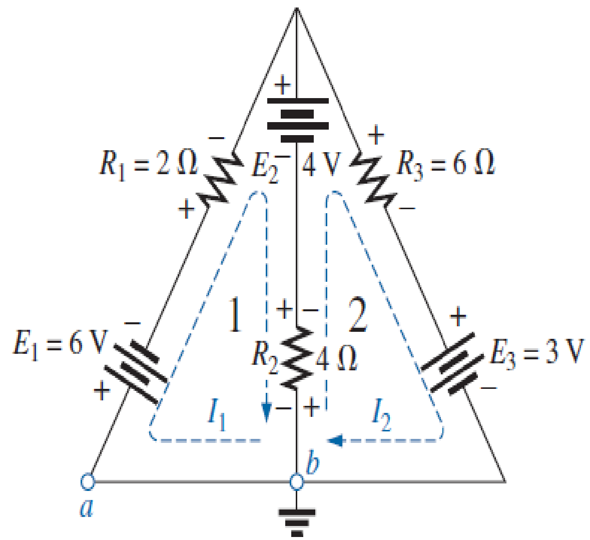
Step 1 and 2:

These are as indicated in the circuit. Note that the polarities of the 4 Ω resistor are different for each loop current.

Step 3:

Kirchhoff's voltage law is applied around each loop in the clockwise direction.

MESH ANALYSIS



loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point a)

$$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point b)

$$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$$

which are rewritten as

$$\left.\begin{array}{r} -10 - 4I_1 - 2I_1 + 4I_2 = 0 \\ +1 + 4I_1 + 4I_2 - 6I_2 = 0 \end{array}\right\} \begin{array}{r} -6I_1 + 4I_2 = +10 \\ +4I_1 - 10I_2 = -1 \end{array}$$

or, by multiplying the top equation by -1, we obtain

$$\begin{array}{r} 6I_1 - 4I_2 = -10 \\ 4I_1 - 10I_2 = -1 \end{array}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.18\ A}$$

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.77\ A}$$

The current in the 4 Ω resistor and 4 V source for loop 1 is:

$$\begin{aligned} I_1 - I_2 &= -2.18\text{ A} - (-0.77\text{ A}) \\ &= -2.18\text{ A} + 0.77\text{ A} \\ &= \mathbf{-1.41\ A} \end{aligned}$$

revealing that it is (1.41 A) in a direction opposite (due to the minus sign) to I1 in loop 1.

Example (4)

Using mesh analysis, determine the currents of the network in Fig. 4-1

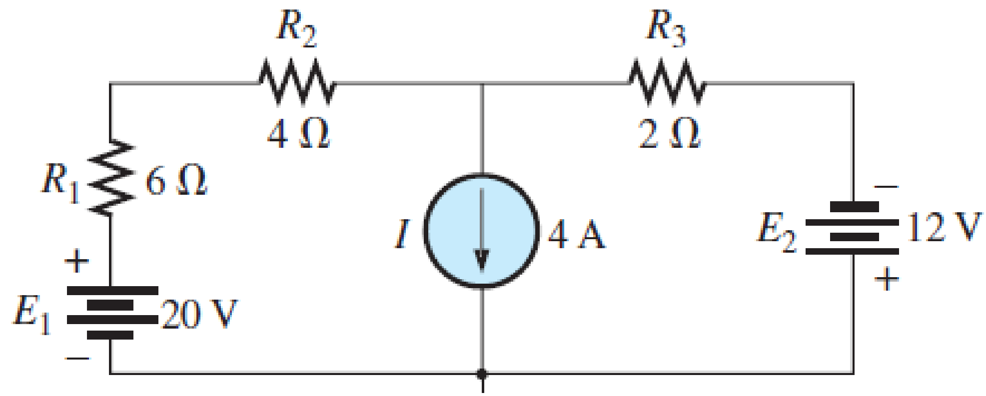


FIG.4-1

Solution:

First, the mesh currents for the network are defined, as shown in Fig. 4-2. Then the current source is mentally removed, as shown in Fig. 4-3, and Kirchhoff's voltage law is applied to the resulting network. The single path now including the effects of two mesh currents is referred to as the path of a Supermesh current.

