# Electrical Circuit-I

## 4th Lecture

## Parallel dc Circuits

By:

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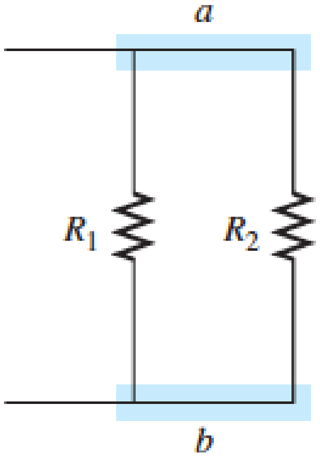
**Ref:** Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

# Parallel Circuits

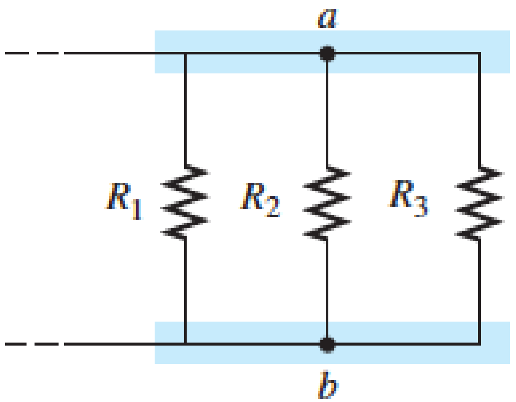
*The term parallel is used so often to describe a physical arrangement between two elements that most individuals are aware of its general characteristics. In general,*

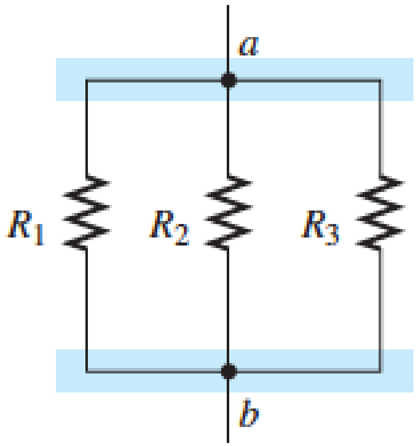
*two elements, branches, or circuits are in parallel if they have two points in common.*

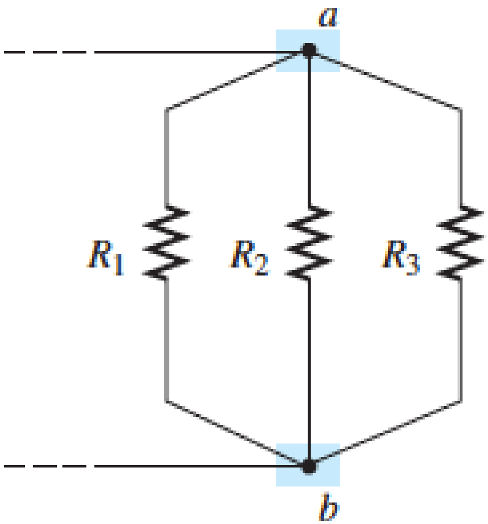



*For instance, in figure below, the two resistors are in parallel because they are connected at points a and b.*

*the parallel combination can appear in a number of ways, as shown in below. In each case, the three resistors are in parallel. They all have points a and b in common.*

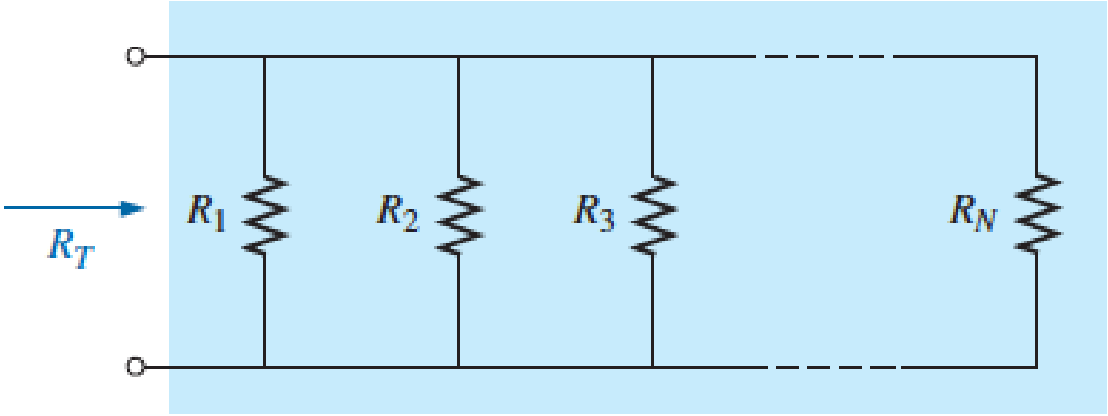


# Total Conductance and Resistance

*For resistors in parallel as shown, the total resistance is determined from the following equation:*




$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

$$R_T = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{R_2} + \cfrac{1}{R_3} + \cdots + \cfrac{1}{R_N}}$$

*Since G = 1/R, the equation can also be written in terms of conductance levels as follows:*

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

*(Siemens, S)*

## *Example* (1)

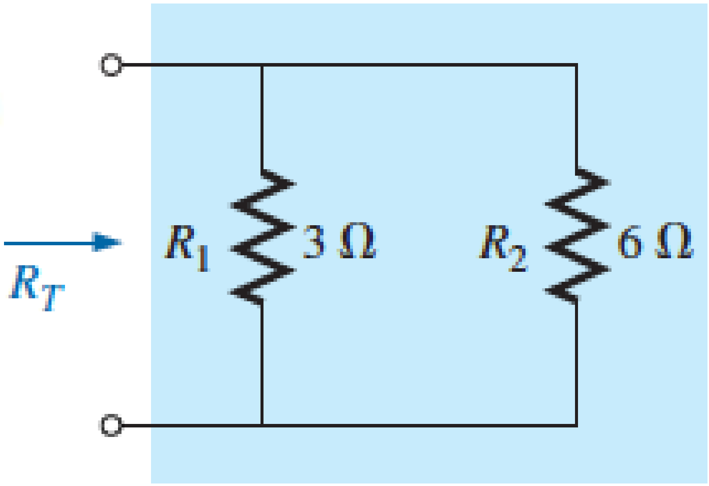
**Determine the total conductance and resistance for the parallel network?**

***Solution:***

$$G_1 = \frac{1}{R_1} = \frac{1}{3\ \Omega} = 0.333\ \text{S}, \quad G_2 = \frac{1}{R_2} = \frac{1}{6\ \Omega} = 0.167\ \text{S}$$

$$G_T = G_1 + G_2 = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$$

$$R_T = \frac{1}{G_T} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$




## *Example* (2)

**Find the total resistance of the parallel resistors**

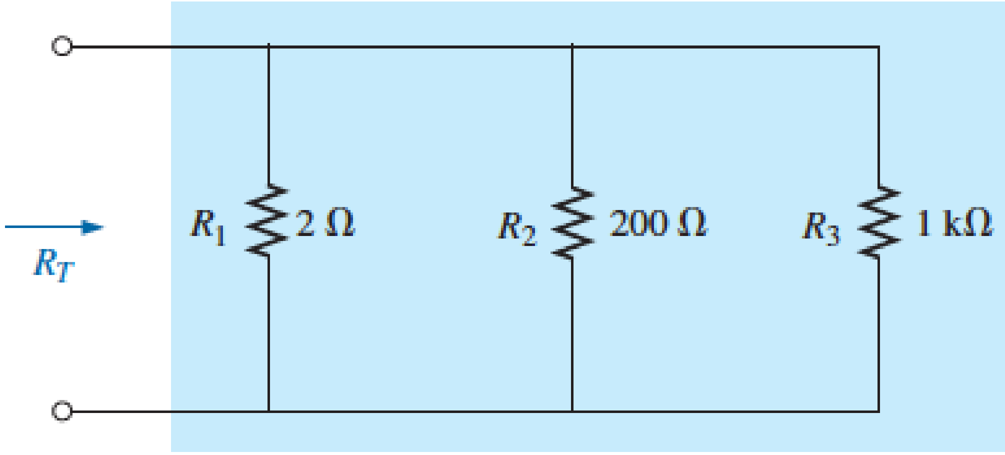
***Solution:***

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{2\ \Omega} + \dfrac{1}{200\ \Omega} + \dfrac{1}{1\ \text{k}\Omega}}$$

