Electrical Circuit-I

5th Lecture

Δ - Y & Y - Δ Conversions

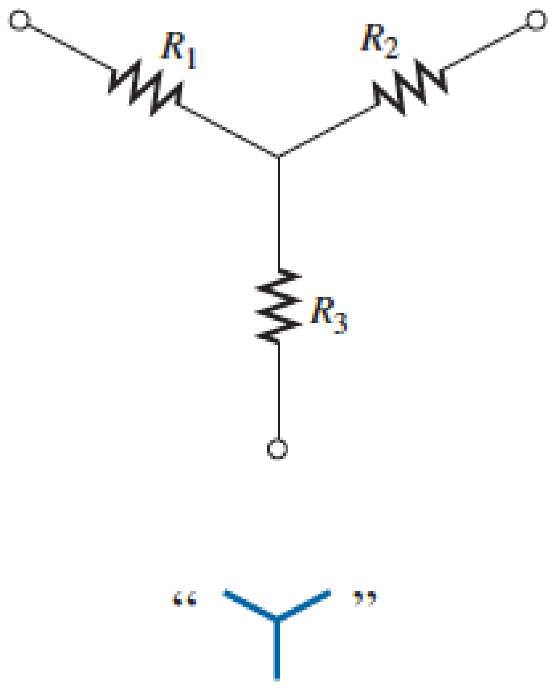
By:

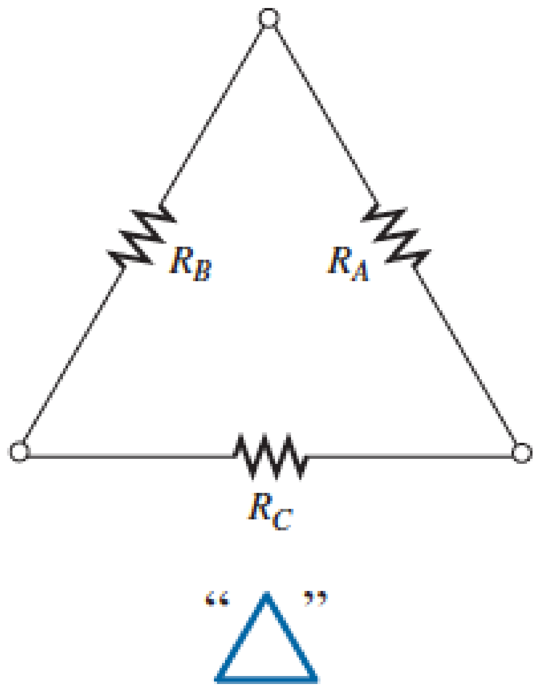
Dr. Ali Albu-Rghaif

Ref: Robert L. Boylestad, *INTRODUCTORY CIRCUIT ANALYSIS*, Pearson Prentice Hall, Eleventh Edition, 2007

Δ - Y & Y - Δ Conversions

Circuit configurations are often encountered in which the resistors do not appear to be in series or parallel. Under these conditions, it may be necessary to convert the circuit from one form to another to solve for any unknown quantities if mesh or nodal analysis is not applied. Two circuit configurations that often account for these difficulties are the wye (Y) and delta (Δ) configurations depicted below





