Electrical Circuit-I

11th Lecture-Tutorial

NODAL ANALYSIS

By:

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Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Example (4)

Determine the nodal voltages V1 and V2 in Fig. 4-1 using the concept of a supernode.

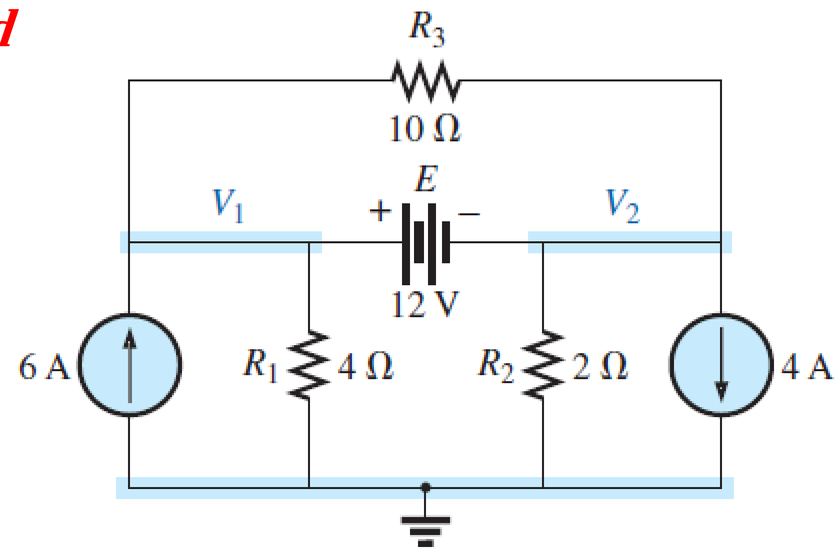


FIG. 4-1

Solution:

- *Replacing the independent voltage source of 12 V with a **short-circuit** equivalent results in the network in Fig. 4-2.*
- *The result is a single supernode for which Kirchhoff's current law must be applied.*
  - *note that the current **I3 leaves** the supernode at V1 and then **enters** the same supernode at V2.*
  - *It must therefore appear **twice** when applying **Kirchhoff's current law**, as shown Fig. 4-2*

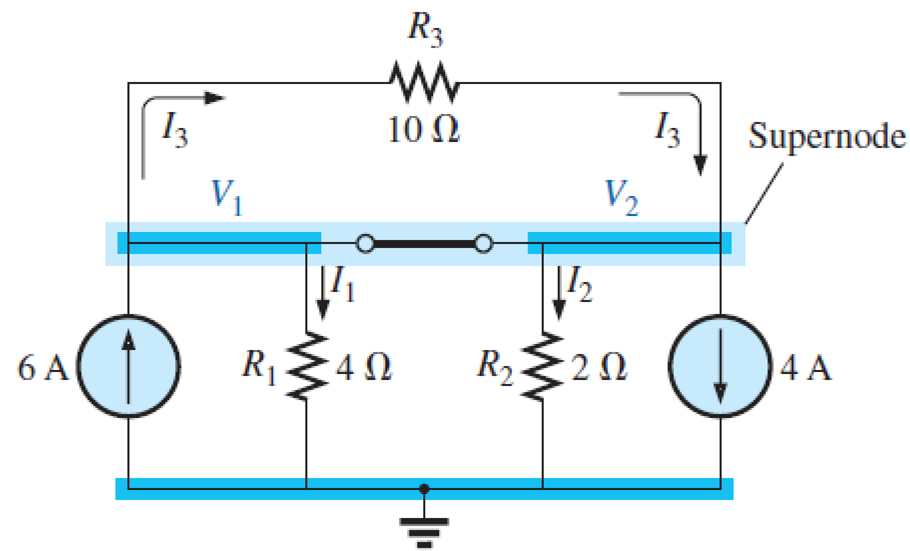
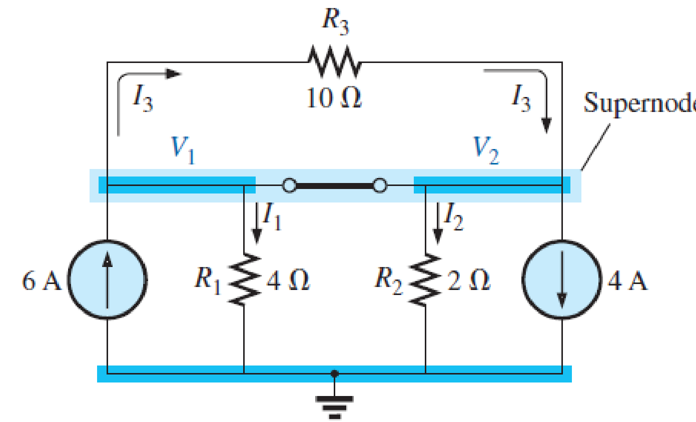