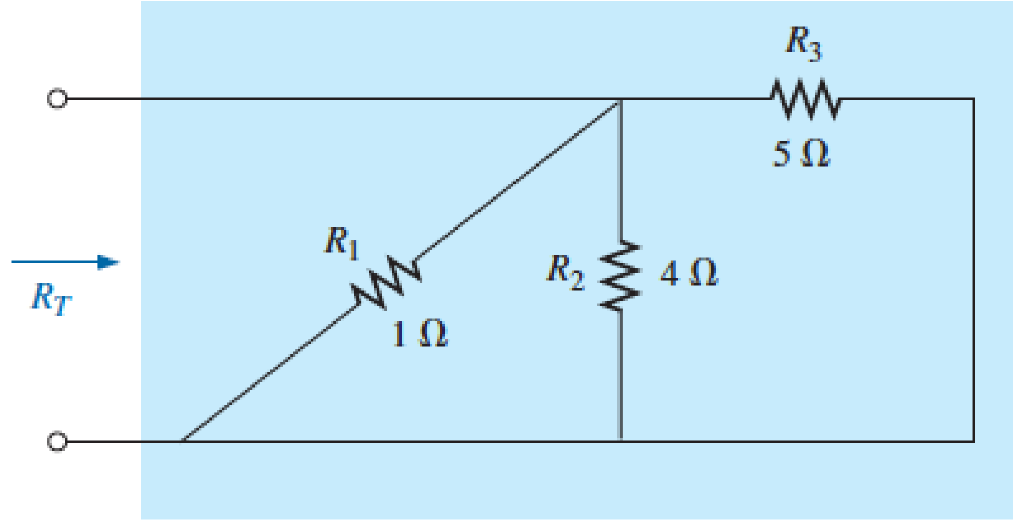
$$= \frac{1}{0.5\ \text{S} + 0.005\ \text{S} + 0.001\ \text{S}}$$

$$= \frac{1}{0.506\ \text{S}} = \mathbf{1.98\ \Omega}$$



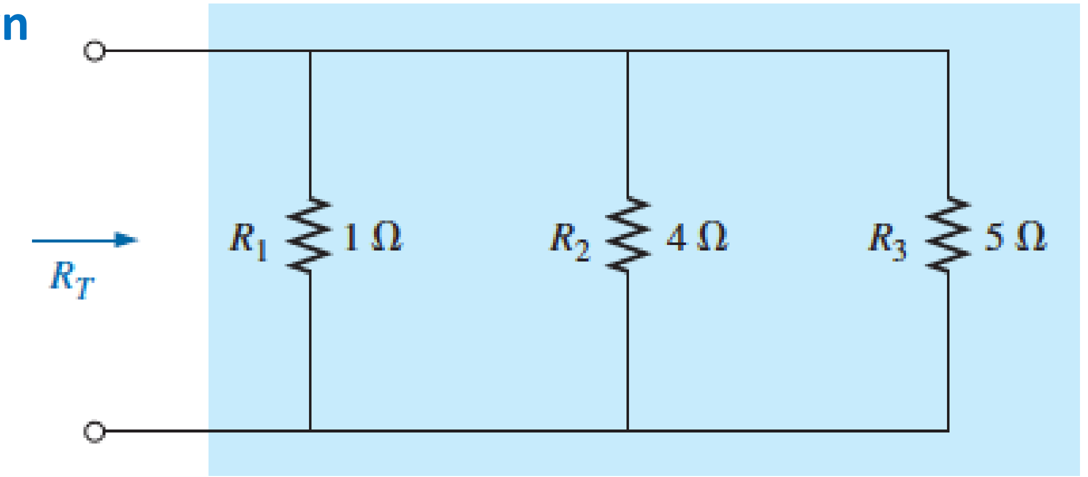
***Example* (3)**

**Find the total resistance of the configuration?**




***Solution:***

**First the network is redrawn as shown**




$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{1\ \Omega} + \dfrac{1}{4\ \Omega} + \dfrac{1}{5\ \Omega}}$$

$$= \frac{1}{1\ \text{S} + 0.25\ \text{S} + 0.2\ \text{S}}$$

$$= \frac{1}{1.45\ \text{S}} \cong \mathbf{0.69\ \Omega}$$

# Parallel Resistors

- *The total resistance of parallel resistors is always less than the value of the smallest resistor.*
- *If the smallest resistor of a parallel combination is much smaller than the other parallel resistors, the total resistance will be very close to the smallest resistor value.*
- *The total resistance of parallel resistors will always drop as new resistors are added in parallel, irrespective of their value.*

*For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For N equal resistors in parallel,*

$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

*In other words,*

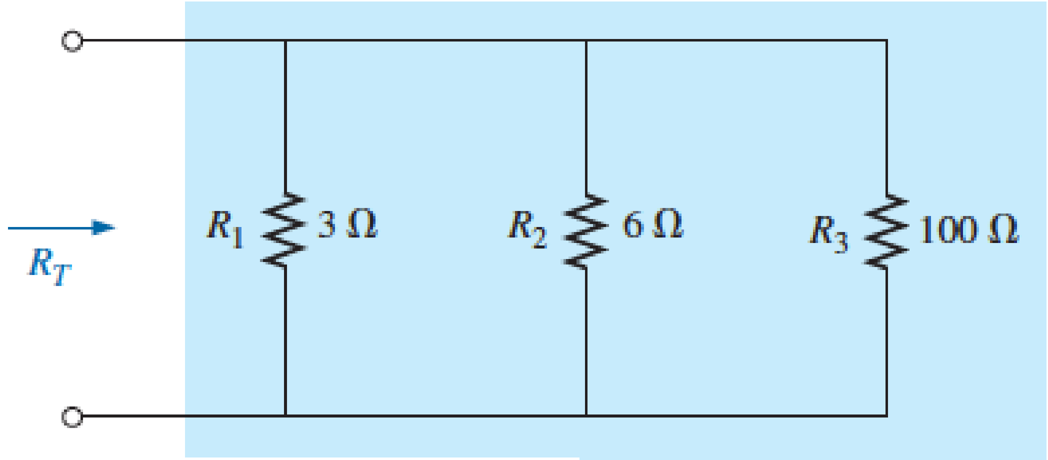
*the total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.*

$$R_T = \frac{R}{N}$$

*Example* (4)

a. What is the effect of adding another resistor of 100 Ω in parallel with the parallel resistors of Example (1) as shown below?

b. What is the effect of adding a parallel 1 Ω resistor to the configuration?