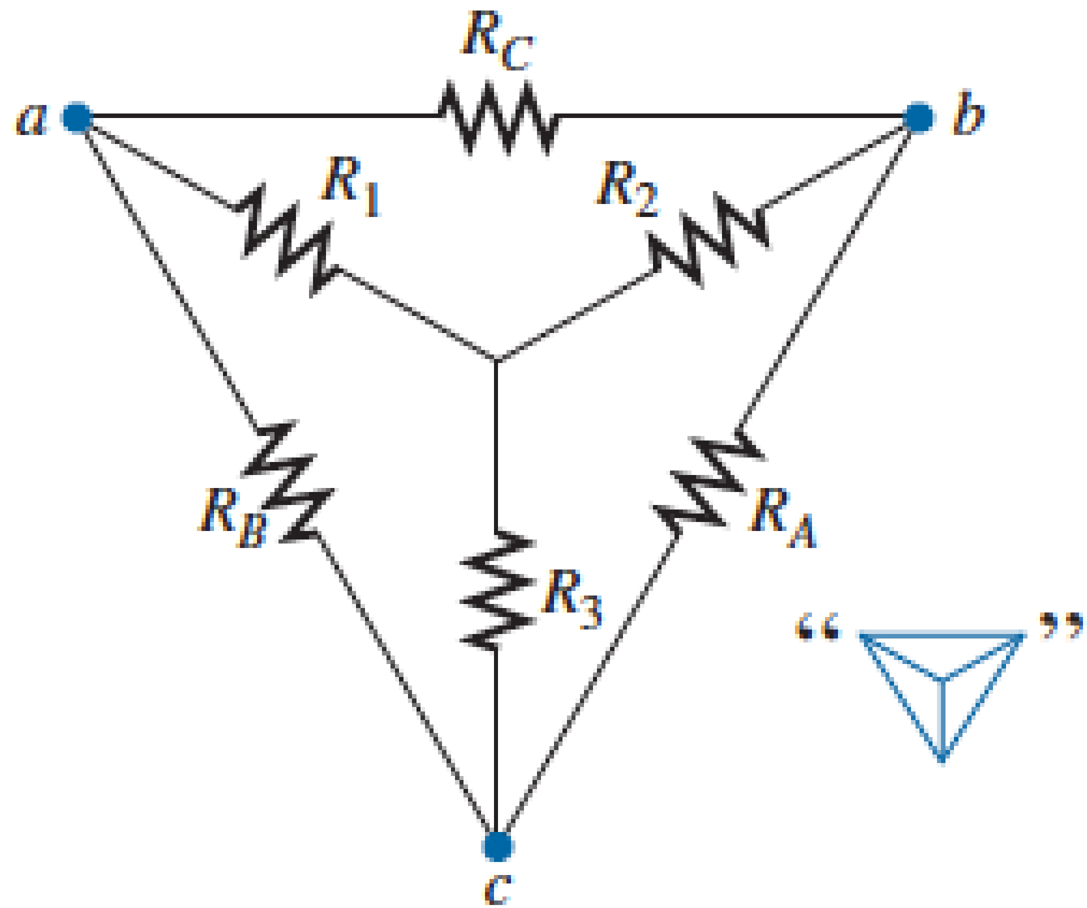
The purpose of this section is to develop the equations for converting from Δ to Y, or vice versa.

