Electrical Circuit-I

3rd Lecture

Series dc Circuits

By:

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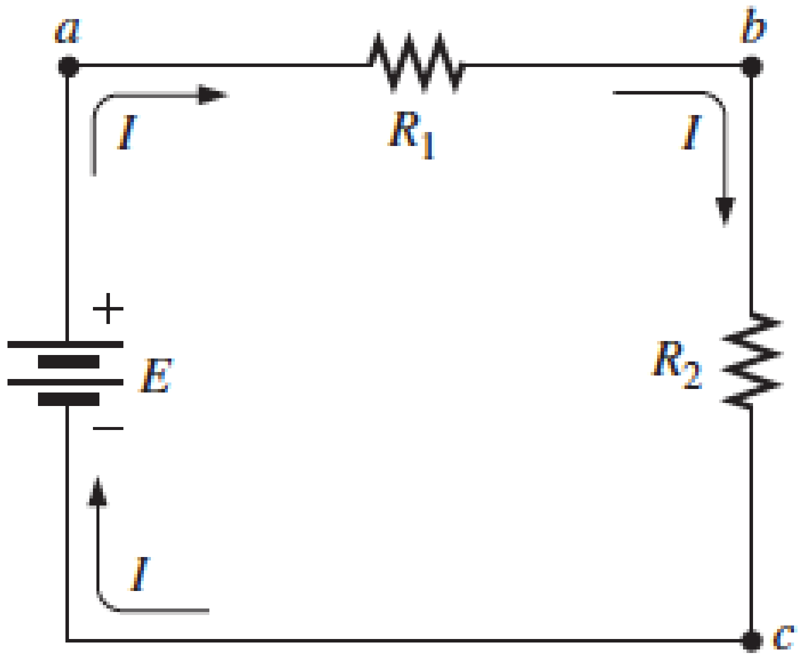
Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Series Circuits

A circuit consists of any number of elements joined at terminal point, providing at least one closed path through which charge can flow.

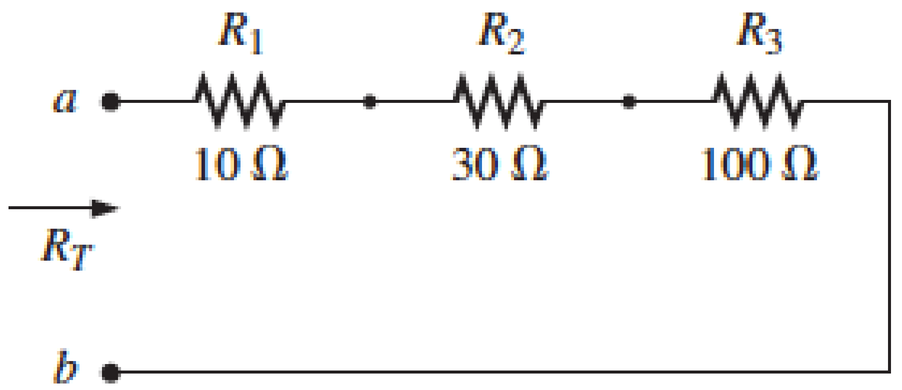
The circuit of Fig. below, has three elements joined at three terminal points (a, b, c) to provide a closed path for the current I.



The current is the same through series elements

Series Resistors

The total resistance of a series configuration is the sum of the resistance levels..



$$R_T = R_1 + R_2 + R_3 + R_4 + \cdots + R_N$$

For the special case where resistors are the same values can be modified as follows:

$$R_T = NR$$

The current is the same at every point in a series circuit, and can be determined using Ohm's law:

$$I_s = \frac{E}{R_T}$$

Series Resistors

The voltage across each resistor with the total resistance, using ohm's law:

$$V_1 = IR_1 , V_2 = IR_2 , V_3 = IR_3 , \ldots V_N = IR_N$$

The power delivered to each resistor can then be determined using any one of three equations as listed below (shown for resistor R_1 only):

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \qquad \text{(watts, W)}$$

The power delivered by the supply can be determined using

$$P_E = EI_s \qquad \text{(watts, W)}$$

The power applied by the dc supply must equal that dissipated by the resistive elements.