***Solution:***

a. $$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{100\,\Omega}}$$

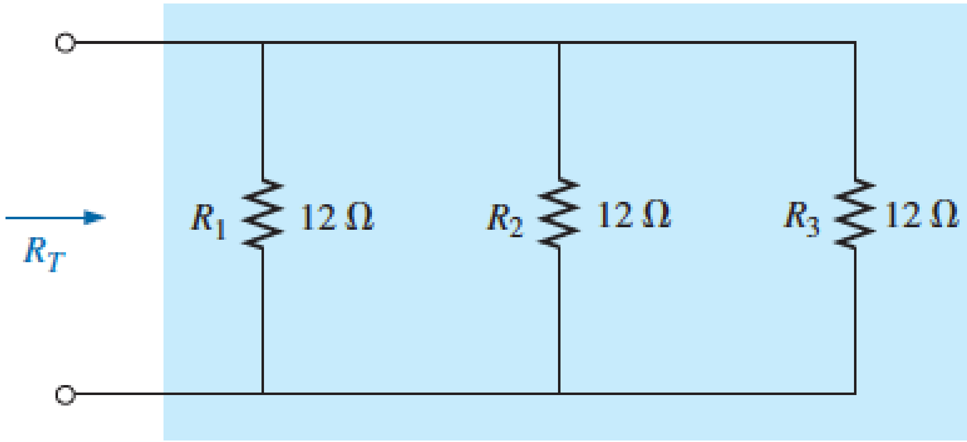
$$= \frac{1}{0.333\text{ S} + 0.167\text{ S} + 0.010\text{ S}} = \frac{1}{0.510\text{ S}} = \mathbf{1.96\ \Omega}$$

b. $$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{1}{\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}+\frac{1}{100\,\Omega}+\frac{1}{1\,\Omega}}$$

$$= \frac{1}{0.333\text{ S} + 0.167\text{ S} + 0.010\text{ S} + 1\text{ S}} = \frac{1}{0.51\text{ S}} = \mathbf{0.66\ \Omega}$$

## *Example* (5)

**Find the total resistance of the parallel resistors?**

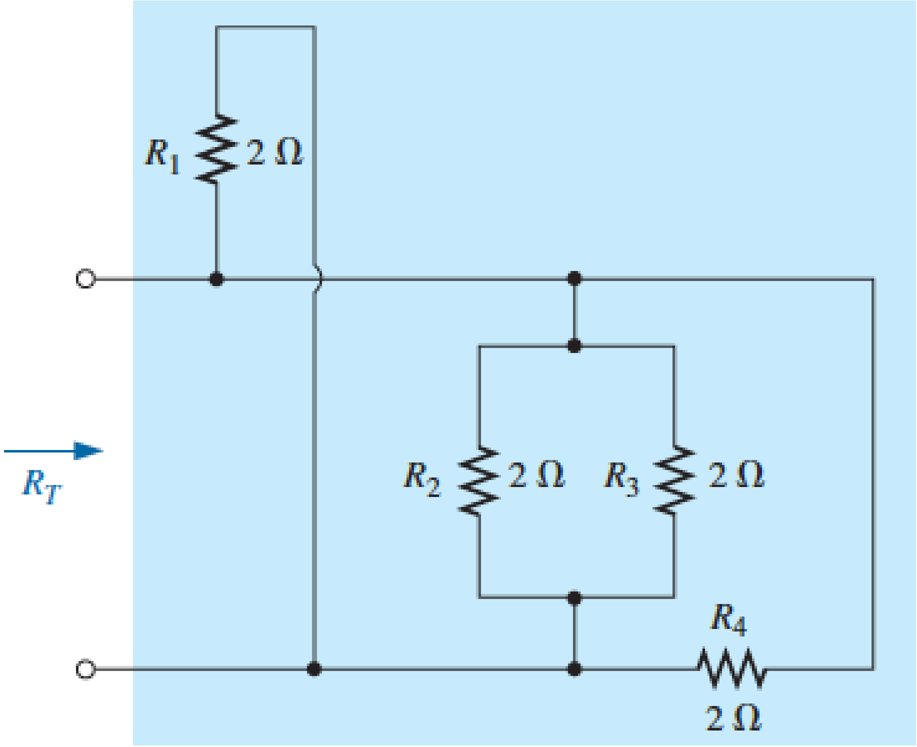



***Solution:***

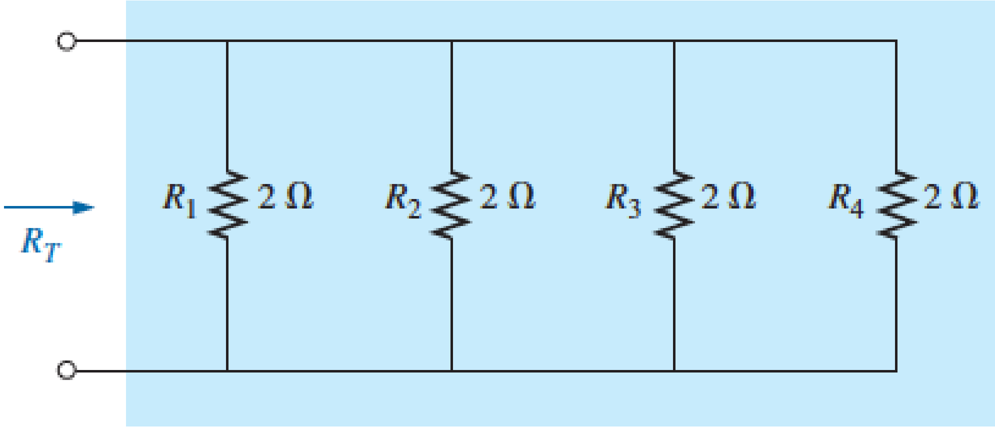
$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = 4\ \Omega$$

## *Example* (6)

**Find the total resistance of the parallel resistors?**




***Solution:***




$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = 0.5\ \Omega$$

# Two Parallel Resistors

*For two parallel resistors, the total resistance is determined by:*

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

*Multiplying the top and bottom of each term of the right side of the equation by the other resistor results in:*

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

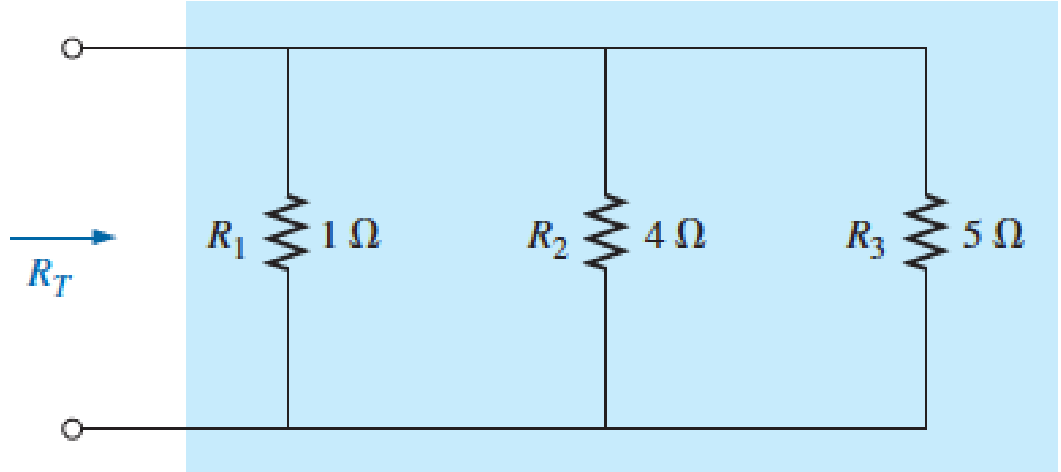
$$\boxed{R_T = \frac{R_1R_2}{R_1 + R_2}}$$

*In words, the equation states that*

*the total resistance of two parallel resistors is simply the product of their values divided by their sum.*

***Example* (7)**

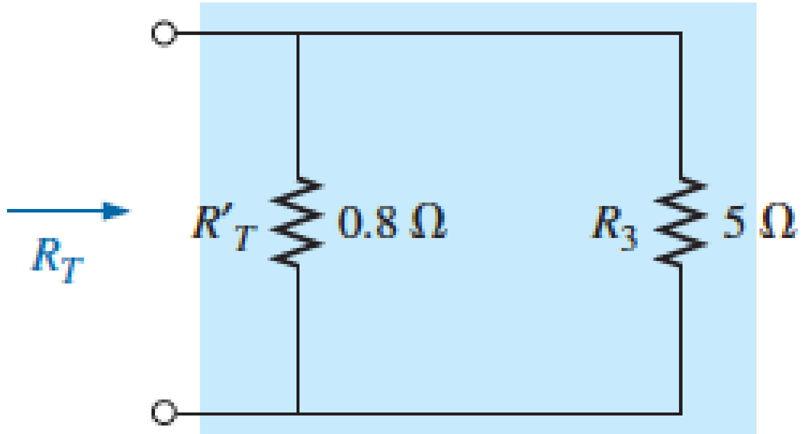
**Determine the total resistance for the parallel combination In example (3)?**