Delta to Star

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$



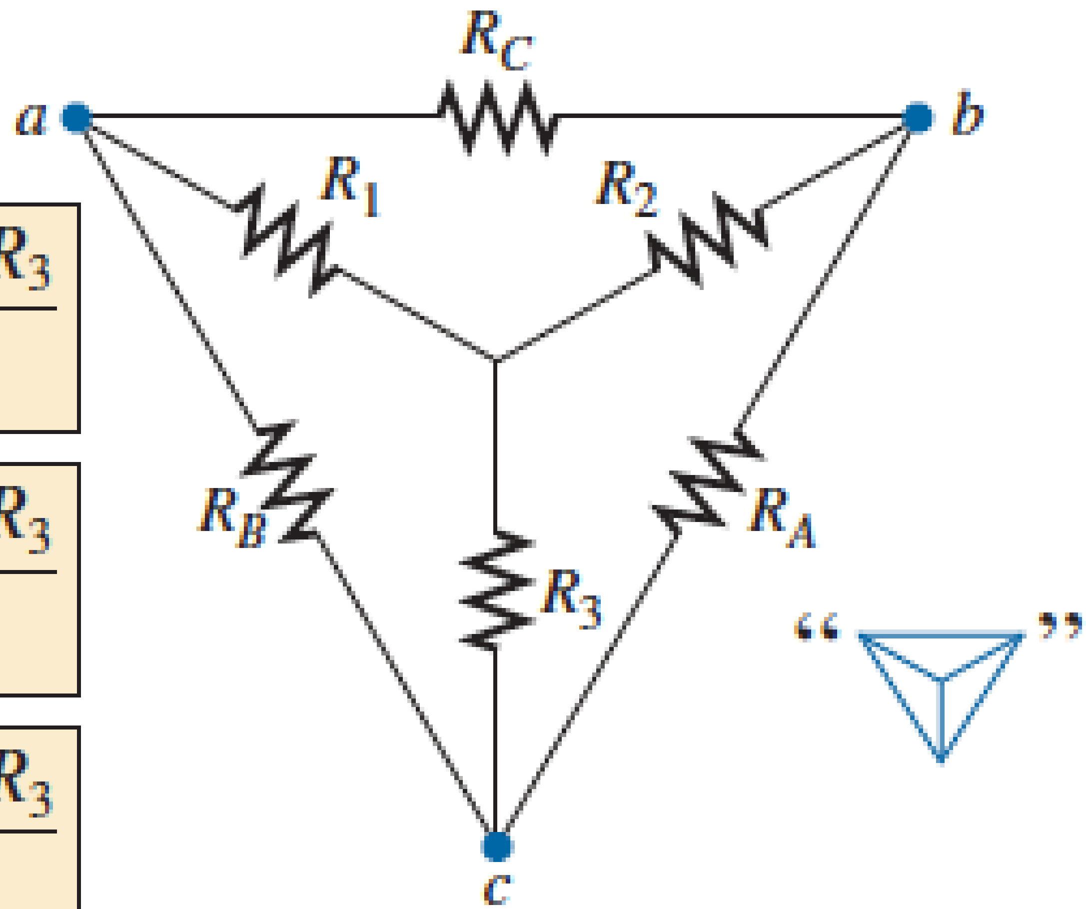
Note that each resistor of the **Y** *is equal to the product of the resistors in the two closest branches of the* **Δ** *divided by the sum of the resistors in the* **Δ**.

Star to Delta

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

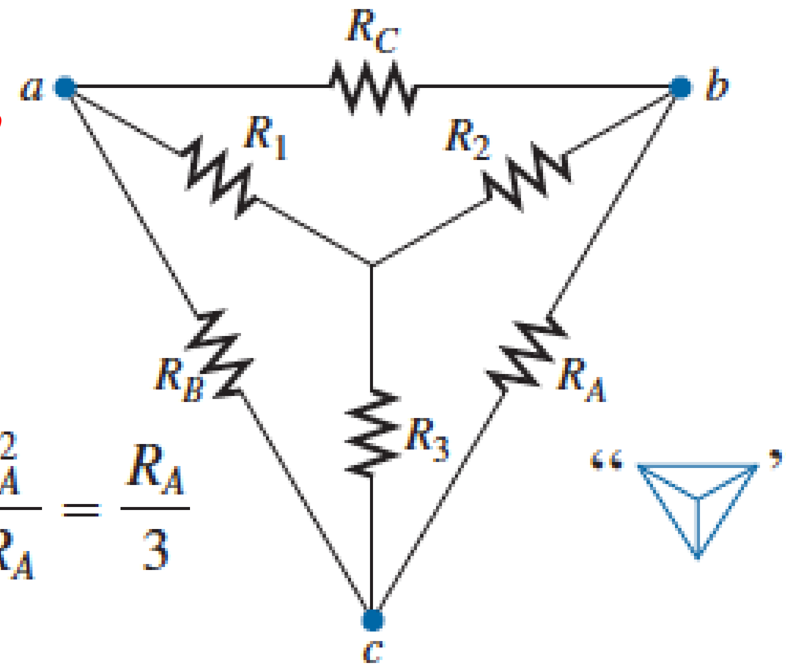
$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$



Note that the value of each resistor of the **Δ** *is equal to the sum of the possible product combinations of the resistances of the* **Y** *divided by the resistance of the* **Y** *farthest from the resistor to be determined.*

Star ⟺ Delta



Let us consider what would occur if all the values of a **Δ** *or* **Y** *were the same. If* $R_A = R_B = R_C$,