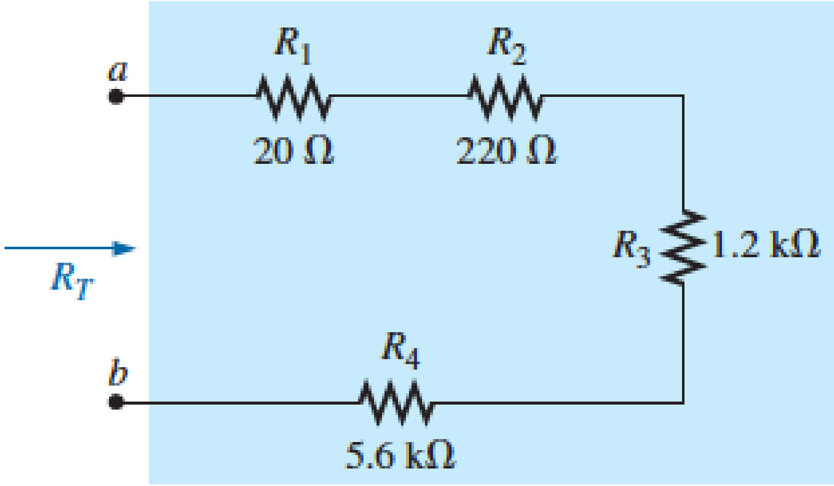
$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

Example (1)

Determine the total resistance for the series resistors?

Solution:

$$R_T = R_1 + R_2 + R_3 + R_4$$
$$R_T = 20\ \Omega + 220\ \Omega + 1.2\ \text{k}\Omega + 5.6\ \text{k}\Omega$$
$$R_T = 7040\ \Omega = \mathbf{7.04\ k\Omega}$$

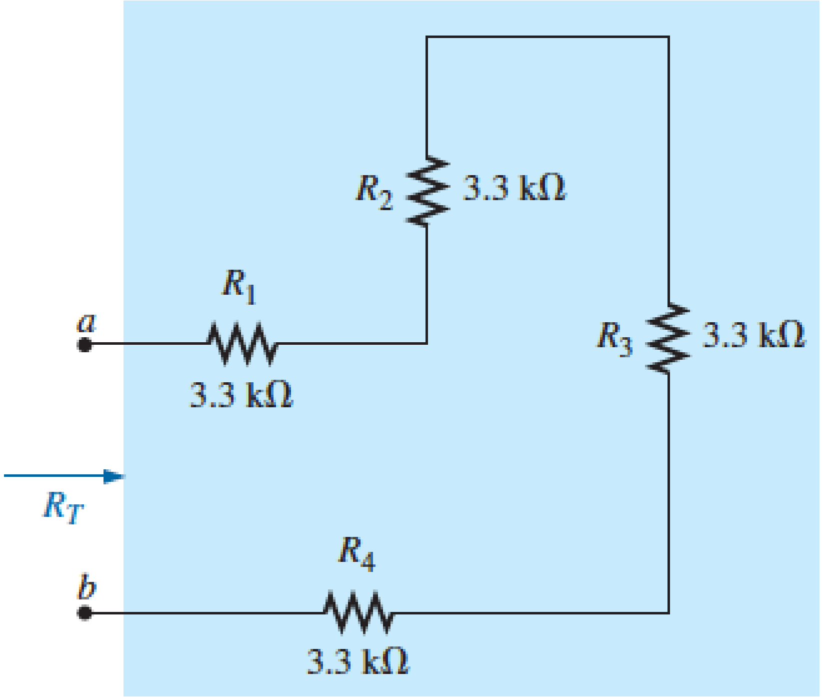


Example (2)