***Solution:***

**First the 1 Ω and 4 Ω resistors are combined, resulting in the reduced network to figure below:**

$$R'_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(1\ \Omega)(4\ \Omega)}{1\ \Omega + 4\ \Omega} = \frac{4}{5}\ \Omega = 0.8$$

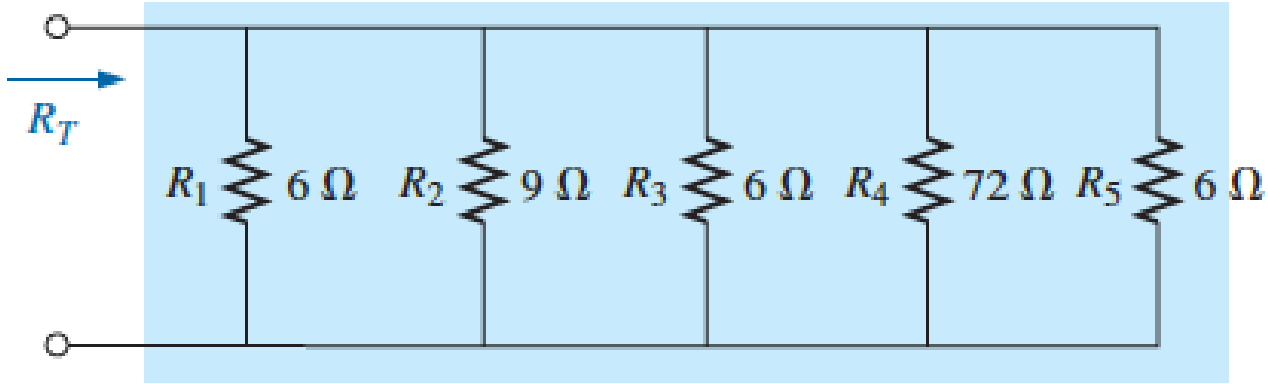



**Then the equivalent value is applied again to find the total resistance:**

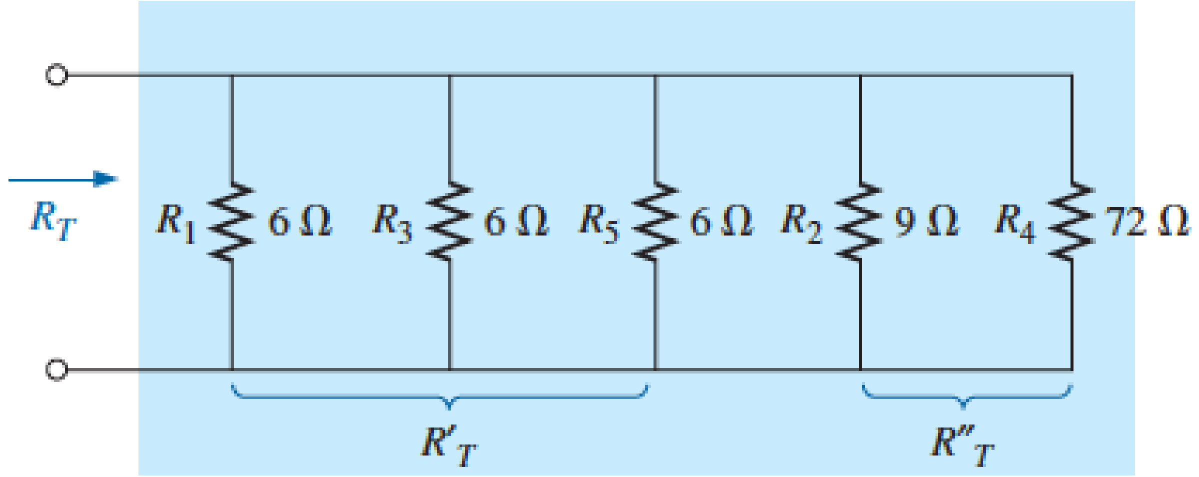
$$R_T = \frac{R'_T R_3}{R'_T + R_3} = \frac{(0.8\ \Omega)(5\ \Omega)}{0.8\ \Omega + 5\ \Omega} = \frac{4}{5.8}\ \Omega = 0.69\ \Omega$$

## *Example* (8)

**Determine the total resistance for the parallel combination?**




## *Solution:*




$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648}{81}\ \Omega = 8\ \Omega$$

$$R_T = \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16}{10}\ \Omega = \mathbf{1.6\ \Omega}$$

# Kirchhoff's Current Law (KCL)

***KCL** states that the algebraic sum of the currents entering and leaving a junction (or region) of a network is **zero**.*

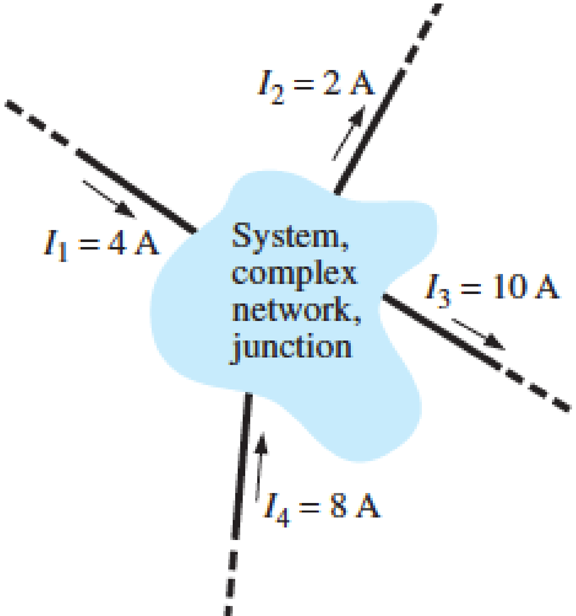
*The law can also be stated in the following way:*
*The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).*

$$\Sigma I_i = \Sigma I_o$$

(*Kirchhoff's current law in symbolic form)*

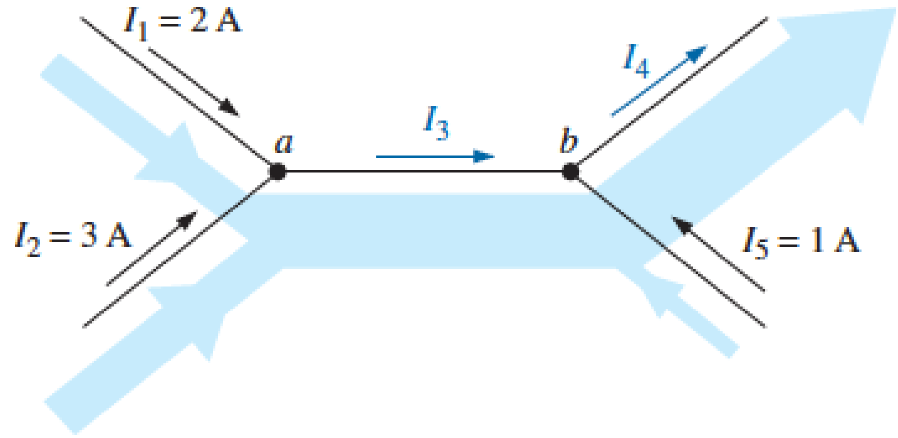
*for example, the shaded area can enclose an entire system or a complex network, or it can simply provide a connection point (junction) for the displayed currents. In each case, the current entering must equal that leaving*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$12\text{ A} = 12\text{ A} \quad \text{(checks)}$$