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

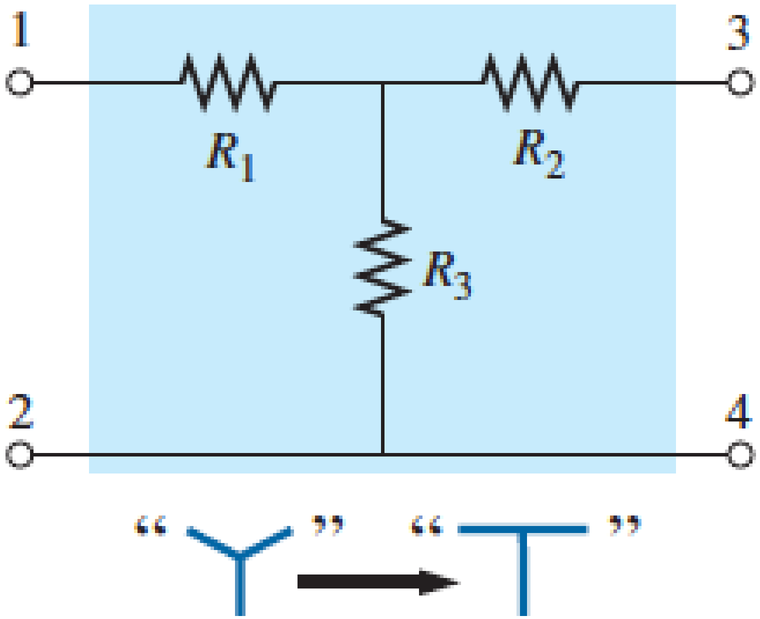
In general, therefore,

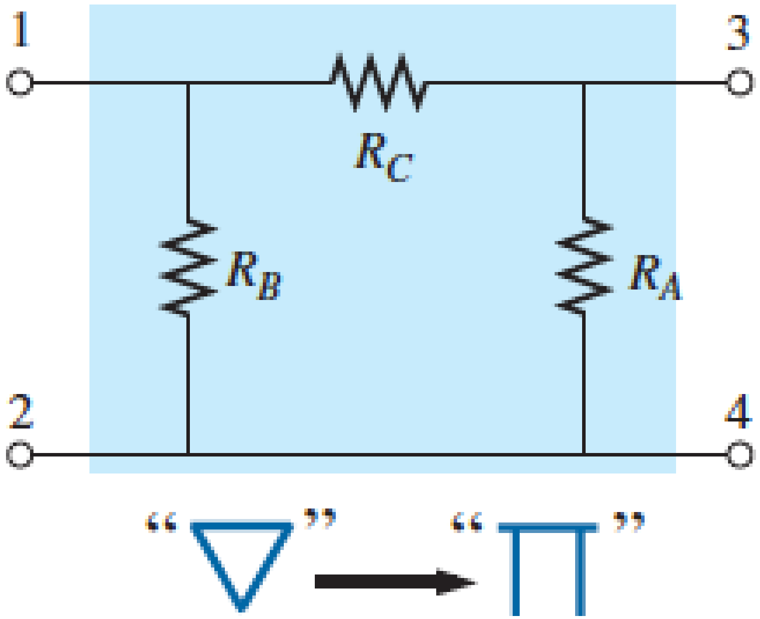
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