FIG. 4-2



$$\Sigma I_i = \Sigma I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or $$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then $$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and $$\frac{V_1}{4\ \Omega} + \frac{V_2}{2\ \Omega} = 2\text{ A}$$

Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - \ \ 1V_2 &= 12 \end{aligned}$$

Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and $$0.75V_2 = 2 - 3 = -1$$

so that $$V_2 = \frac{-1}{0.75} = -\mathbf{1.33\ V}$$

and $$V_1 = V_2 + 12\text{ V} = -1.33\text{ V} + 12\text{ V}$$

$$V_1 = \mathbf{+10.67\ V}$$

The current of the network can then be determined as follows:

$$I_1\downarrow = \frac{V}{R_1} = \frac{10.67\text{ V}}{4\ \Omega} = \mathbf{2.67\ A}$$

$$I_2\uparrow = \frac{V_2}{R_2} = \frac{1.33\text{ V}}{2\ \Omega} = \mathbf{0.67\ A}$$

$$\underset{\rightarrow}{I_3} = \frac{V_1 - V_2}{10\ \Omega} = \frac{10.67\text{ V} - (-1.33\text{ V})}{10\ \Omega}$$

$$\underset{\rightarrow}{I_3} = \frac{12\ \Omega}{10\ \Omega} = \mathbf{1.2\ A}$$

Example (5)

Write the nodal equations for the network in Fig. 5-1

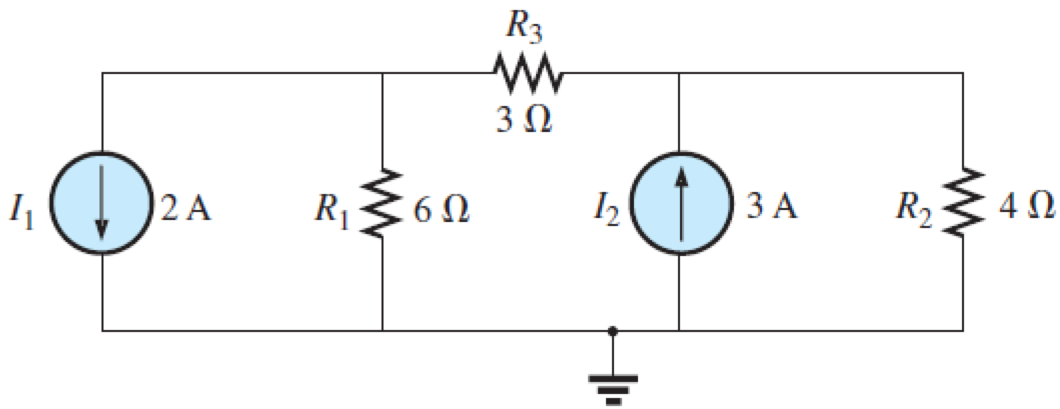


FIG. 5-1

Solution:

Step 1 :