# *Example* (9)

**Determine currents $I_3$ and $I_4$ using Kirchhoff's current law.**




***Solution:***

**At node a:**

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = 5\text{ A}$$

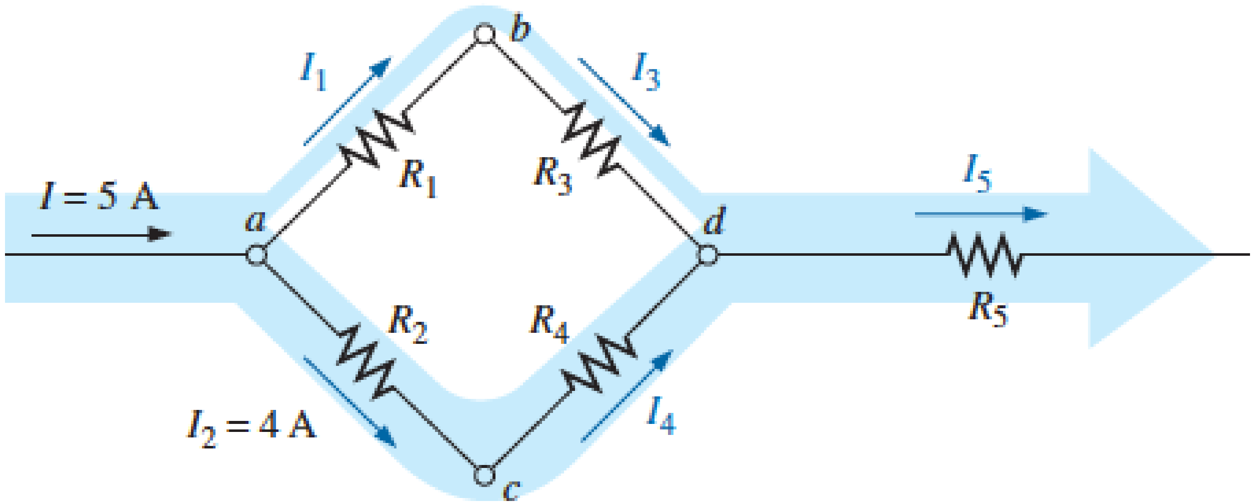
**At node b, using the result just obtained:**

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = 6\text{ A}$$

*Example* (10)

Determine currents $I_1$ , $I_3$, $I_4$ , and $I_5$ for the network using Kirchhoff's current law.

*Solution:*




At node a:

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = 1\text{ A}$$

At node b:

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = 1\text{ A}$$

At node c:

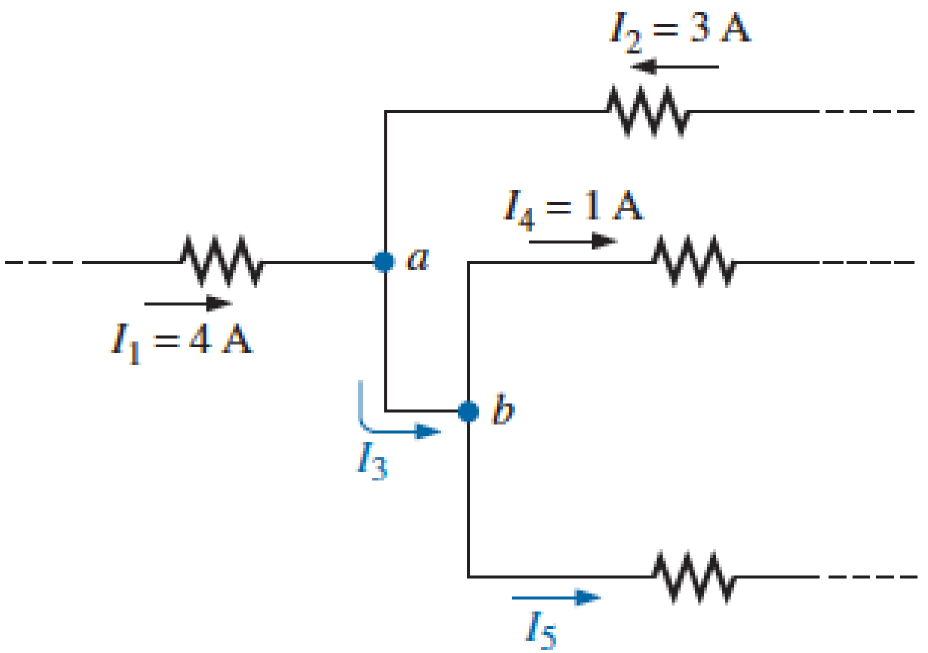
$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
$$I_4 = I_2 = 4\text{ A}$$

At node d:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = 5\text{ A}$$

*Example* (11)

Determine currents $I_3$ and $I_5$ for the network using Kirchhoff's current law.




***Solution:***

At node a:

$$\Sigma I_i = \Sigma I_o$$

$$I_1 + I_2 = I_3$$

$$4\text{ A} + 3\text{ A} = I_3 = \mathbf{7\text{ A}}$$

At node b:

$$\Sigma I_i = \Sigma I_o$$

$$I_3 = I_4 + I_5$$

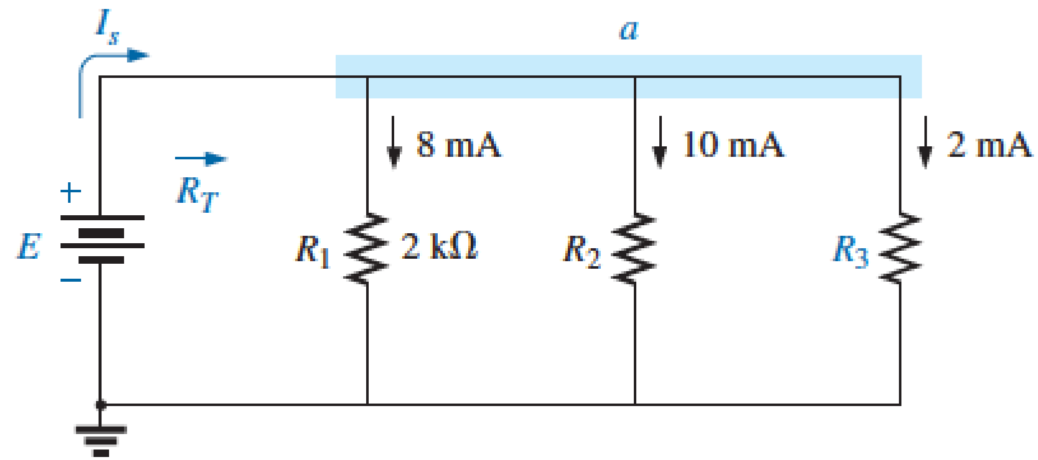
$$7\text{ A} = 1\text{ A} + I_5$$

$$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}$$

***Example*** **(12)**

**For the parallel dc network:**

a. Determine the source current $I_s$
b. Find the source voltage $E$
c. Determine $R_3$
d. Calculate $R_T$



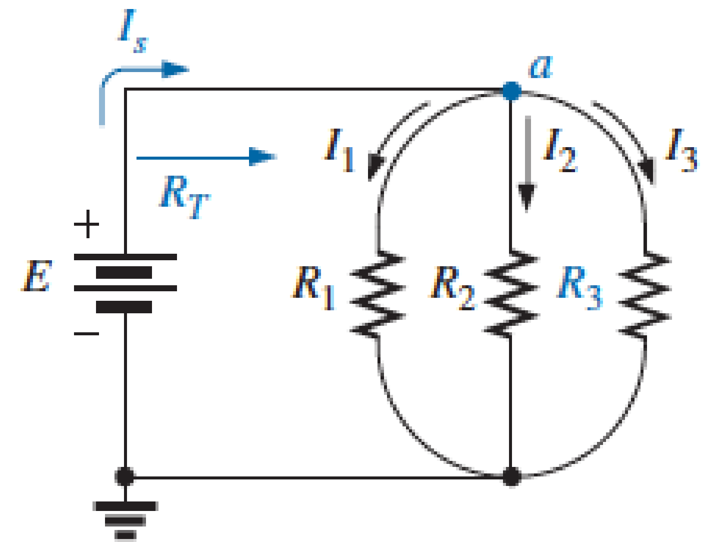

***Solution:***

a:

$$\Sigma I_i = \Sigma I_o$$

$$I_s = I_1 + I_2 + I_3$$

$$I_s = 8 \text{ mA} + 10 \text{ mA} + 2 \text{ mA} = \mathbf{20 \text{ mA}}$$




b:

$$E = V_1 = I_1 R_1 = (8 \text{ mA})(2 \text{ k}\Omega) = \mathbf{16 \text{ V}}$$

c:

$$R_3 = \frac{V_3}{I_3} = \frac{E}{I_3} = \frac{16 \text{ V}}{2 \text{ mA}} = \mathbf{8 \text{ k}\Omega}$$

d:

$$R_T = \frac{E}{I_s} = \frac{16 \text{ V}}{20 \text{ mA}} = \mathbf{0.8 \text{ k}\Omega}$$