$$\boxed{R_\Delta = 3R_Y}$$

Star ⟺ Delta

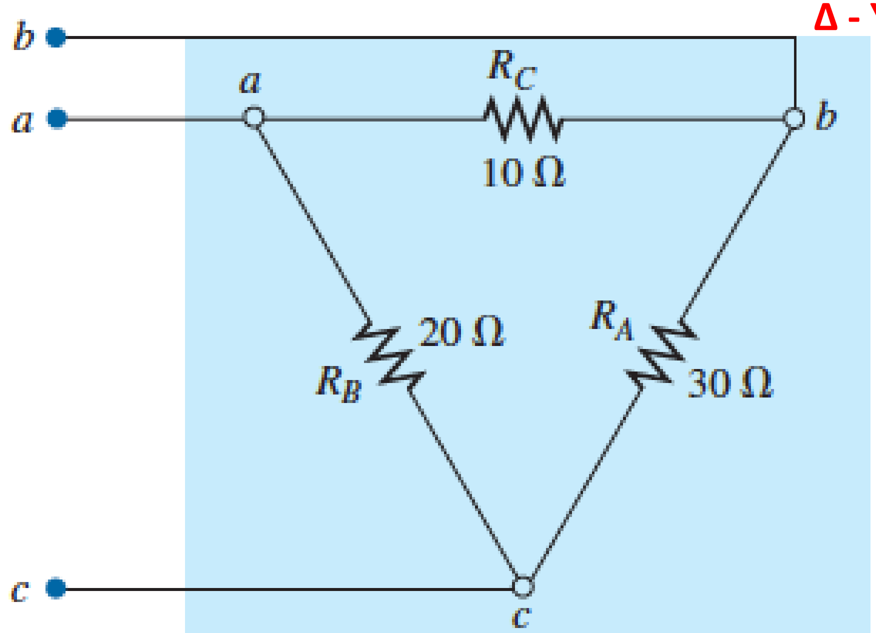
The **Y** *and the* **Δ** *often appear as shown below. They are then referred to as a tee (***T***) and a pi (***π***) network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the* **Y** *and* **Δ** *transformation.*





Example **(1)**

Convert the Δ in Fig. to a Y

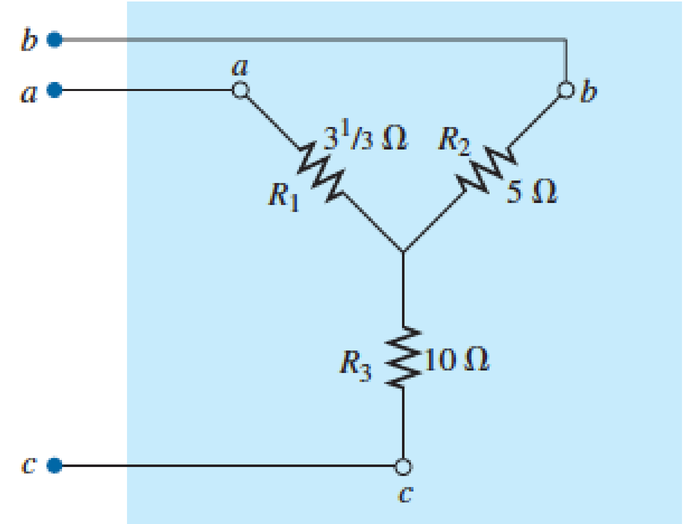


Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = 3\tfrac{1}{3}\ \Omega$$

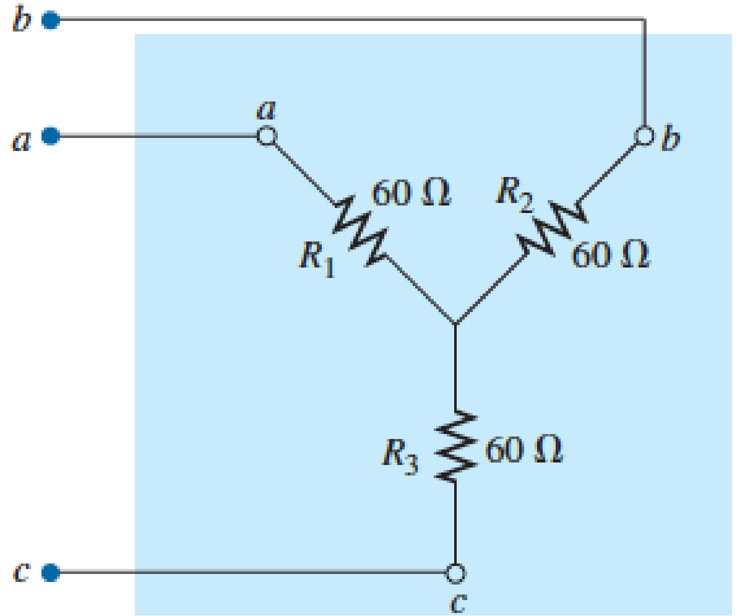
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = 5\ \Omega$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = 10\ \Omega$$