Find the total resistance of the series resistors

Solution:

$$R_T = NR$$
$$= (4)(3.3\ \text{k}\Omega) = \mathbf{13.2\ k\Omega}$$



Example (3)

For the series circuit:

a. Find the total resistance R_T.

b. Calculate the resulting source current **I**.

c. Determine the voltage across each resistor.

d. Calculate the power dissipated by R_1, R_2 , R_3

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d)

Solution:

a. $R_T = R_1 + R_2 + R_3$

$= 2\ \Omega + 1\ \Omega + 5\ \Omega$

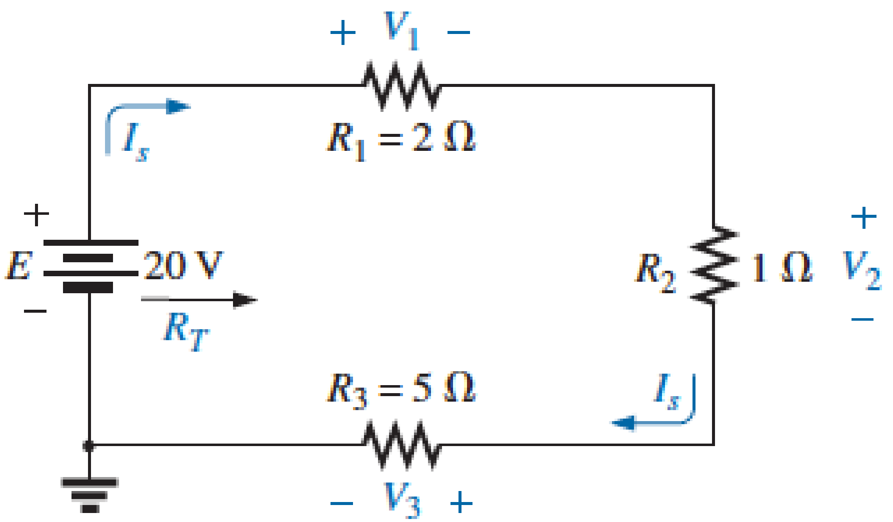
$R_T = 8\ \Omega$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = 2.5\ \text{A}$

c. $V_1 = I_1R_1 = I_sR_1 = (2.5\ \text{A})(2\ \Omega) = 5\ \text{V}$

$V_2 = I_2R_2 = I_sR_2 = (2.5\ \text{A})(1\ \Omega) = 2.5\ \text{V}$

$V_3 = I_3R_3 = I_sR_3 = (2.5\ \text{A})(5\ \Omega) = 12.5\ \text{V}$



***Example* (3)**

For the series circuit:

d. Calculate the power dissipated by R_1, R_2 , R_3

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d)

Solution:

d. $P_1 = V_1 I \quad = (5V)(2.5\,A) \quad = 12.5\,W$

$P_2 = I^2 R_2 \quad = (2.5\,A)^2 (1\Omega) \quad = 6.25\,W$

$P_3 = {V_3}^2 / R_3 = (12.5\,V)^2/(5\Omega) \quad = 31.25\,W$

e. $P_E \quad = EI \quad = (20V)(2.5A) \quad = 50\,W$

$P_{del} = P_1 + P_2 + P_3$

$50\,W = 12.5\,W + 6.25\,W + 31.5\,W$

$50\,W = 50\,W$ (checks)