Redraw the figure with assigned subscripted voltages in Fig. 5-2

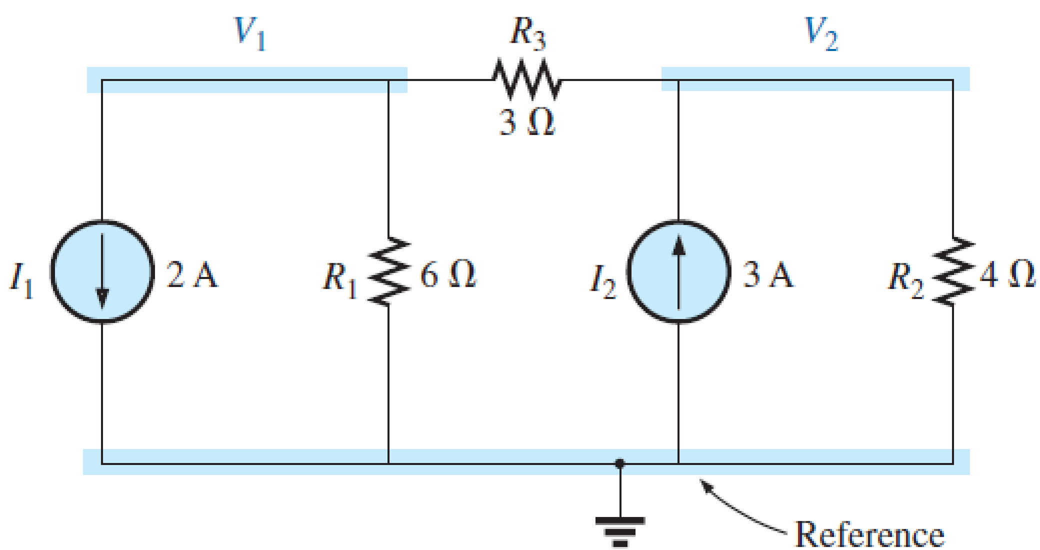
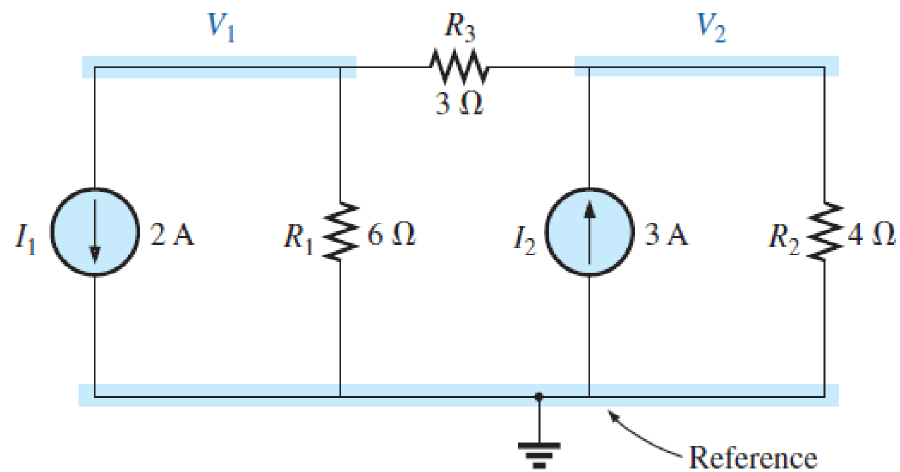


FIG. 5-2

Step 2 to 4:

Drawing current from node 1

$$V_1: \underbrace{\left(\frac{1}{6\ \Omega} + \frac{1}{3\ \Omega}\right)}_{\text{Sum of conductances connected to node 1}} V_1 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\text{Mutual conductance}} V_2 = -2\ \text{A}$$



Supplying current to node 2

$$V_2: \underbrace{\left(\frac{1}{4\ \Omega} + \frac{1}{3\ \Omega}\right)}_{\text{Sum of conductances connected to node 2}} V_2 - \underbrace{\left(\frac{1}{3\ \Omega}\right)}_{\text{Mutual conductance}} V_1 = +3\ \text{A}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

Example (6)

Find the voltage across the 3 Ω resistor in Fig. 6-1 by nodal analysis.