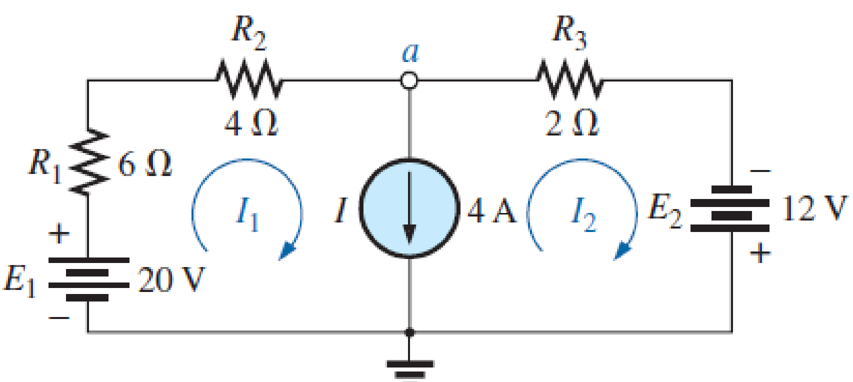


FIG.4-2

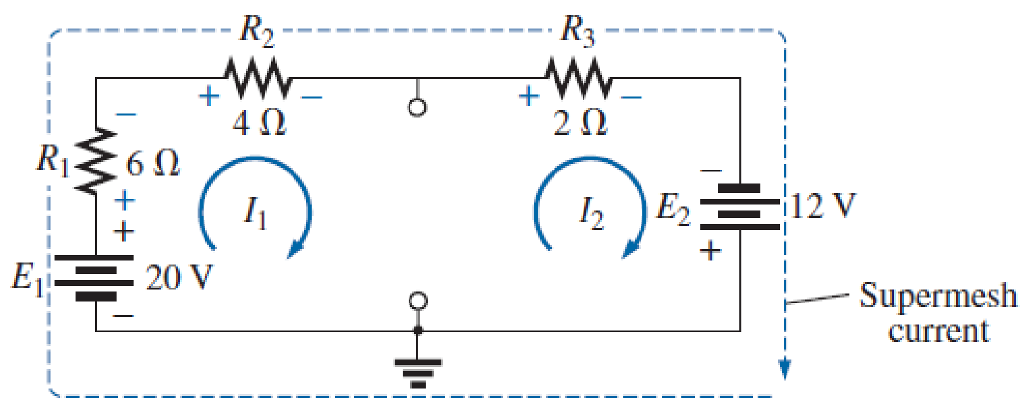
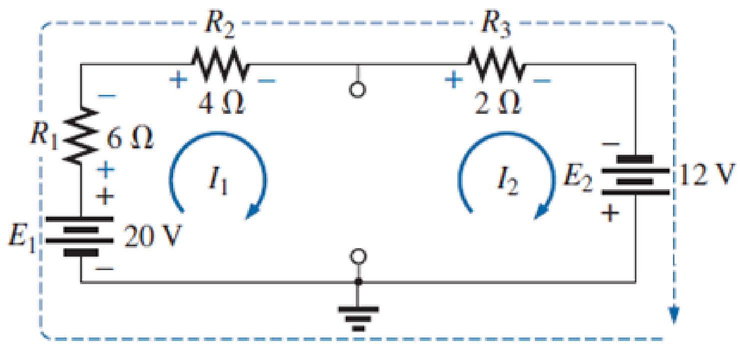


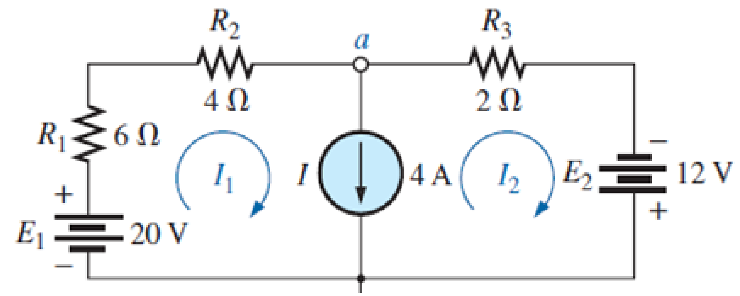
FIG.4-3

Applying Kirchhoff's voltage law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$
$$10I_1 + 2I_2 = 32$$



Node a is then used to relate the mesh currents and the current source using Kirchhoff's current law:



$$I_1 = I + I_2$$

The result is two equations and two unknowns:

$$\begin{aligned} 10I_1 + 2I_2 &= 32 \\ I_1 - I_2 &= 4 \end{aligned}$$

Applying determinants:

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{(32)(-1) - (2)(4)}{(10)(-1) - (2)(1)} = \frac{40}{12} = \mathbf{3.33\ A}$$

$$I_2 = I_1 - I = 3.33\text{ A} - 4\text{ A} = \mathbf{-0.67\ A}$$

Example (5)

Using mesh analysis, determine the currents for the network in Fig. 5-1

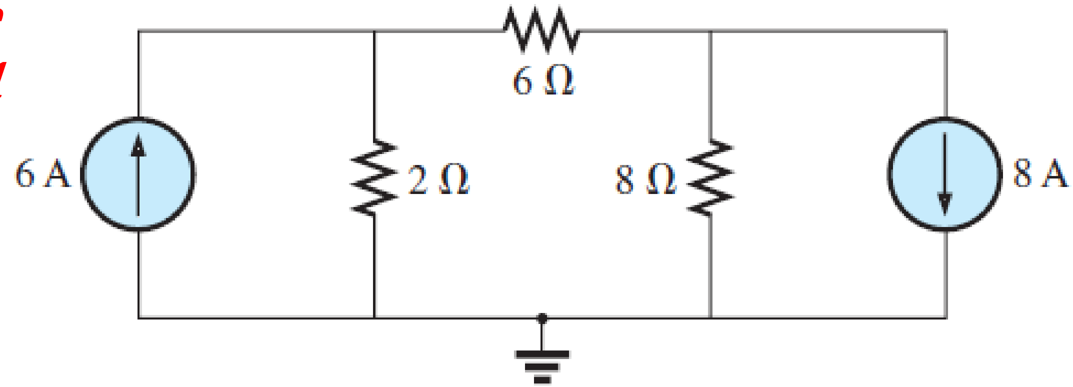


FIG.5-1

Solution:

The mesh currents are defined in Fig. 5-2. The current sources are removed, and the single supermesh path is defined in Fig. 5-3.

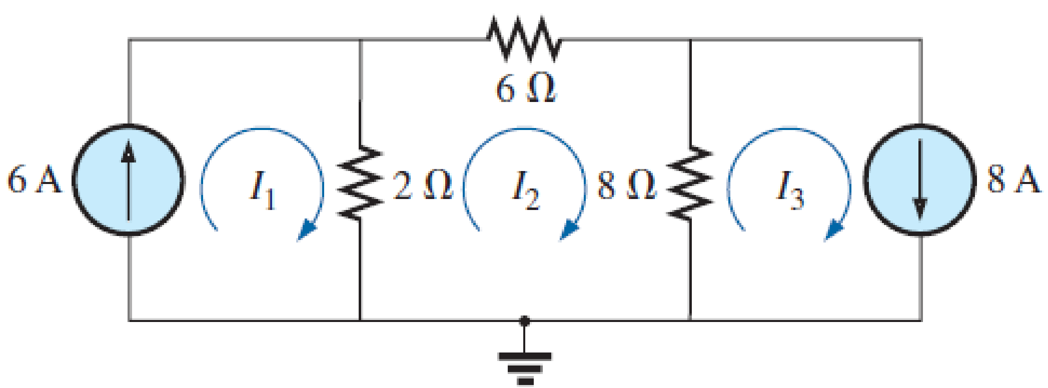


FIG.5-2

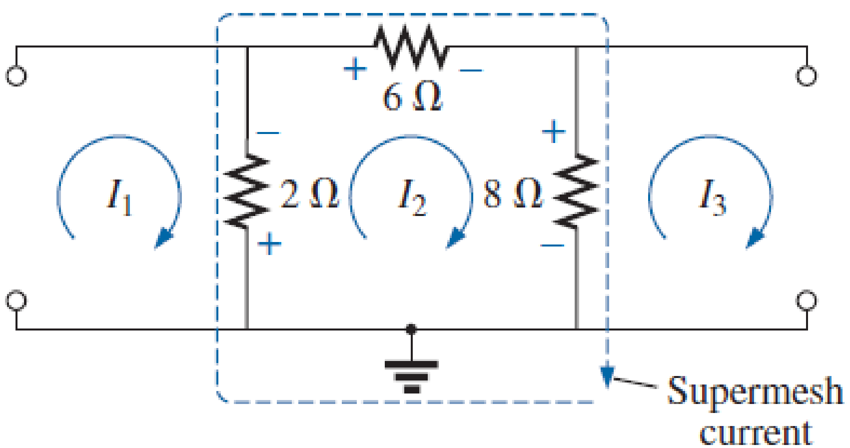
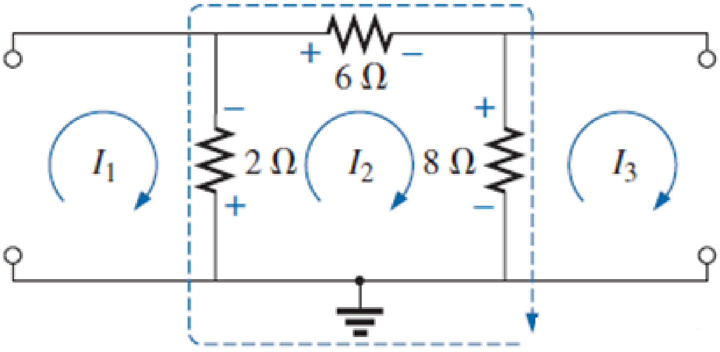


FIG.5-3

Applying Kirchhoff's voltage law around the supermesh path:



$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

Introducing the relationship between the mesh currents and the current sources:

$$I_1 = 6\ \text{A}$$
$$I_3 = 8\ \text{A}$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$
$$2(6\ \text{A}) - 16I_2 + 8(8\ \text{A}) = 0$$

and $$I_2 = \frac{76\ \text{A}}{16} = \mathbf{4.75\ A}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\ \text{A} - 4.75\ \text{A} = \mathbf{1.25\ A}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\ \text{A} - 4.75\ \text{A} = \mathbf{3.25\ A}$$