Example **(2)**

Convert the Y in Fig. to a Δ



Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

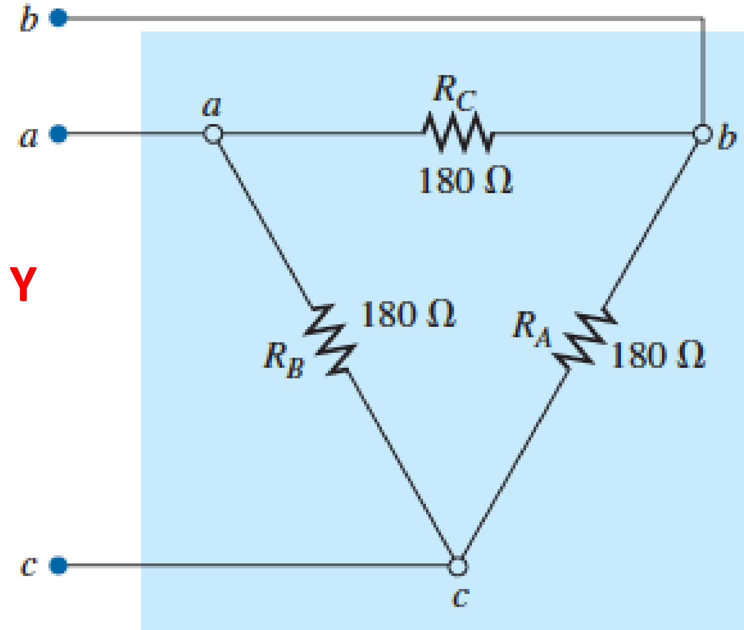
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

However, the three resistors for the Y are equal

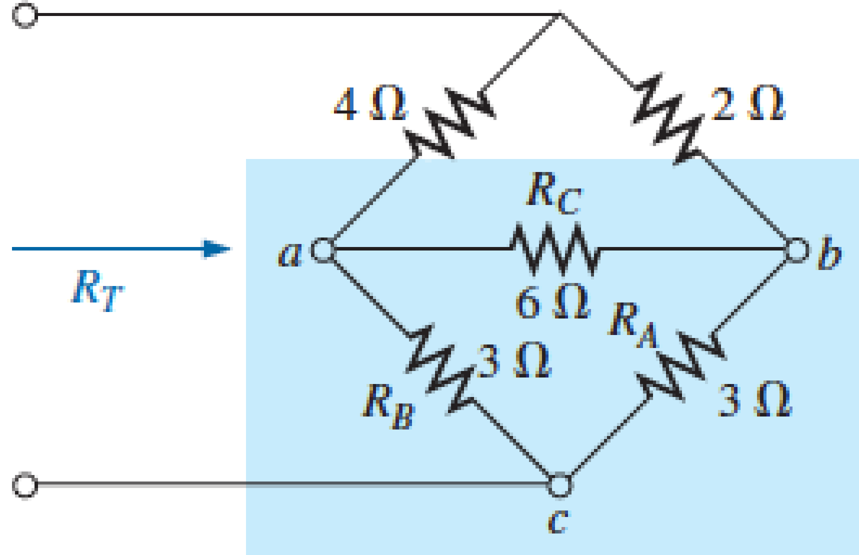
$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

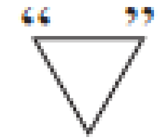
$$R_B = R_C = \mathbf{180\ \Omega}$$



Example (3)

Find the total resistance of the network, where:
$R_A = 3\Omega, R_B = 3\Omega, and R_C = 6\Omega$ **?**





Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

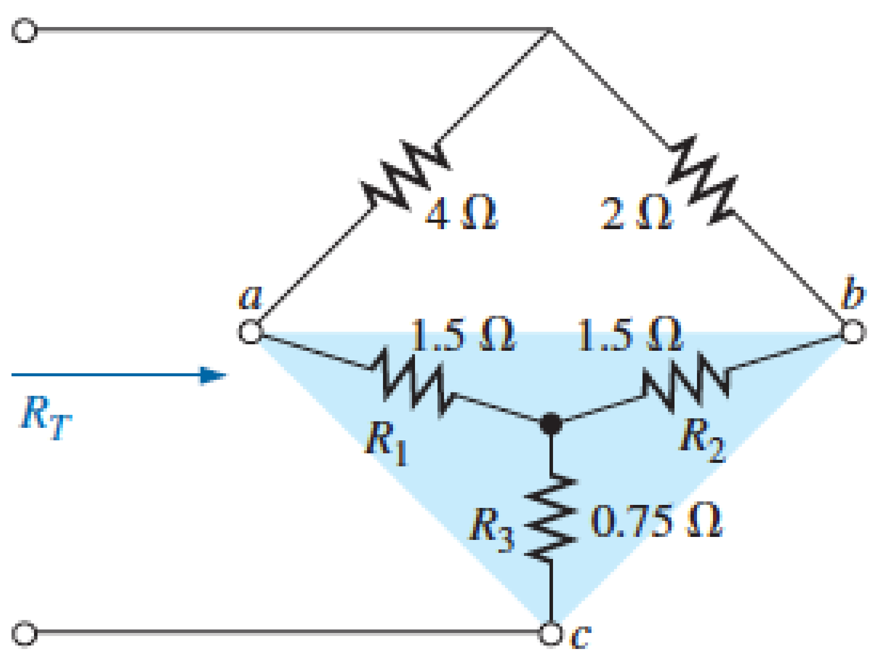
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Example (3)

Replacing the Δ by the Y, as shown



$$R_T = 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)}$$

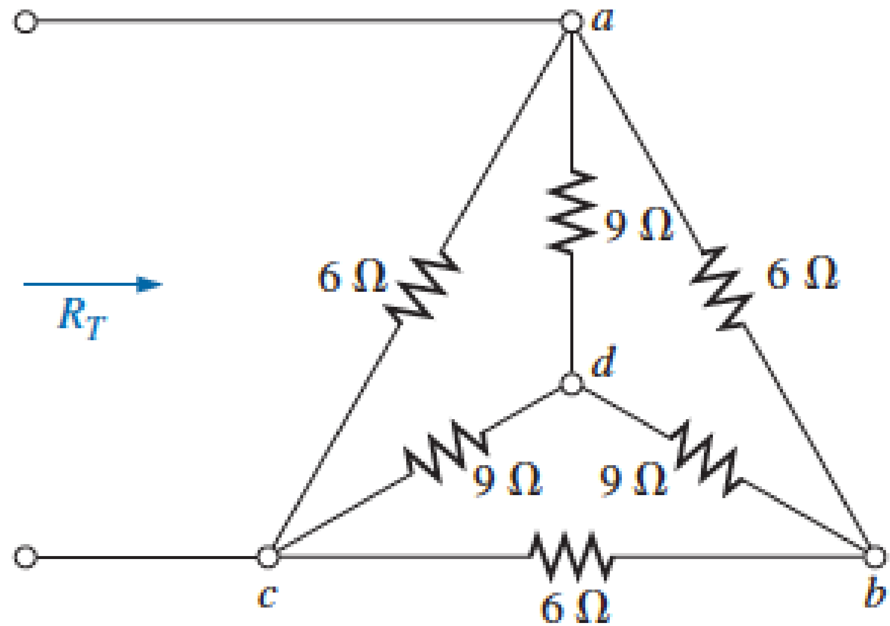
$$= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega}$$