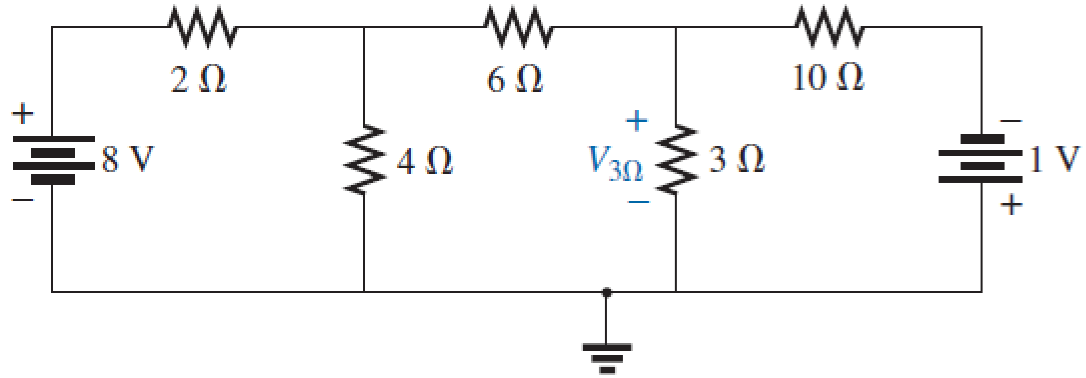


FIG. 6-1

Solution:

Converting sources and choosing nodes (Fig. 6- 2), we have

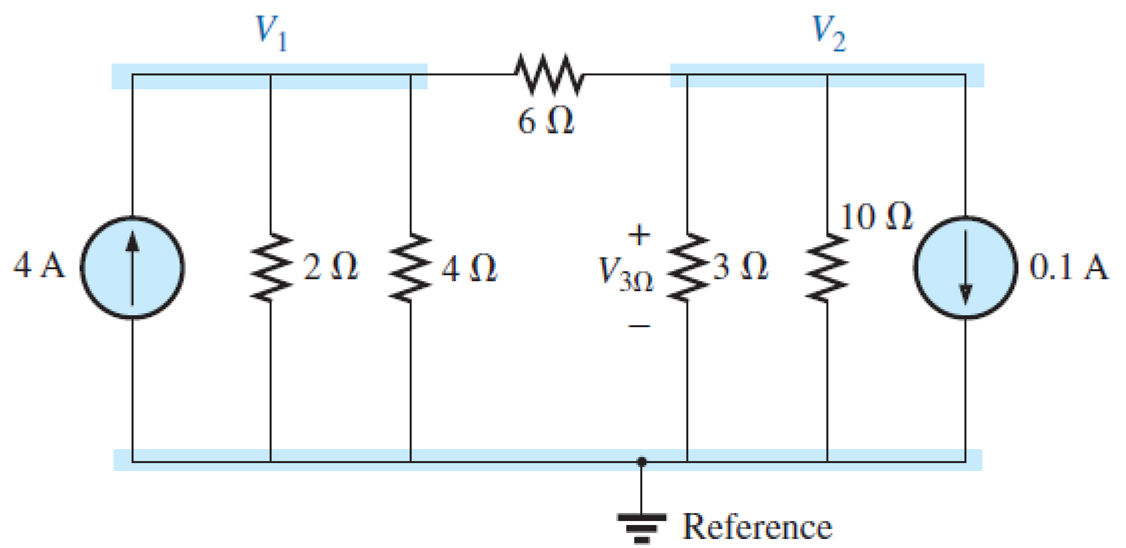
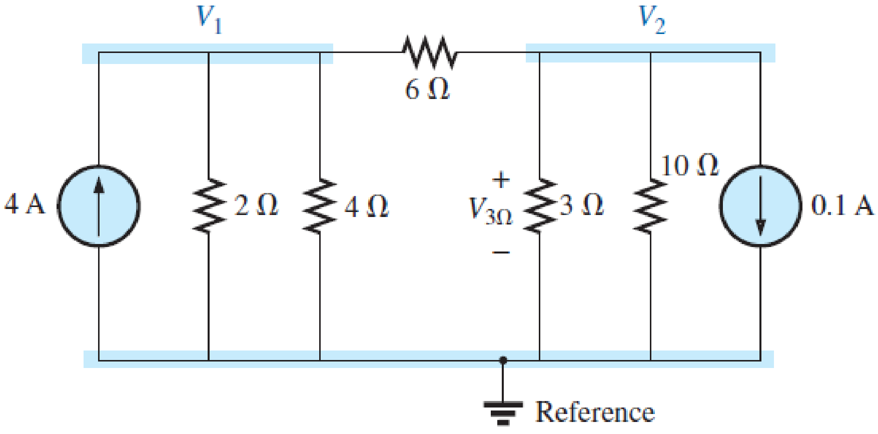