# Current Divider Rule (CDR)

*We now introduce the equally powerful current divider rule (CDR) for finding the current through a resistor in a parallel circuit.*

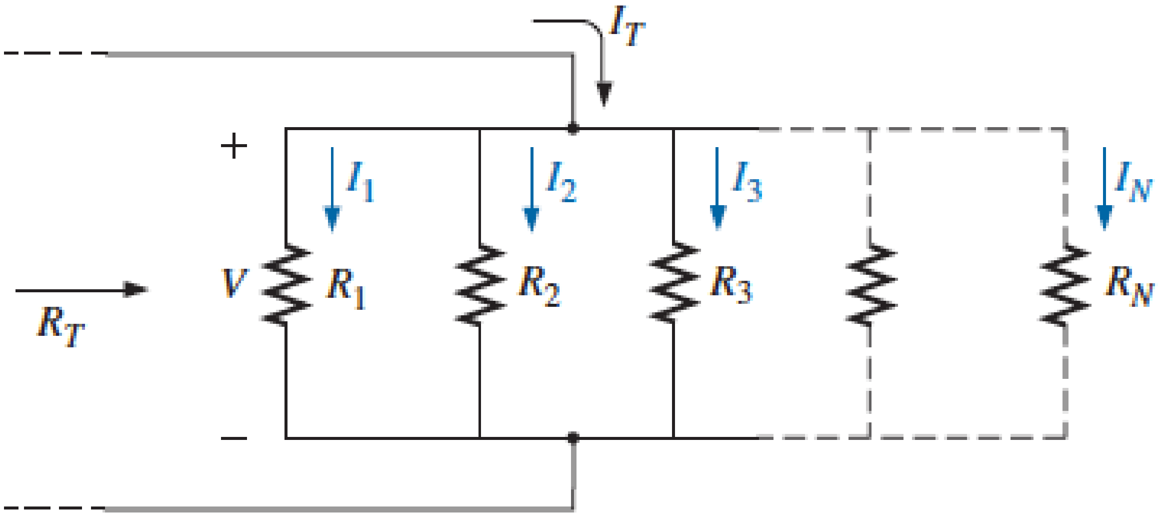
- *For two parallel elements of equal value, the current will divide equally.*
- *For parallel elements with different values, the smaller the resistance, the greater the share of input current.*
- *For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.*

*The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.*

$$I_x = \frac{R_T}{R_x} I_T$$

# Current Divider Rule (CDR)

*The current $I_T$ splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration*



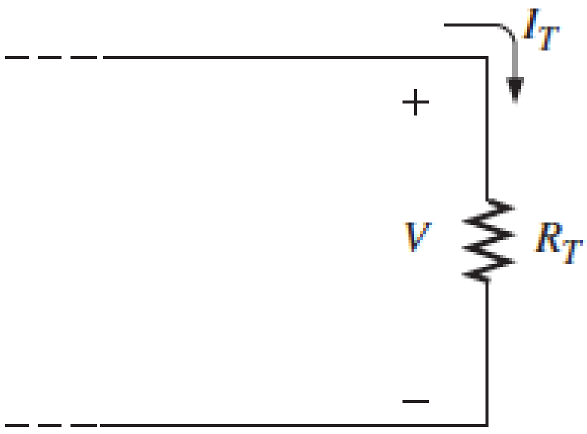

*The current $I_T$ can then be determined using Ohm's law:*

$$I_T = \frac{V}{R_T}$$

*Since the voltage V is the same across parallel elements, the following is true:*

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

*where the product $I_x R_x$ refers to any combination in the series.*
*Substituting for V in the above equation for $I_T$, we have*



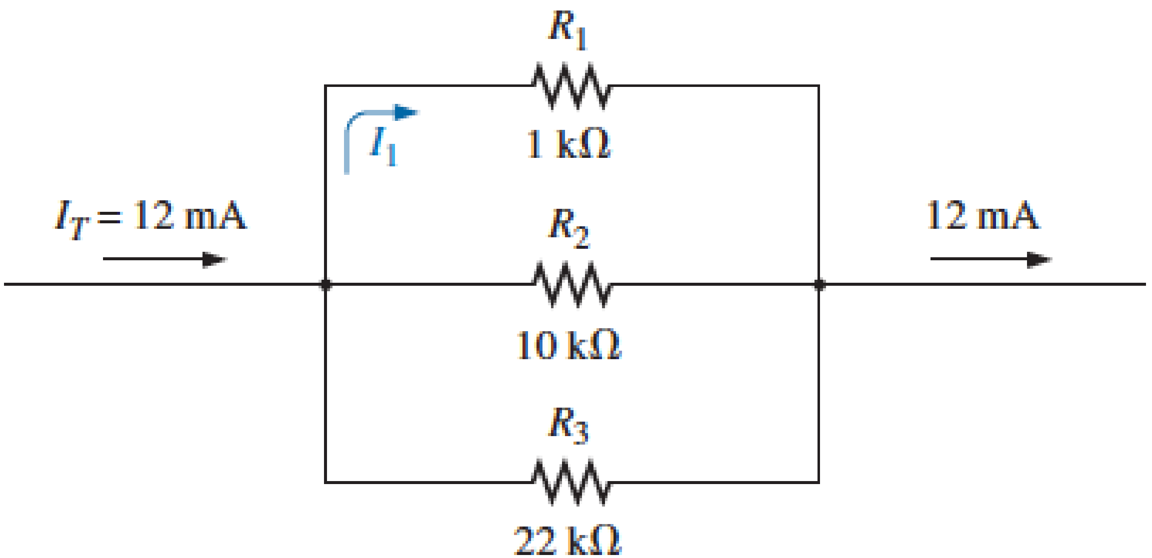

$$I_T = \frac{I_xR_x}{R_T}$$

**Current Divider Rule:**

$$I_x = \frac{R_T}{R_x}I_T$$

***Example* (13)**

**For the parallel network, determine current $I_1$, Using the current divider rule?**




***Solution:***

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$= \frac{1}{\dfrac{1}{1\ \text{k}\Omega} + \dfrac{1}{10\ \text{k}\Omega} + \dfrac{1}{22\ \text{k}\Omega}}$$

$$= \frac{1}{1 \times 10^{-3} + 100 \times 10^{-6} + 45.46 \times 10^{-6}}$$

$$= \frac{1}{1.145 \times 10^{-3}} = \mathbf{873.01\ \Omega}$$

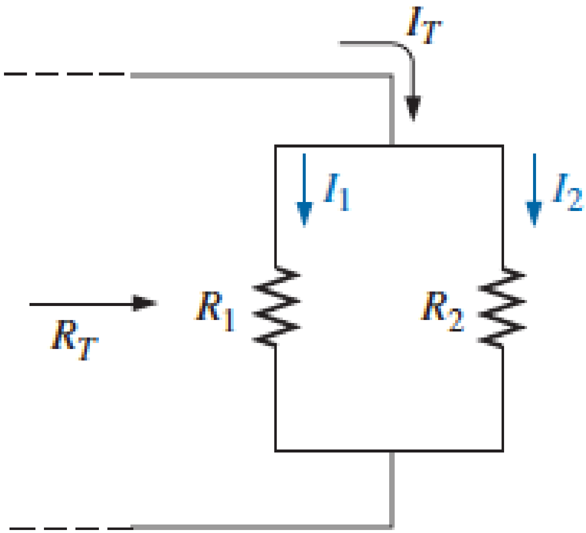
$$I_1 = \frac{R_T}{R_1} I_T$$

$$= \frac{(873.01\ \Omega)}{1\ \text{k}\Omega}(12\ \text{mA}) = (0.873)(12\ \text{mA}) = \mathbf{10.48\ mA}$$