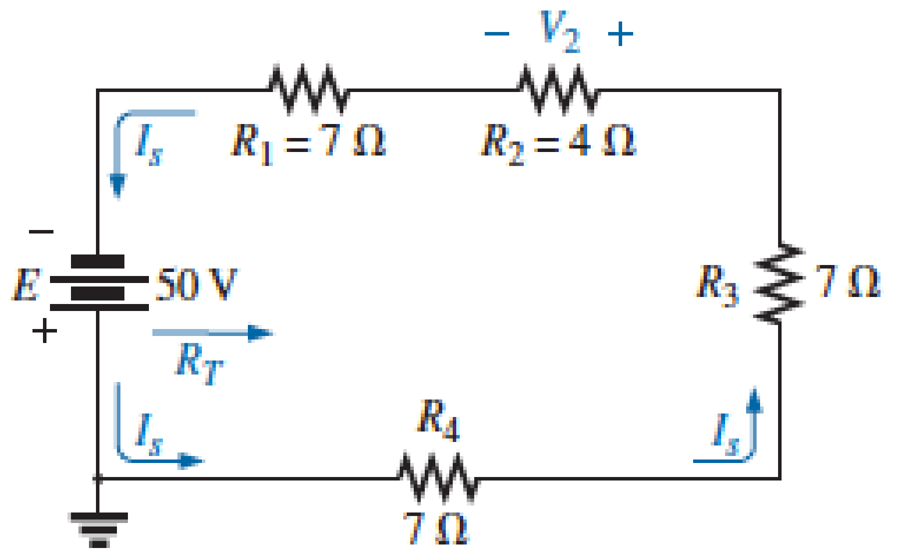
***Example* (4)**

For the series circuit:

a. Find the **total** resistance R_T.

b. Determine the **source** current I_S.

c. Find the **voltage** across resistor R_2.



Solution:

a.

$$R_T = R_2 + NR$$
$$= 4\ \Omega + (3)(7\ \Omega)$$
$$= 4\ \Omega + 21\ \Omega$$
$$R_T = \mathbf{25\ \Omega}$$

b.

$$I_s = \frac{E}{R_T} = \frac{50\text{ V}}{25\ \Omega} = \mathbf{2\ A}$$

c.

$$V_2 = I_2R_2 = I_sR_2 = (2\text{ A})(4\ \Omega)$$

$$V_2 = \mathbf{8\ V}$$

Kirchhoff's Voltage Law (KVL)

KVL *states that the algebraic sum of the potential rises and drops around a closed path (or closed loop) is* ***zero****.*

A closed loop*: is any continuous path that leaves a point in one direction and return to that same point from another direction without leaving the circuit.*

As shown in the circuit below, we can trace a continuous path that leaves point ***a*** *through* R_1 *and returns through* ***E*** *without leaving the circuit. Therefore,* ***abcda*** *is a closed loop. For us to be able to apply* ***KVL****, the summation of potential rises and drops must be made in one direction around the closed loop.*

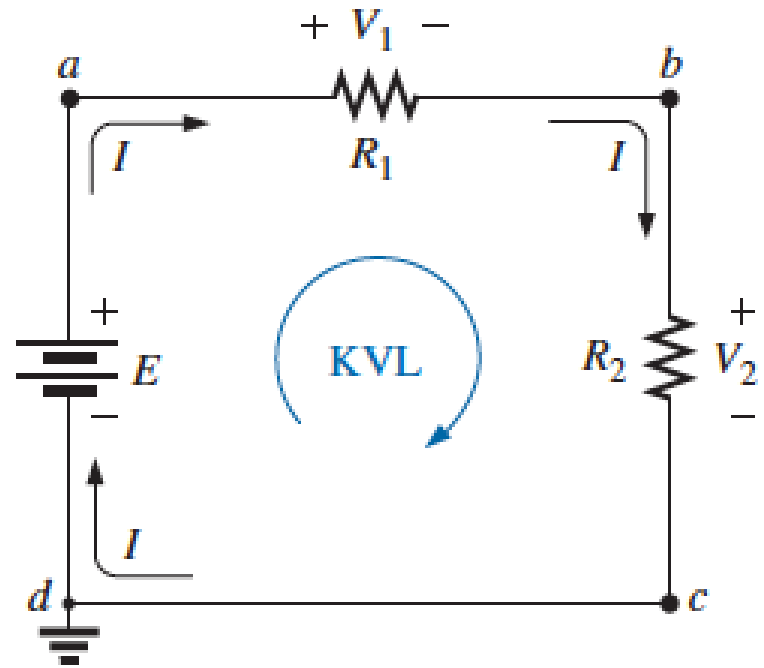
In symbolic form it can be written as

$$\Sigma_C V = 0$$

(Kirchhoff's voltage law in symbolic form)