FIG. 6-2

$$\left(\frac{1}{2\ \Omega}+\frac{1}{4\ \Omega}+\frac{1}{6\ \Omega}\right)V_1-\left(\frac{1}{6\ \Omega}\right)V_2=+4\text{ A}$$

$$\left(\frac{1}{10\ \Omega}+\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}\right)V_2-\left(\frac{1}{6\ \Omega}\right)V_1=-0.1\text{ A}$$



$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

resulting in

$$11V_1-2V_2=+48$$

$$-5V_1+18V_2=-3$$

$$V_2=V_{3\Omega}=\frac{\begin{vmatrix}11 & 48\\ -5 & -3\end{vmatrix}}{\begin{vmatrix}11 & -2\\ -5 & 18\end{vmatrix}}=\frac{-33+240}{198-10}=\frac{207}{188}=\mathbf{1.10\ V}$$

Example (7)

Write the nodal equations and find the voltage across the 2 Ω resistor for the network in Fig. 7-1

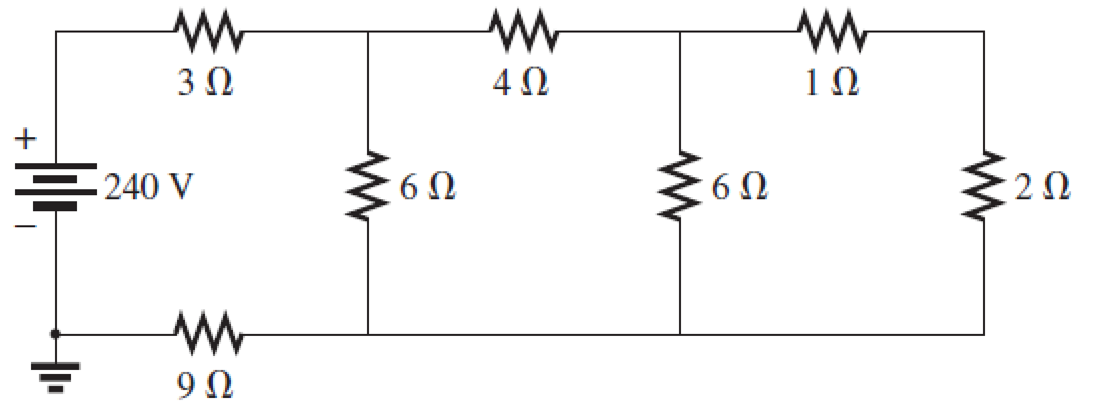
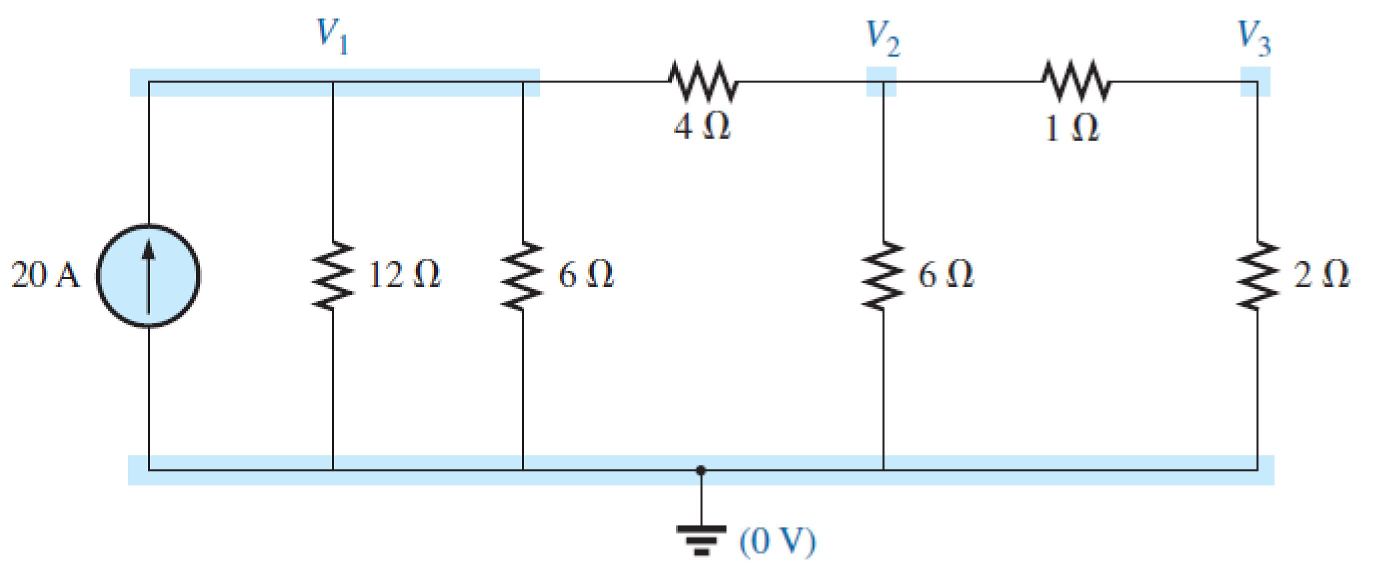


