Electrical Circuit-I

11th Lecture

NODAL ANALYSIS

By:

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Ref: Robert L. Boylestad, "***Introductory Circuit Analysis***", Pearson Prentice Hall, Eleventh Edition, 2007

NODAL ANALYSIS

- ***the number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.***

  (N - 1) nodal voltages that need to be determined

- ***the number of equations required to solve for all the nodal voltages of a network is 1 less than the total number of independent nodes.***

  (N -1) independent equations be written to find the nodal voltages

NODAL ANALYSIS

Nodal Analysis Procedure

1) *Determine the number of nodes within the network.*

2) *Pick a reference node, and label each remaining node with a subscripted value of voltage: V1, V2, and so on.*

3) *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.*

4) *Solve the resulting equations for the nodal voltages.*

Example (1)

Apply nodal analysis to the networ[k] in Fig. 1-1

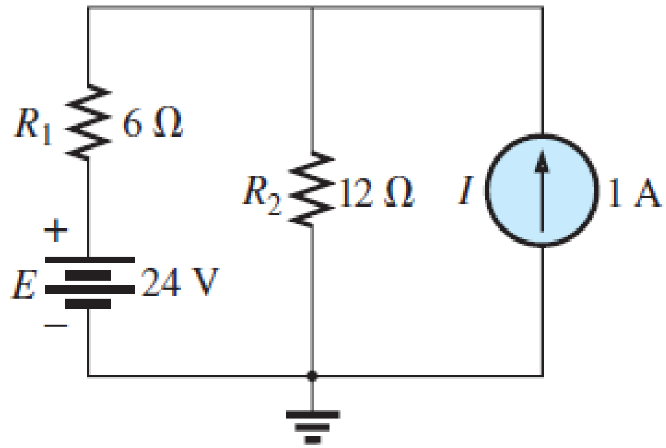


FIG. 1-1

Solution:

Step 1 and 2:

The network has two nodes, as shown in Fig. 1-2. The lower node is defined as the reference node at ground potential (zero volts), and the other node as V1, the voltage from node 1 to ground.

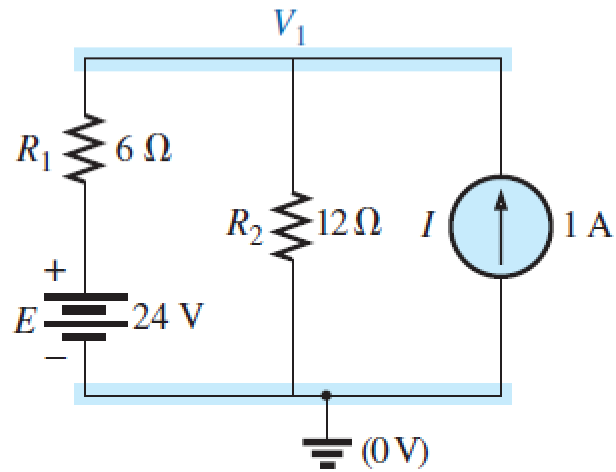


FIG. 1-2

Step 3:

I1 and I2 are defined as leaving the node in Fig. 1-3, and Kirchhoff's current law is applied as follows:

$$I = I_1 + I_2$$

The current I2 is related to the nodal voltage V1 by Ohm's law:

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

The current I1 is also determined by Ohm's law as follows:

$$I_1 = \frac{V_{R_1}}{R_1}$$

$$V_{R_1} = V_1 - E$$

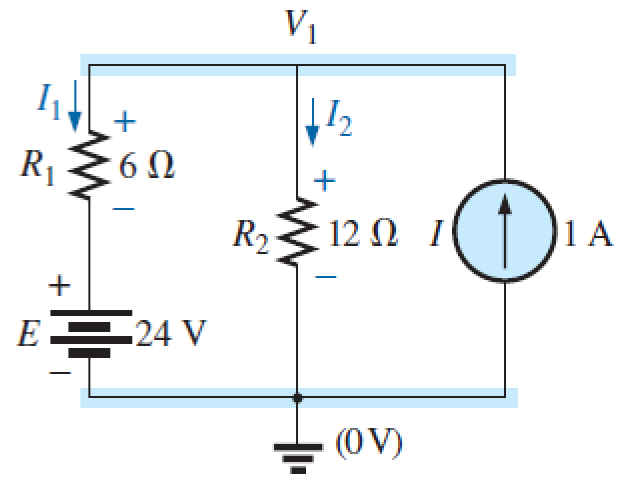


FIG. 1-3

Substituting into the Kirchhoff's current law equation.