Writing out the sequence with the voltages and the signs results in the following:

$$+E - V_1 - V_2 = 0$$
$$E = V_1 + V_2$$



Kirchhoff's Voltage Law (KVL)

The applied voltage of a series dc circuit will equal the sum of the voltage drops of the circuit.

Kirchhoff's voltage law can also be written in the following form:

$$\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}$$

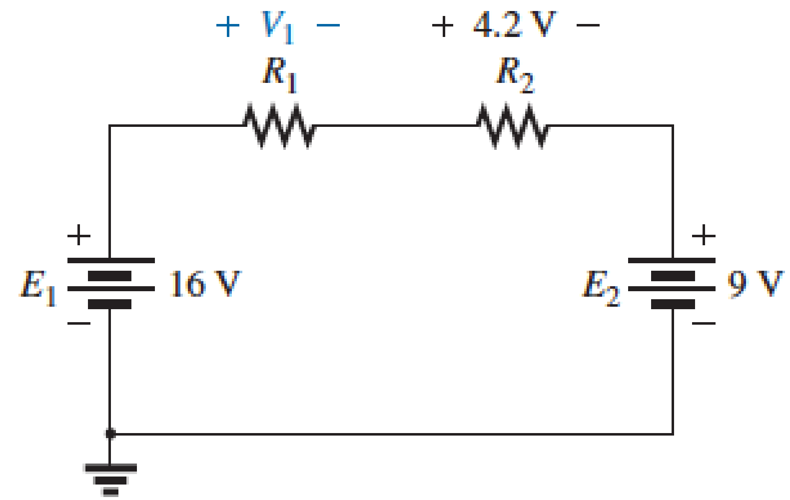
revealing that
the sum of the voltage rises around a closed path will always equal the sum of the voltage drops.

To demonstrate that the direction that you take around the loop has no effect on the results, let's take the counter-clockwise path and compare results.
The resulting sequence appears as

$$-E + V_2 + V_1 = 0$$
$$E = V_1 + V_2$$

Example (5)

Use Kirchhoff's voltage law to determine the unknown voltage for the circuit

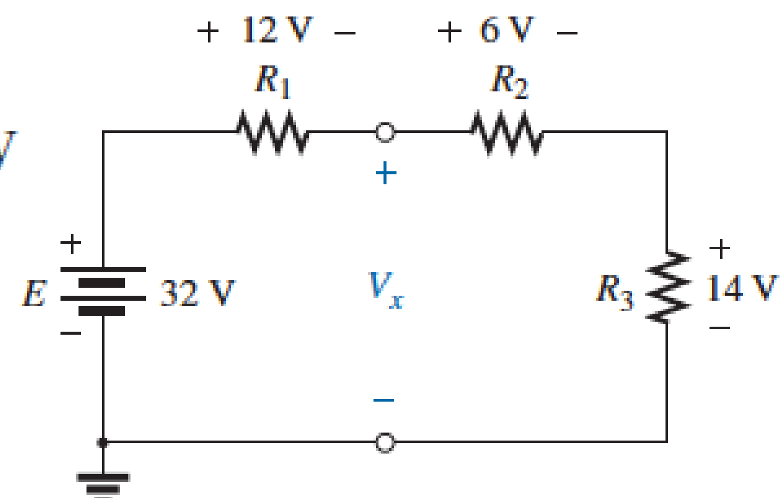


Solution:

$$+E_1 - V_1 - V_2 - E_2 = 0$$
$$V_1 = E_1 - V_2 - E_2$$
$$= 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$V_1 = \mathbf{2.8\text{ V}}$$

Example (6)