$$= 0.75\ \Omega + 2.139\ \Omega$$

$$R_T = \mathbf{2.89\ \Omega}$$

***Example* (4)**

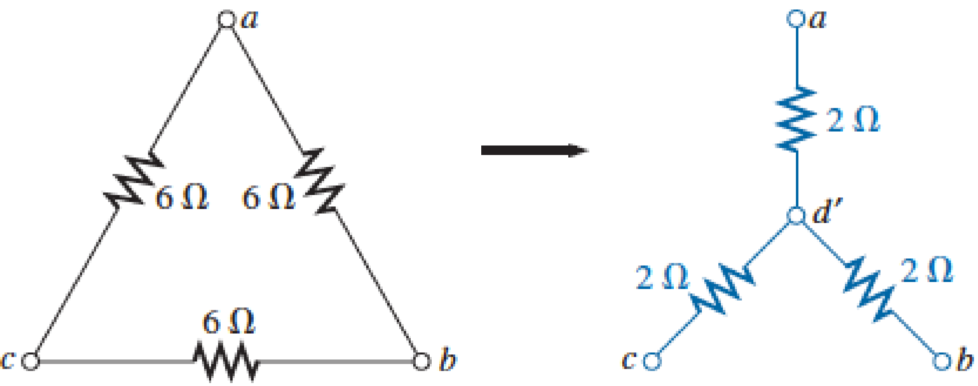
Find the total resistance of the network,



Solution:

Since all the resistors of the Δ or Y are the same

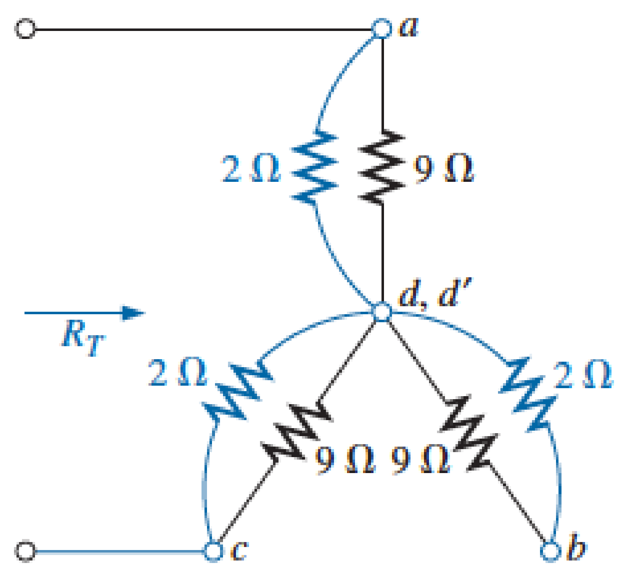
a. Converting the Δ to a Y:



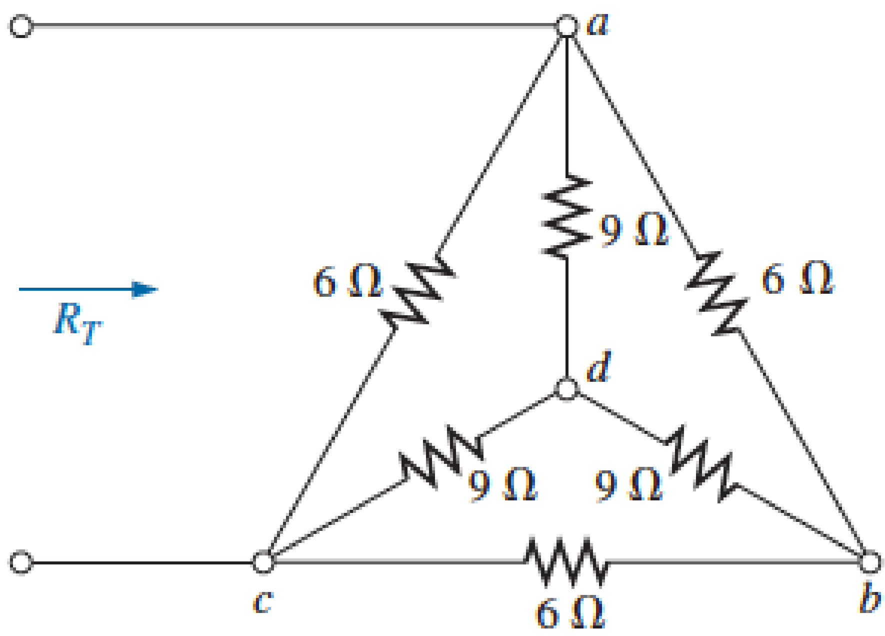
$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = 3.27\ \Omega$$

The network then appears as shown



Example (4)



b. Converting the Y to a Δ :

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.91\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.91\ \Omega)}{3} = \mathbf{3.27\ \Omega}$$