$$I = \frac{V_1 - E}{R_1} + \frac{V_1}{R_2}$$

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$$

$$V_1\left(\frac{1}{6\,\Omega} + \frac{1}{12\,\Omega}\right) = \frac{24\text{ V}}{6\,\Omega} + 1\text{ A} = 4\text{ A} + 1\text{ A}$$

$$V_1\left(\frac{1}{4\,\Omega}\right) = 5\text{ A}$$

$$V_1 = \mathbf{20\ V}$$

The currents I1 and I2 can then be determined using the preceding equations:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\text{ V} - 24\text{ V}}{6\,\Omega} = \frac{-4\text{ V}}{6\,\Omega} = \mathbf{-0.67\ A}$$

$$I_2 = \frac{V_1}{R_2} = \frac{20\text{ V}}{12\,\Omega} = \mathbf{1.67\ A}$$

Example (2)

Apply nodal analysis to the network in Fig. 2-1

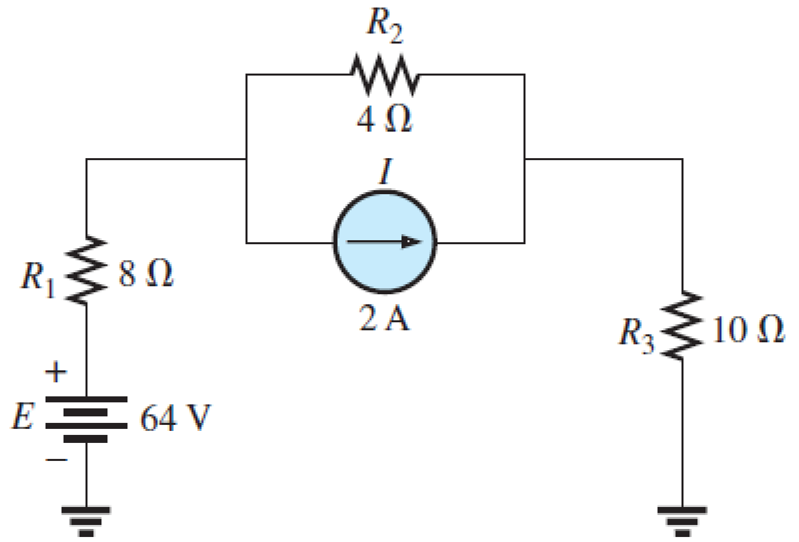


FIG. 2-1

Solution:

Step 1 and 2:

The network has three nodes, as defined in Fig. 2-2, with the bottom node again defined as the reference node (at ground potential, or zero volts), and the other nodes as V1 and V2.

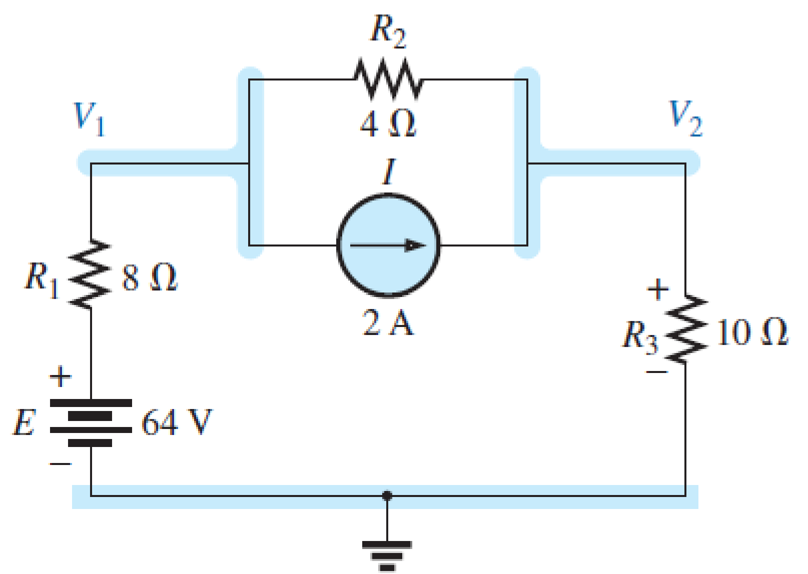


FIG. 2-2

Step 3:

For node V1, the currents are defined as shown in Fig. 2-3 and Kirchhoff's current law is applied::

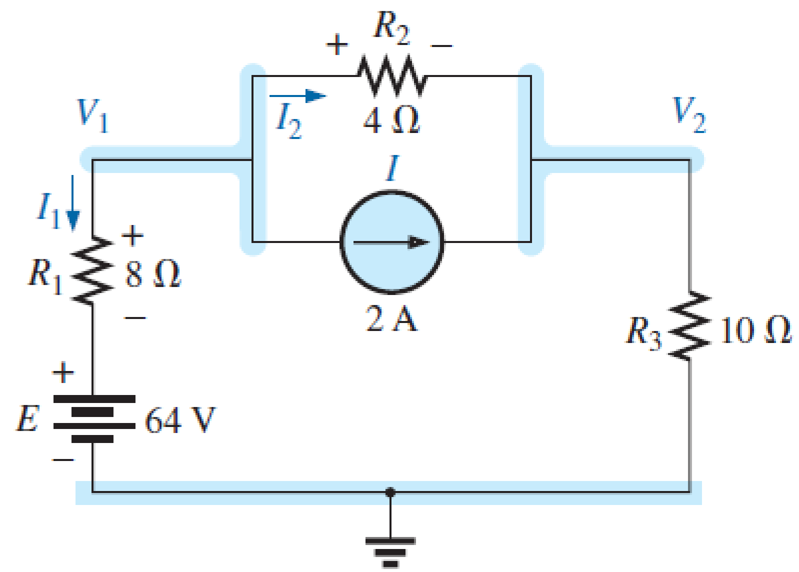


FIG. 2-3

$$0 = I_1 + I_2 + I$$

with $$I_1 = \frac{V_1 - E}{R_1}$$

and $$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

so that $$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

or $$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

and $$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

Substituting values:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = -2\ \text{A} + \frac{64\ \text{V}}{8\ \Omega} = 6\ \text{A}$$

For node V2 the currents are defined as shown in Fig. 2-4, and Kirchhoff's current law is applied:

$$I = I_2 + I_3$$

with
$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

or
$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

and
$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

Substituting values:

$$V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) - V_1\left(\frac{1}{4\ \Omega}\right) = 2\ \text{A}$$

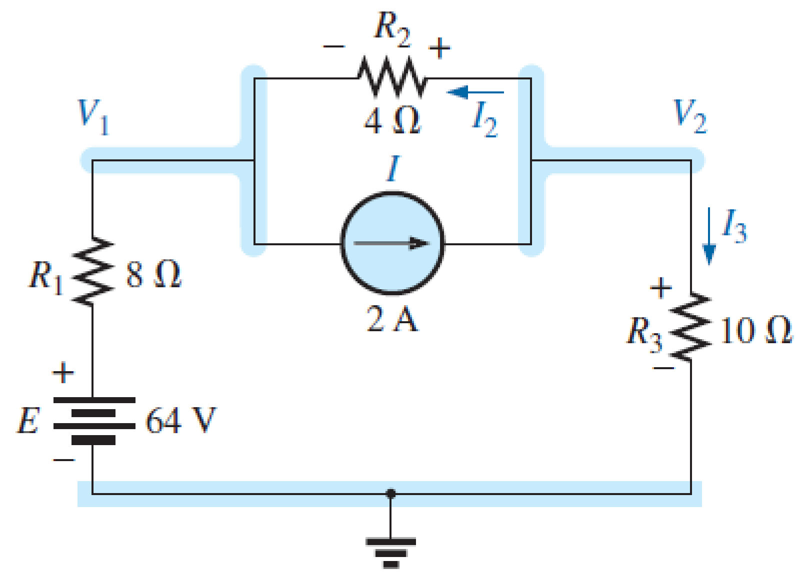


FIG. 2-4

Step 4:

The result is two equations and two unknowns:

$$V_1\left(\frac{1}{8\ \Omega} + \frac{1}{4\ \Omega}\right) - V_2\left(\frac{1}{4\ \Omega}\right) = 6\ \text{A}$$

$$-V_1\left(\frac{1}{4\ \Omega}\right) + V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) = 2\ \text{A}$$

which become:

$$0.375V_1 - 0.25V_2 = 6$$
$$-0.25V_1 + 0.35V_2 = 2$$

Using determinants:

$$V_1 = \mathbf{37.82\ V}$$
$$V_2 = \mathbf{32.73\ V}$$

Since E is greater than V1, the current I1 flows from ground to V1 and is equal to:

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64\ \text{V} - 37.82\ \text{V}}{8\ \Omega} = \mathbf{3.27\ A}$$

The positive value for V2 results in a current IR3 from node V2 to ground equal to:

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.73\ \text{V}}{10\ \Omega} = \mathbf{3.27\ A}$$

Since V1 is greater than V2, the current IR2 flows from V1 to V2 and is equal to:

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.82\ \text{V} - 32.73\ \text{V}}{4\ \Omega} = \mathbf{1.27\ A}$$

Example (3)

Determine the nodal voltages for the network in Fig. 3-1

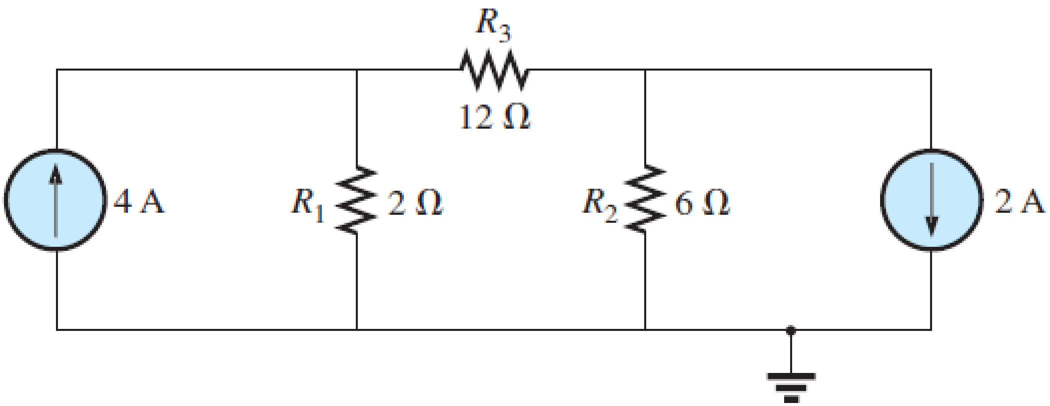


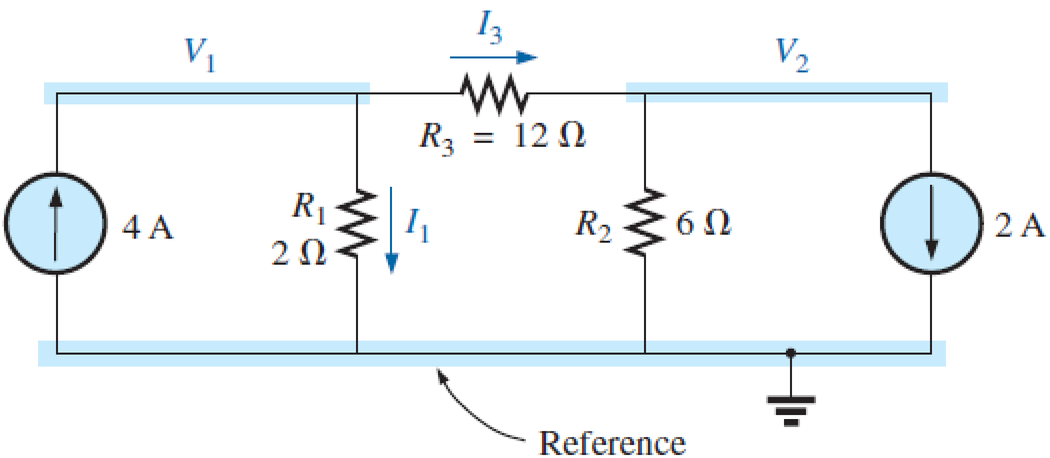
FIG. 3-1

Solution:

Step 1 and 2:

As indicated in Fig. 3-2

FIG. 3-2



Step 3:

Included in Fig. 3-2 for the node V1. Applying Kirchhoff's current law:

$$4\text{ A} = I_1 + I_3$$

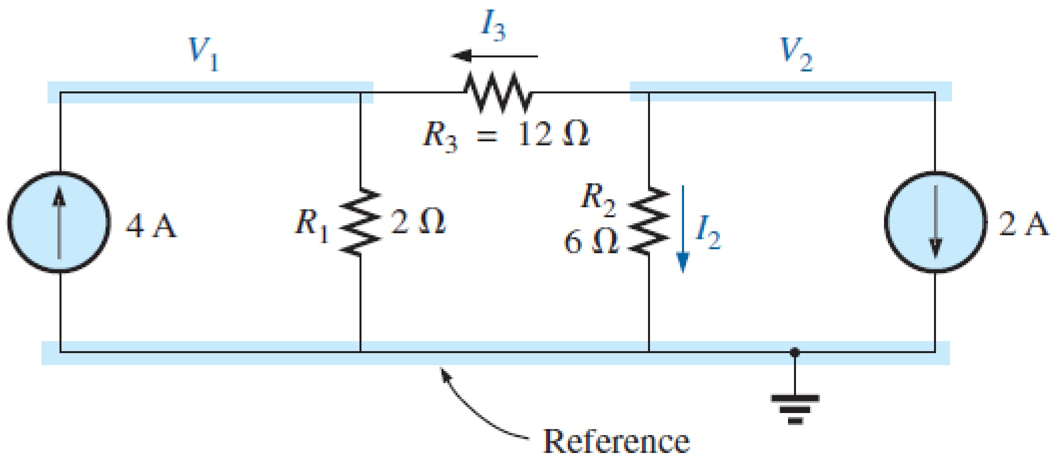
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\ \Omega} + \frac{V_1 - V_2}{12\ \Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\ \Omega}+\frac{1}{12\ \Omega}\right)-V_2\left(\frac{1}{12\ \Omega}\right)=4\ \text{A}$$

For node V2, the currents are defined as in Fig. 3-3.

FIG. 3-3



Applying Kirchhoff's current law:

$$0 = I_3 + I_2 + 2\ \text{A}$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\ \text{A} = 0 \longrightarrow \frac{V_2 - V_1}{12\ \Omega} + \frac{V_2}{6\ \Omega} + 2\ \text{A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\ \Omega}+\frac{1}{6\ \Omega}\right)-V_1\left(\frac{1}{12\ \Omega}\right)=-2\ \text{A}$$

Step 4:
The result is two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega}+\frac{1}{12\,\Omega}\right)-V_2\left(\frac{1}{12\,\Omega}\right)&=+4\text{ A}\\ V_2\left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}\right)-V_1\left(\frac{1}{12\,\Omega}\right)&=-2\text{ A}\end{aligned}\right\} \Rightarrow \left.\begin{aligned}\frac{7}{12}V_1-\frac{1}{12}V_2&=+4\\ -\frac{1}{12}V_1+\frac{3}{12}V_2&=-2\end{aligned}\right\} \quad \begin{aligned}7V_1-\quad V_2&=48\\ -1V_1+3V_2&=-24\end{aligned}$$

$$V_1=\frac{\begin{vmatrix}48 & -1\\ -24 & 3\end{vmatrix}}{\begin{vmatrix}7 & -1\\ -1 & 3\end{vmatrix}}=\frac{120}{20}=+\mathbf{6\ V}$$

$$V_2=\frac{\begin{vmatrix}7 & 48\\ -1 & -24\end{vmatrix}}{20}=\frac{-120}{20}=\mathbf{-6\ V}$$

Since V1 is greater than V2, the current through R3 passes from V1 to V2.
Its value is

$$I_{R_3}=\frac{V_1-V_2}{R_3}=\frac{6\text{ V}-(-6\text{ V})}{12\,\Omega}=\frac{12\text{ V}}{12\,\Omega}=\mathbf{1\ A}$$

The fact that V1 is positive results in a current IR1 from V1 to ground equal to

$$I_{R_1}=\frac{V_{R_1}}{R_1}=\frac{V_1}{R_1}=\frac{6\text{ V}}{2\,\Omega}=\mathbf{3\ A}$$

Finally, since V2 is negative, the current IR2 flows from ground to V2 and is equal to

$$I_{R_2}=\frac{V_{R_2}}{R_2}=\frac{V_2}{R_2}=\frac{6\text{ V}}{6\,\Omega}=\mathbf{1\ A}$$