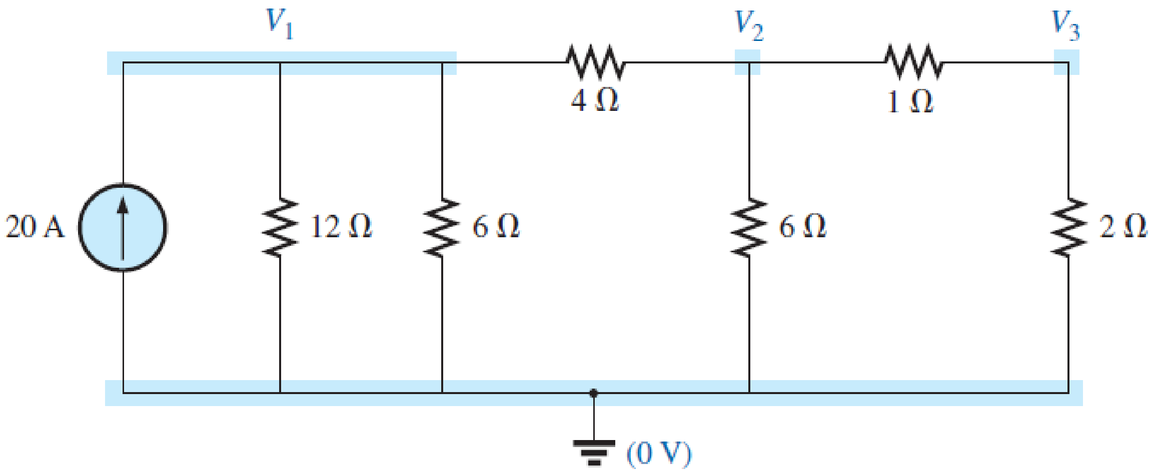
FIG. 7-1

Solution:

The nodal voltages are chosen as shown in Fig. 7- 2

FIG. 7-2





$$V_1: \left(\frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{4\ \Omega}\right)V_2 + 0 = 20\ \text{A}$$

$$V_2: \left(\frac{1}{4\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{1\ \Omega}\right)V_2 - \left(\frac{1}{4\ \Omega}\right)V_1 - \left(\frac{1}{1\ \Omega}\right)V_3 = 0$$

$$V_3: \left(\frac{1}{1\ \Omega} + \frac{1}{2\ \Omega}\right)V_3 - \left(\frac{1}{1\ \Omega}\right)V_2 + 0 = 0$$

and

$$0.5V_1 - 0.25V_2 + 0 = 20$$

$$-0.25V_1 + \frac{17}{12}V_2 - 1V_3 = 0$$

$$0 - 1V_2 + 1.5V_3 = 0$$

$$V_3 = V_{2\Omega} = \mathbf{10.67\ V}$$