Use Kirchhoff's voltage law to determine the unknown voltage for the circuit



Solution:

$$+E - V_1 - V_x = 0$$
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V} = \mathbf{20\text{ V}}$$

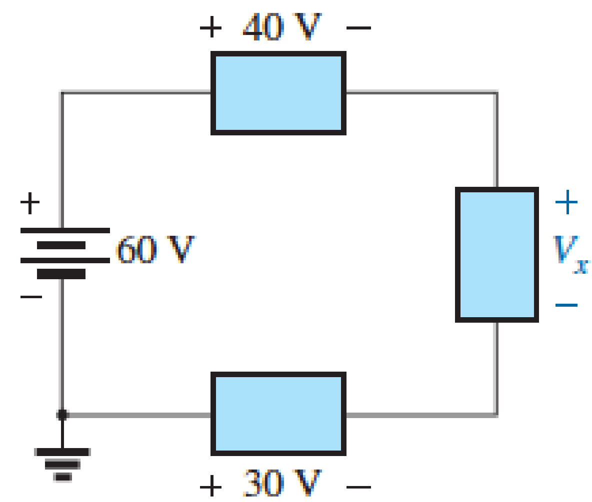
For the clockwise path, including resistor R_3, the following results:

$$+V_x - V_2 - V_3 = 0$$
$$V_x = V_2 + V_3$$
$$= 6\text{ V} + 14\text{ V}$$
$$V_x = \mathbf{20\text{ V}}$$

Example (7)

Use Kirchhoff's voltage law to determine the unknown voltage for the circuit

Solution:



$$+60\text{ V} - 40\text{ V} - V_x + 30\text{ V} = 0$$

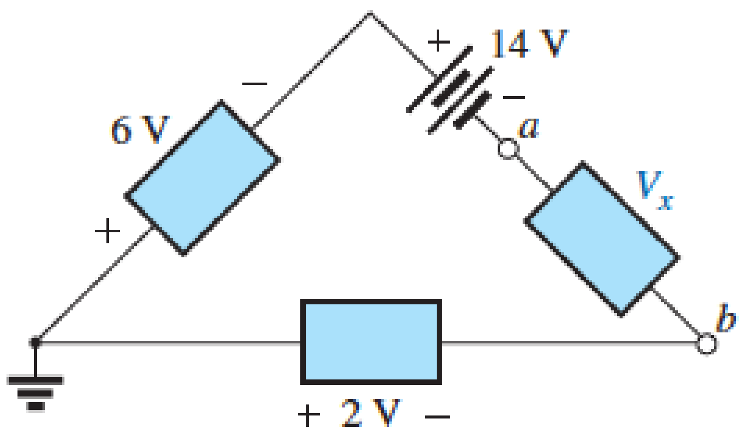
$$V_x = 60\text{ V} + 30\text{ V} - 40\text{ V} = 90\text{ V} - 40\text{ V}$$

$$V_x = \mathbf{50\text{ V}}$$

Example (8)

Use Kirchhoff's voltage law to find thevoltage V_x for the circuit

Solution:



$$-6\text{ V} - 14\text{ V} - V_x + 2\text{ V} = 0$$

$$V_x = -20\text{ V} + 2\text{ V}$$

$$V_x = \mathbf{-18\text{ V}}$$

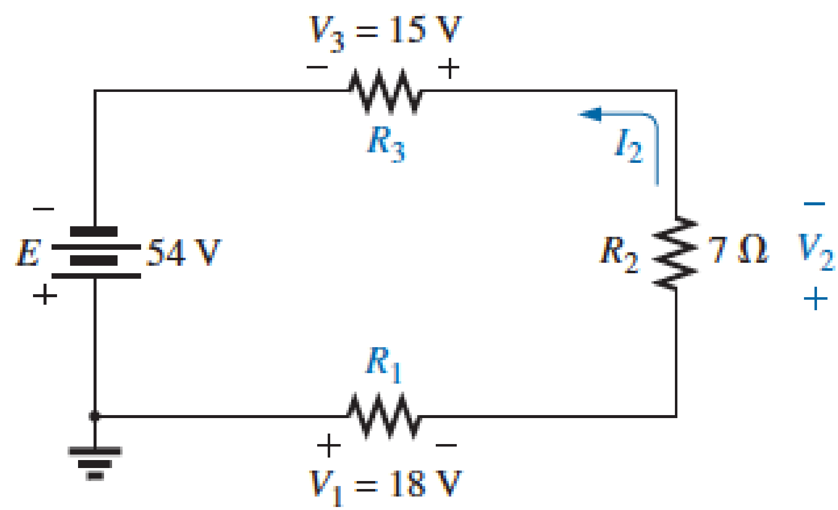
Example **(9)**

For the series circuit:

a. Determine V_2 using Kirchhoff's voltage law.

b. Determine current I_2.

c. Find R_1 and R_3.



Solution:

a. Applying Kirchhoff's voltage law in the clockwise direction

$$-E + V_3 + V_2 + V_1 = 0$$

and $$E = V_1 + V_2 + V_3 \text{ (as expected)}$$

so that $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V}$$

and $$V_2 = \mathbf{21\ V}$$

b. $$I_2 = \frac{V_2}{R_2} = \frac{21\text{ V}}{7\ \Omega}$$

$$I_2 = \mathbf{3A}$$

c. $$R_1 = \frac{V_1}{I_1} = \frac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$$

with $$R_3 = \frac{V_3}{I_3} = \frac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$$

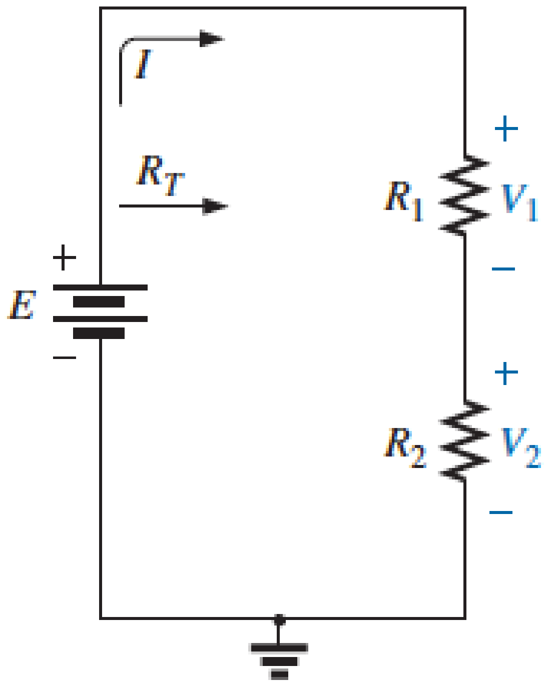
Voltage Divider Rule (VDR)

The voltage divider rule (VDR) permits the determination of the voltage across a series resistor without first having to determine the current of the circuit.

The rule itself can be derived by analysing the simple series circuit shown:

$$R_T = R_1 + R_2$$

$$I_s = I_1 = I_2 = \frac{E}{R_T}$$



Apply Ohm's law to each resistor:

$$V_1 = I_1 R_1 = \left(\frac{E}{R_T}\right) R_1 = R_1 \frac{E}{R_T}$$

$$V_2 = I_2 R_2 = \left(\frac{E}{R_T}\right) R_2 = R_2 \frac{E}{R_T}$$

The resulting format for V1 and V2 is

$$V_x = R_x \frac{E}{R_T}$$

Voltage Divider Rule (VDR)

$$V_x = R_x \frac{E}{R_T}$$

where V_x is the voltage across the resistor R_x, E is the impressed voltage across the series elements, and R_T is the total resistance of the series circuit.

The voltage divider rule states that:

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.