# Special Case: Two Parallel Resistors

*For the case of two parallel resistors as shown below, the total resistance is determined by*

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$




*Substituting $R_T$ into Eq. for current $I_1$ results in*

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T$$

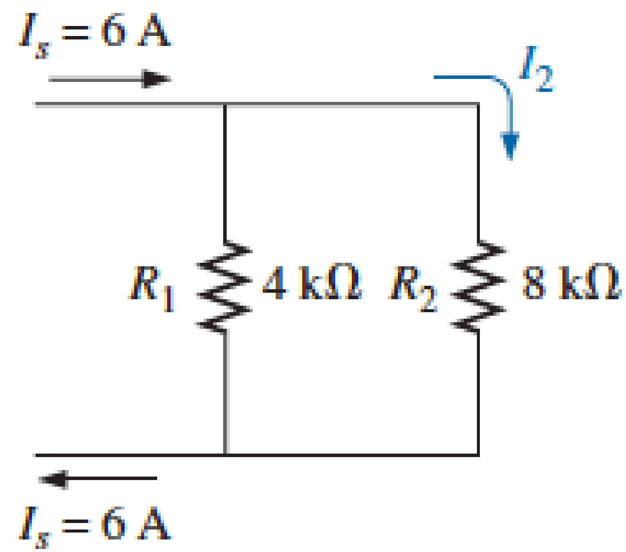
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

*for two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors.*

## *Example* (14)

**Determine current $I_2$ for the network using the current divider rule.**

***Solution:***




$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

$$= \left(\frac{4\ \text{k}\Omega}{4\ \text{k}\Omega + 8\ \text{k}\Omega}\right) 6\ \text{A} = (0.333)(6\ \text{A}) = \mathbf{2\ A}$$

***or***

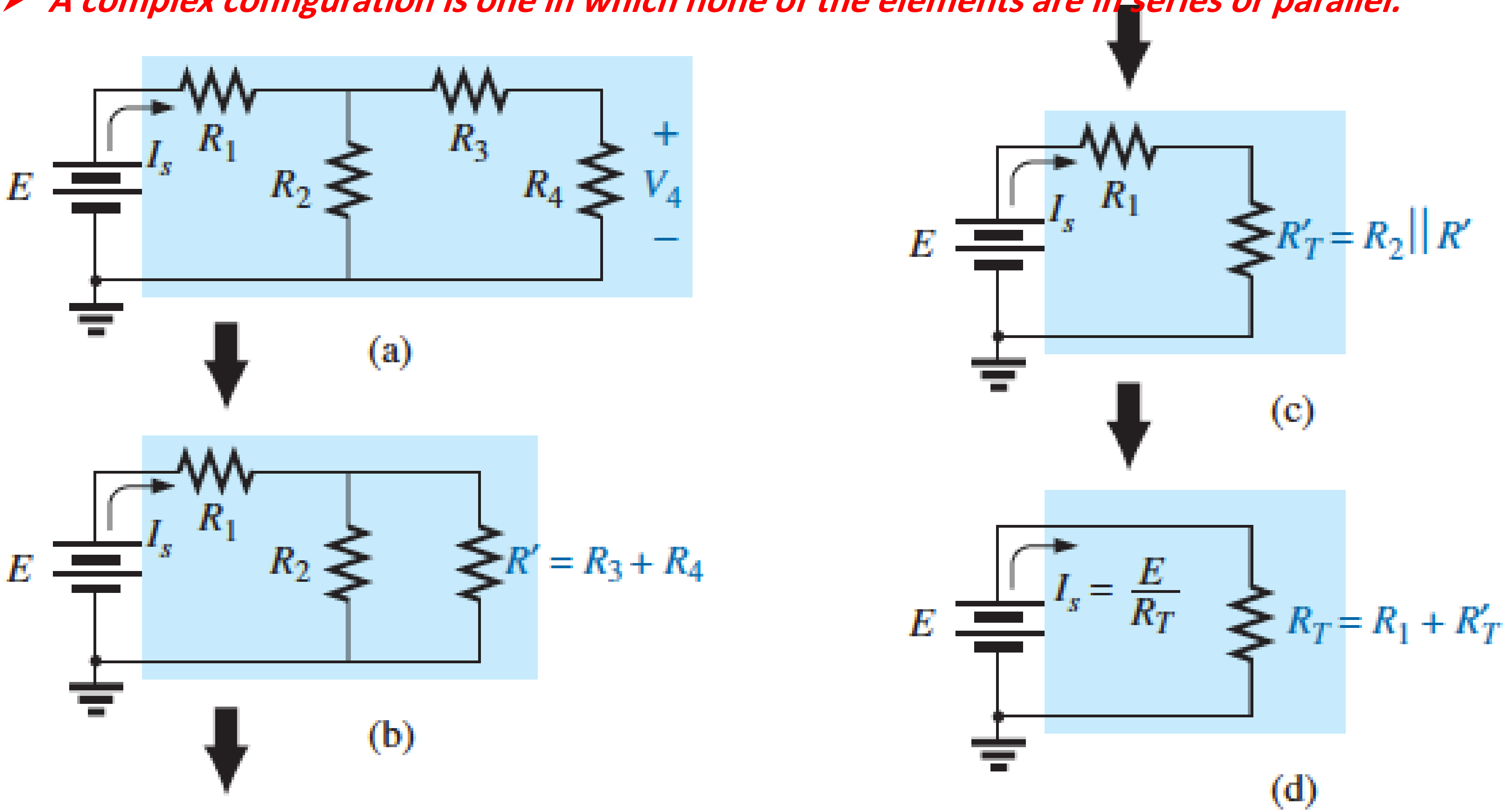
$$I_2 = \frac{R_T}{R_2} I_T$$

$$R_T = 4\ \text{k}\Omega \parallel 8\ \text{k}\Omega = \frac{(4\ \text{k}\Omega)(8\ \text{k}\Omega)}{4\ \text{k}\Omega + 8\ \text{k}\Omega} = 2.667\ \text{k}\Omega$$

$$I_2 = \left(\frac{2.667\ \text{k}\Omega}{8\ \text{k}\Omega}\right) 6\ \text{A} = (0.333)(6\ \text{A}) = \mathbf{2\ A}$$

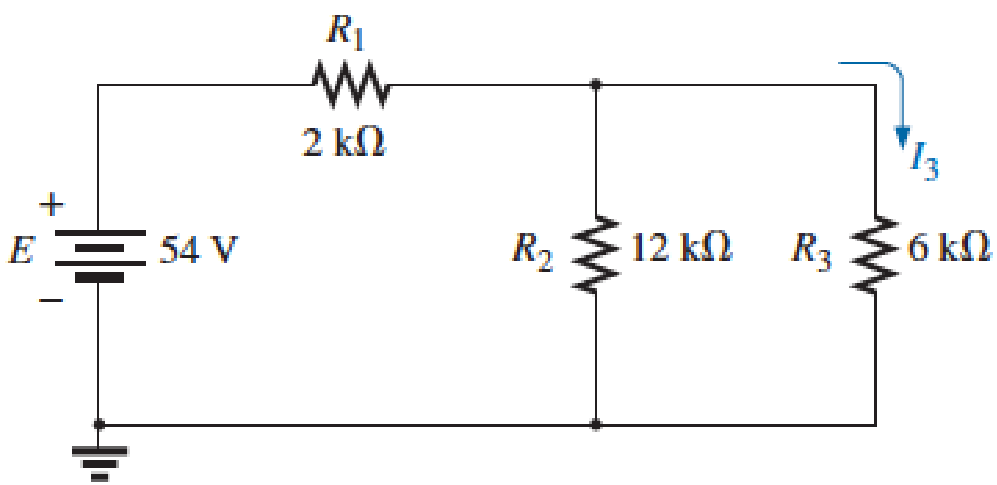
# Series-Parallel Network

- *A series-parallel configuration is one that is formed by a combination of series and parallel elements.*
- *A complex configuration is one in which none of the elements are in series or parallel.*