Example (10)

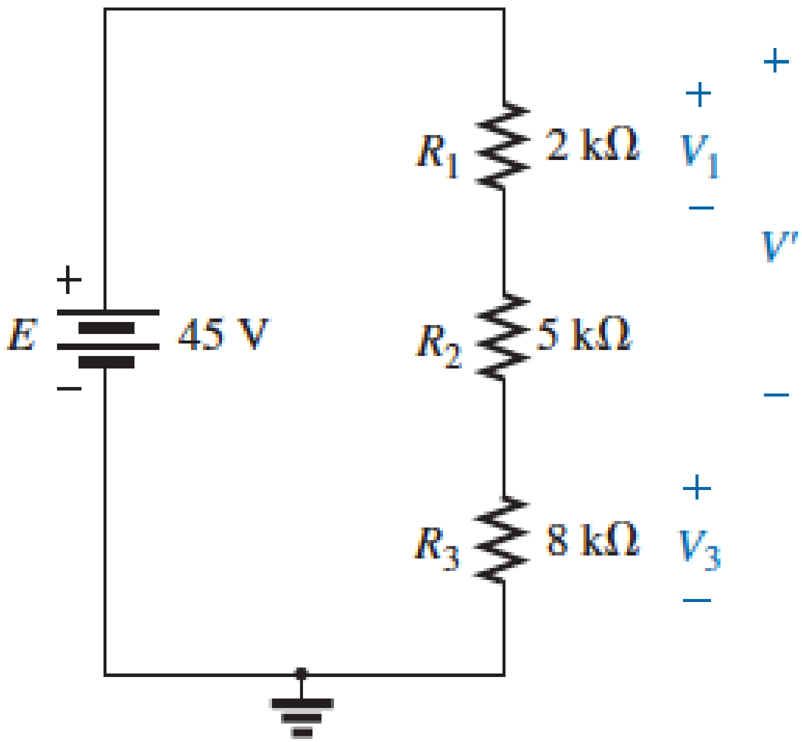
Using the voltage divider rule, determine voltages V_1 and V_3 for the series circuit?

Solution:

$$R_T = R_1 + R_2 + R_3$$
$$= 2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega$$
$$R_T = 15\,\text{k}\Omega$$

$$V_1 = R_1\frac{E}{R_T} = 2\ \text{k}\Omega\left(\frac{45\ \text{V}}{15\,\text{k}\Omega}\right) = \mathbf{6\ V}$$

$$V_3 = R_3\frac{E}{R_T} = 8\ \text{k}\Omega\left(\frac{45\ \text{V}}{15\ \Omega}\right) = \mathbf{24\ V}$$



Example (11)

For the series circuit:

a. Without making any calculations, how much larger would you expect the voltage across R2 to be compared to that across R_1?

b. Find the voltage V_1 using only the voltage divider rule.

c. Using the conclusion of part (a), determine the voltage across R_2.

d. Use the voltage divider rule to determine the voltage across R_2, and compare your answer to your conclusion in part (c).

e. How does the sum of V_1 and V_2 compare to the applied voltage?

Solution:

a. Since resistor R_2 is three times R_1, it is expected that $V_2 = 3V_1$.

b. $V_1 = R_1\dfrac{E}{R_T} = 20\,\Omega\left(\dfrac{64\text{ V}}{20\,\Omega + 60\,\Omega}\right) = 20\,\Omega\left(\dfrac{64\text{ V}}{80\,\Omega}\right) = \mathbf{16\text{ V}}$

c. $V_2 = 3V_1 = 3(16\text{ V}) = \mathbf{48\text{ V}}$

d. $V_2 = R_2\dfrac{E}{R_T} = (60\,\Omega)\left(\dfrac{64\text{ V}}{80\,\Omega}\right) = \mathbf{48\text{ V}}$

The results are an exact match.

e. $E = V_1 + V_2$
$64\text{ V} = 16\text{ V} + 48\text{ V} = \mathbf{64\text{ V}}$ (checks)