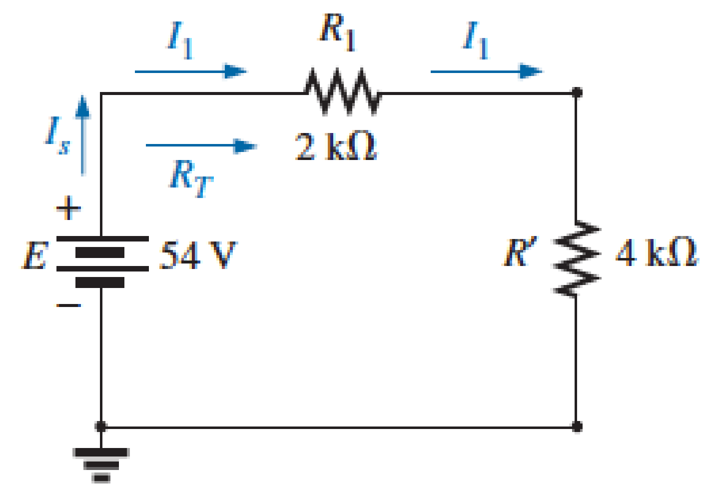


***Example* (15)**

**Find current $I_3$ for the series-parallel network?**




***Solution:***




$$R' = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(12\ \text{k}\Omega)(6\ \text{k}\Omega)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = 4\ \text{k}\Omega$$

$$R_T = R_1 + R' = 2\ \text{k}\Omega + 4\ \text{k}\Omega = 6\ \text{k}\Omega$$

$$I_s = \frac{E}{R_T} = \frac{54\ \text{V}}{6\ \text{k}\Omega} = 9\ \text{mA}$$

$$I_1 = I_s = 9\ \text{mA}$$

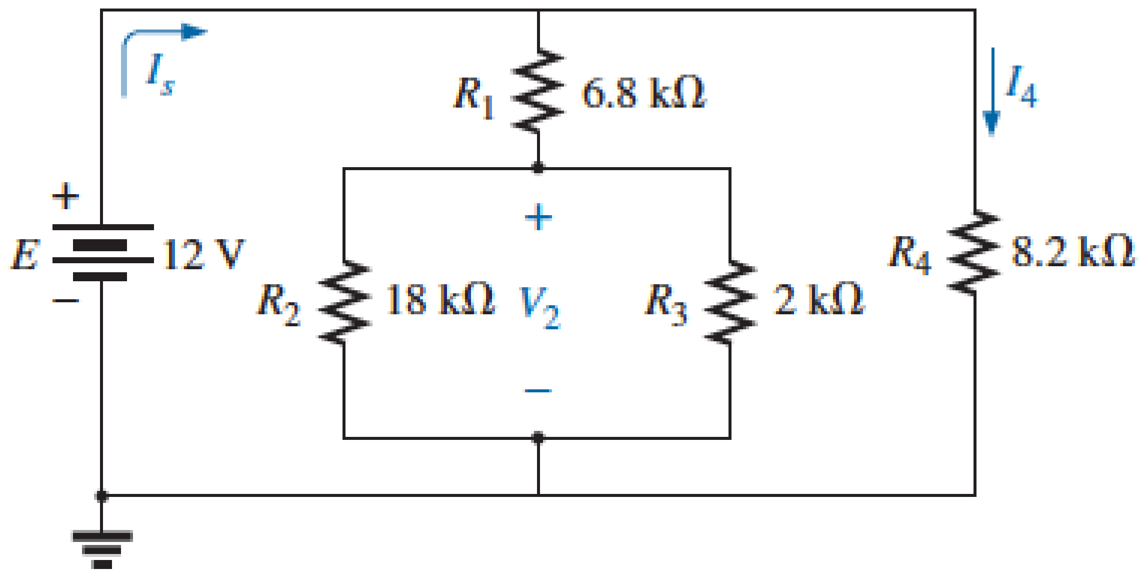
$$I_3 = \left(\frac{R_2}{R_2 + R_3}\right) I_1 = \left(\frac{12\ \text{k}\Omega}{12\ \text{k}\Omega + 6\ \text{k}\Omega}\right) 9\ \text{mA} = \mathbf{6\ mA}$$

## *Example* (16)

**For the network, determine currents $I_4$ and $I_s$ and voltage $V_2$.**

***Solution:***

$$I_4 = \frac{V_4}{R_4} = \frac{E}{R_4} = \frac{12\ \text{V}}{8.2\ \text{k}\Omega} = \mathbf{1.46\ m}$$

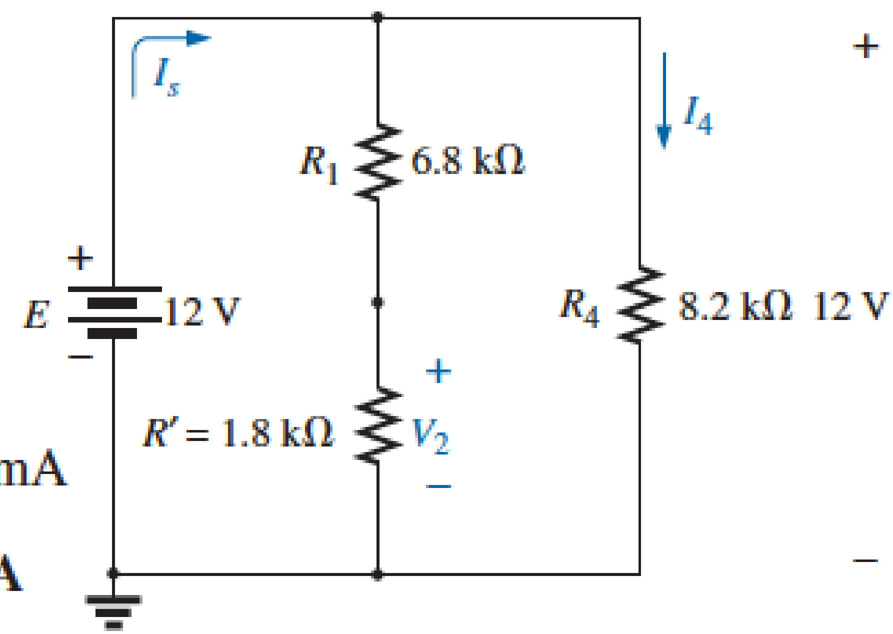



$$R' = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$= \frac{(18\ \text{k}\Omega)(2\ \text{k}\Omega)}{18\ \text{k}\Omega + 2\ \text{k}\Omega} = 1.8\ \text{k}\Omega$$

$$V_2 = \left(\frac{R'}{R' + R_1}\right)E =$$

$$= \left(\frac{1.8\ \text{k}\Omega}{1.8\ \text{k}\Omega + 6.8\ \text{k}\Omega}\right)12\ \text{V} = \mathbf{2.51\ V}$$




$$I_1 = \frac{E}{R_1 + R'} = \frac{12\ \text{V}}{6.8\ \text{k}\Omega + 1.8\ \text{k}\Omega} = 1.40\ \text{mA}$$

$$I_s = I_1 + I_4 = 1.40\ \text{mA} + 1.46\ \text{mA} = \mathbf{2.86\ mA}$$